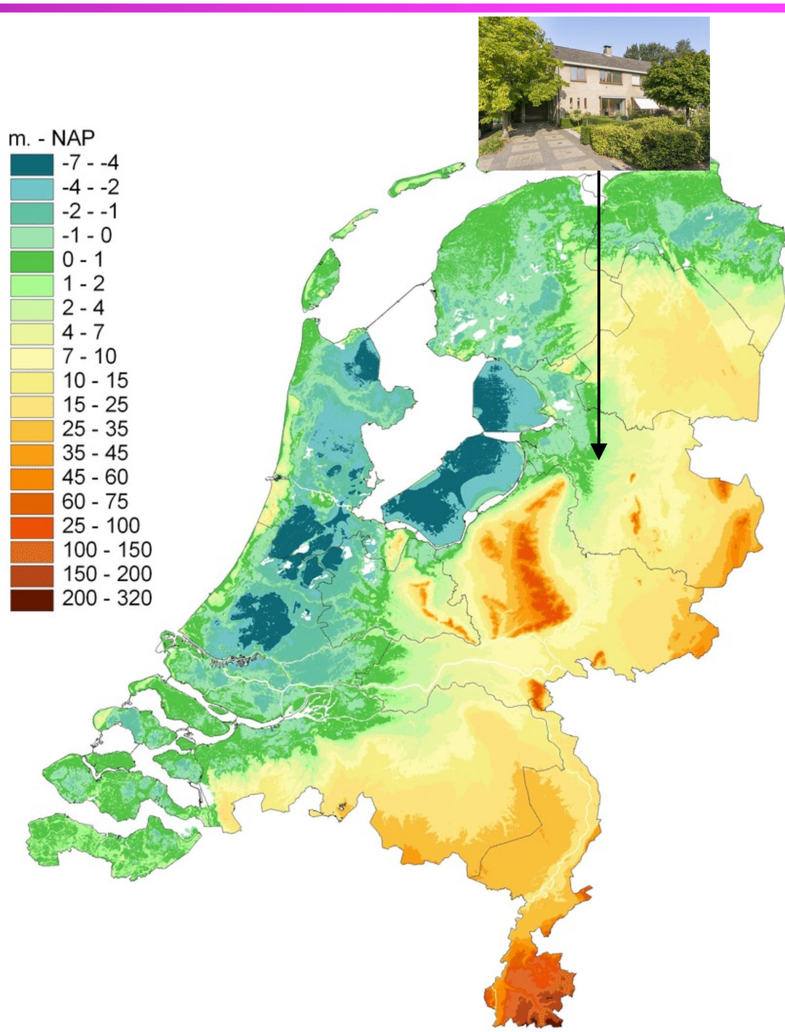
Classical Mechanics
Phy 235, Review, Exam 1.

Frank L. H. Wolfs
Department of Physics and Astronomy
University of Rochester

Exam # 1

- Exam # 1 will take place on Thursday September 23 at 8.00 am in B&L 109.
- The exam will cover the material in Chapters 1 – 4.
- The exam will have 4 questions:
  - Three questions will be analytical questions.
  - One question will be conceptual questions (including concepts related to the Yankees or the Netherlands or KLM).
- You will be provided with an equation sheet.

Good to know for Exam # 1. Soo much below sea level.



The Dutch measure water level in units of **NAP**:

Normaal Amsterdams Peil.

Used in most of Western Europe to measure water levels.

You can no longer trust sea levels but you can trust the level of Amsterdam.

Official reference height.
Normaal Amsterdams Peil (NAP)

City hall in Amsterdam

Preparing for Exam # 1

- Take the practice exam as if it was a real exam: take 80 minutes to complete it. Compare your work to the posted solutions to help you focus on specific areas.
- Office hours on Monday 9/22:
  - Laurel: 3 – 5 pm
  - Anagha: 5 – 6 pm
  - Waly: 6 – 7 pm.
- There will be no recitations on Tuesday 9/23 and Thursday 9/25.
- There will be no office hours on Wednesday 9/24 and Thursday 9/25.

Time management

- Work no more than 10 – 15 minutes on each problem.
- Even if not finished, move on to the next problem.
- This will leave 20 minutes at the end to finish your problems and/or make correction.
- We can only give credit for what you write (not what you think).
- We can only give credit for what we can read (write neatly).

Warning.

- I cannot cover everything I discussed in lectures 1 – 6 in this review.
- If I skip over certain topics, it does not mean you should not understand that material.
- Your TAs will not see the exam until you see it.

Overview

- Chapter 1: Math. No specific question focused just on this Chapter. Concepts presented in Chapter 1 will of course be used on the exam.
- Chapter 2: Newtonian Mechanics and Reference frames.
- Chapter 3: Harmonic motion (linear oscillations).
- Chapter 4: Non-linear oscillations.

Chapter 2.

Newton's Laws

- First law:
  - A body remains at rest or in uniform motion unless acted upon by a force.
  - Note: uniform motion requires constant speed and constant direction.
- Second law:
  - A body acted upon by a force moves in such a manner that the time rate of change of its linear momentum equals the force.
- Third law:
  - If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.

Reference Systems

- Inertial reference frame:
  - A reference frame in which Newton's laws are valid.
- Specific requirements:
  - The equation of motion of a single particle should be independent of the origin of the coordinate system.
  - The equation of motion of a single particle should be independent of the orientation of the coordinate system.
  - Time must be homogeneous.
- Accelerating reference frames are not good inertial reference frames (e.g. accelerating airplane).
  - The earth is a non-inertial reference frame since it rotates around its axis, since it rotates around the sun, and since the sun rotates around the center of the Milky-Way.

Conservation Laws.

- The following conservation laws are a direct consequence of Newton's laws:
  - Conservation of linear momentum: the the total force is 0 N.
  - Conservation of angular momentum: then the total torque is 0 Nm.
  - Conservation of energy: in a conservative force field that is constant in time. The requirements can be written as:

