Quantum Mechanics I - Module 6

1. Define the following operators:

$$L^2 = L_x^2 + L_y^2 + L_z^2 \qquad L_\pm = L_x \pm iL_y$$

Work out the following commutators. Try to use as many shortcuts as possible!

(a) $[L_z, x]$
(b) $[L_z, y]$
(c) $[L_z, z]$
(d) $[L_z, p_x]$
(e) $[L_z, p_y]$
(f) $[L_z, p_z]$
(g) $[L_x, L_y]$
(h) $[L^2, L_x]$
(i) $[L_z, L_\pm]$
(j) $[L^2, L_\pm]$
(k) $[L_z, r^2]$
(l) $[L_z, p^2]$

2. Use the previous question to show that the Hamiltonian

$$H = \frac{p^2}{2m} + V(r)$$

commutes with all three components of L provided that V depends only on r.

3. Considering the hydrogen atom, what do you think the following equation means?

$$\boxed{H|211\rangle = E_2|211\rangle}$$

Write this equation as you are used to seeing it.