

**Classical Mechanics**  
**Phy 235, Review, Exam 1.**

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**Exam # 1**

- Exam # 1 will take place on Thursday September 23 at 8.00 am in B&L 109.
- The exam will cover the material in Chapters 1 – 4.
- The exam will have 4 questions:
  - Three questions will be analytical questions.
  - One question will be conceptual questions (including concepts related to the Yankees or the Netherlands or KLM).
- You will be provided with an equation sheet.

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**Good to know for Exam # 1.**  
**Soo much below sea level.**

The Dutch measure water level in units of **NAP**:  
**Normaal Amsterdams Peil.**  
 Used in most of Western Europe to measure water levels.  
 You can no longer trust sea levels but you can trust the level of Amsterdam.

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## Official reference height. Normaal Amsterdams Peil (NAP)



City hall in  
Amsterdam

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## Preparing for Exam # 1

- Take the practice exam as if it was a real exam: take 80 minutes to complete it. Compare your work to the posted solutions to help you focus on specific areas.
- Office hours on Monday 9/22:
  - Laurel: 3 – 5 pm
  - Anagha: 5 – 6 pm
  - Waly: 6 – 7 pm.
- There will be no recitations on Tuesday 9/23 and Thursday 9/25.
- There will be no office hours on Wednesday 9/24 and Thursday 9/25.

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## Time management

- Work no more than 10 – 15 minutes on each problem.
- Even if not finished, move on to the next problem.
- This will leave 20 minutes at the end to finish your problems and/or make correction.
- We can only give credit for what you write (not what you think).
- We can only give credit for what we can read (write neatly).

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## Warning.

- I cannot cover everything I discussed in lectures 1 – 6 in this review.
- If I skip over certain topics, it does not mean you should not understand that material.
- Your TAs will not see the exam until you see it.

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## Overview

- Chapter 1: Math. No specific question focused just on this Chapter. Concepts presented in Chapter 1 will of course be used on the exam.
- Chapter 2: Newtonian Mechanics and Reference frames.
- Chapter 3: Harmonic motion (linear oscillations).
- Chapter 4: Non-linear oscillations.

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## Chapter 2.

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## Newton's Laws

- **First law:**
  - A body remains at rest or in uniform motion unless acted upon by a force.
  - Note: uniform motion requires constant speed and constant direction.
- **Second law:**
  - A body acted upon by a force moves in such a manner that the time rate of change of its linear momentum equals the force.
- **Third law:**
  - If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.

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## Reference Systems

- **Inertial reference frame:**
  - A reference frame in which Newton's laws are valid.
- **Specific requirements:**
  - The equation of motion of a single particle should be independent of the origin of the coordinate system.
  - The equation of motion of a single particle should be independent of the orientation of the coordinate system.
  - Time must be homogeneous.
- **Accelerating reference frames are not good inertial reference frames (e.g. accelerating airplane).**
  - The earth is a non-inertial reference frame since it rotates around its axis, since it rotates around the sun, and since the sun rotates around the center of the Milky-Way.

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## Conservation Laws.

- The following conservation laws are a direct consequence of Newton's laws:
  - Conservation of linear momentum: the the total force is 0 N.
  - Conservation of angular momentum: then the total torque is 0 Nm.
  - Conservation of energy: in a conservative force field that is constant in time. The requirements can be written as:

$$\vec{F} = -\vec{\nabla}U$$

$$\frac{\partial U}{\partial t} = 0$$

- The relation  $\vec{F} = -\vec{\nabla}U$  allows us to quickly check is a force is conservative since

$$\vec{\nabla} \times \vec{F} = -\vec{\nabla} \times \vec{\nabla}U = 0$$

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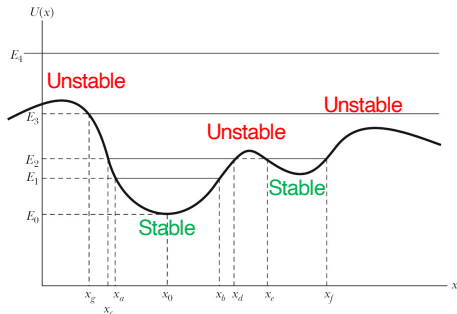
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### Predicting motion based on $U$ .



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### Problem 2.37

A particle of mass  $m$  has a speed  $v = \alpha/x$ , where  $x$  is its displacement. Find the force  $F(x)$  responsible for this motion.

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### Chapter 3.

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## Harmonic motion.

- **Harmonic motion:**
  - Motion around a position of stable equilibrium.
- **Simple harmonic motion:**
  - At small distances around the equilibrium position, the force is approximately equal to  $-kx$ .
  - The total energy of the system is constant. The kinetic and potential energy will be time dependent.
- **Damped and driven harmonic motion:**
  - Damped harmonic motion occurs when friction or drag forces are acting on the system. Energy is dissipated and the system will gradually come to rest.
  - Driven harmonic motion adds a driving force in order to compensate for damping losses.

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## Solving Second-Order Differential Equations.

- **General form:**
$$\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = f(x)$$
- If you find two linearly independent solutions, every other solution will be a linear combination of these two solutions.
- The general solution has two constants, defined by the initial conditions.
- **Homogeneous equation:**
  - $f(x)$  is equal to 0.
  - Simple harmonic motion when  $a = 0$ .
- **Inhomogeneous equation:**
  - $f(x)$  is not equal to 0.

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## Homogeneous Equation

- Consider a damping force  $-bv$  and a restoring force  $-kx$ . The equation of motion for such system is:  $ma = -bv - kx$ .
- This provides us with the homogeneous equation:
$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega_0^2 x = 0$$
- Try the following solution:  $x = e^{rt}$ .
- This is a valid solution if  $r^2 + 2\beta r + \omega_0^2 = 0$ . This requires:
$$r = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$
- Three different scenarios:
  - $\beta^2 > \omega_0^2$ : over damping. Two values of  $r$ .
  - $\beta^2 = \omega_0^2$ : critical damping. One value of  $r$ . Second solution is  $te^{-\beta t}$ .
  - $\beta^2 < \omega_0^2$ : under damping. Two values of  $r$ ;  $r$  is a complex number

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### Inhomogeneous Equation

- Consider a damping force  $-bv$ , a restoring force  $-kx$ , and a driving force  $f(t)$ . The equation of motion for such system is:  $ma = -bv - kx + f(t)$ .
- The equation of motion can be rewritten as:
 
$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m}x = \frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega_0^2 x = f(t)$$
- Suppose:
  - $v$  is a solution of the inhomogeneous equation (this is called the **particular solution**).
  - $u$  is the general solution of the homogeneous equation (this is called the **complementary solution**).
 then  $u + v$  is the general solution of the inhomogeneous equation.

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### Visualizing Harmonic Motion. y vs x for different restoring forces.

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### Visualizing Harmonic Motion. Phase Diagrams.

Different paths correspond to different energies.

Different paths cannot cross. Paths are followed in a clockwise direction.

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### Problem 3.12.

A simple pendulum of mass  $m$  is suspended from a fixed point by a weightless rod of length  $l$ . Obtain the equation of motion for small angles.

Discuss the motion when it takes place in a viscous medium with a retarding force  $2m\sqrt{gl}\theta$ .

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### Chapter 4.

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### Non-linear oscillations.

- **Linear differential equations:**

- Terms are proportional to acceleration, velocity, and position:

$$\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = f(x)$$

- **Non-linear differential equations:**

- Include terms that non-linear in term of acceleration, velocity, and position.

- **Non-linear terms are divided in two groups:**

- Symmetric around the equilibrium position. This requires terms proportional to  $\epsilon^3$ . If  $\epsilon > 0$ : soft system. If  $\epsilon < 0$ : hard system.
- Asymmetric around the equilibrium position. This requires terms proportional to  $r^2$ .

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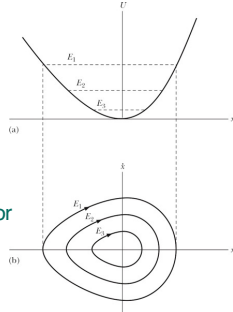
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## Phase Diagrams. Asymmetric for asymmetric potentials.



Closed contours for motion around stable equilibrium positions.

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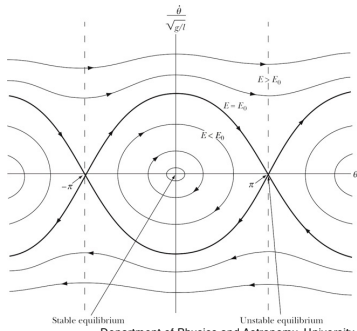
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## Phase diagram for the plane pendulum.



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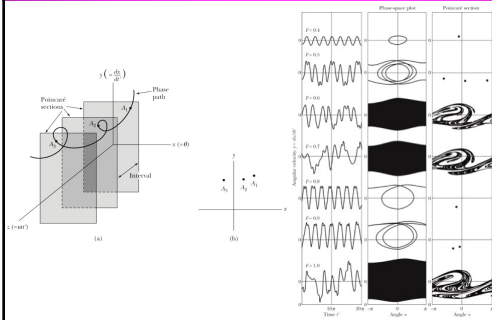
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## Visualizing chaos. Poincare plots.



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### Logistic equations. Creating chaos with maps.

$\alpha = 2.9$

(a)

$\alpha = 5.1$

(b)

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### Visualizing chaos. Bifurcation diagrams.

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### The Lyapunov exponent $\lambda$ .

- The development of chaos can be studied by examining the Lyapunov exponent  $\lambda$ :

$$\lambda = \lim_{n \rightarrow \infty} \left( \frac{1}{n} \sum_{i=1}^{n-1} \left| \frac{df(\alpha, x_i)}{dx} \right| \right)$$

- This exponent is a measure of the difference between solutions when we make a small change in the initial conditions:
  - If  $\lambda < 0$ : stable solutions.
  - If  $\lambda = 0$ : doubling of the number of solutions
  - If  $\lambda > 0$ : chaos.

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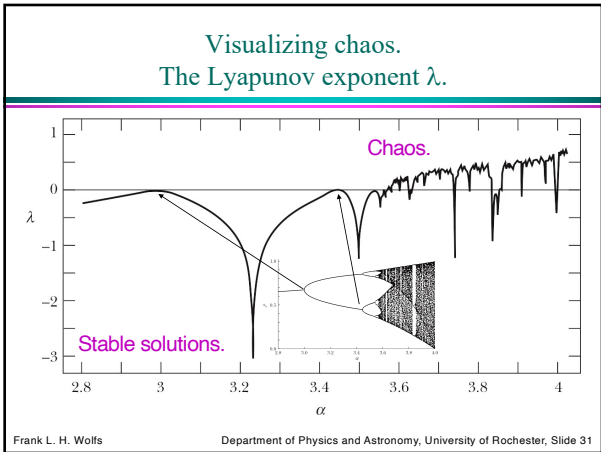
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### No need to always solve differential equations. Problem 4.9.

Investigate the motion of an undamped particle, subject to a force of the form:

$$F(x) = \begin{cases} -kx & |x| < a \\ -(k + \delta)x + \delta a & |x| > a \end{cases}$$

$k$  and  $\delta$  are positive constants.

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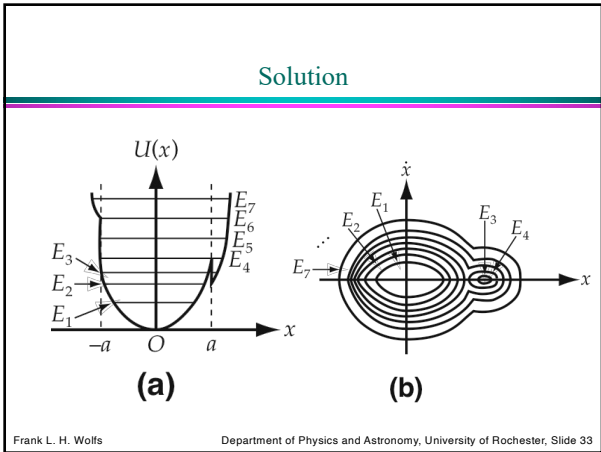
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**ENOUGH FOR TODAY?**

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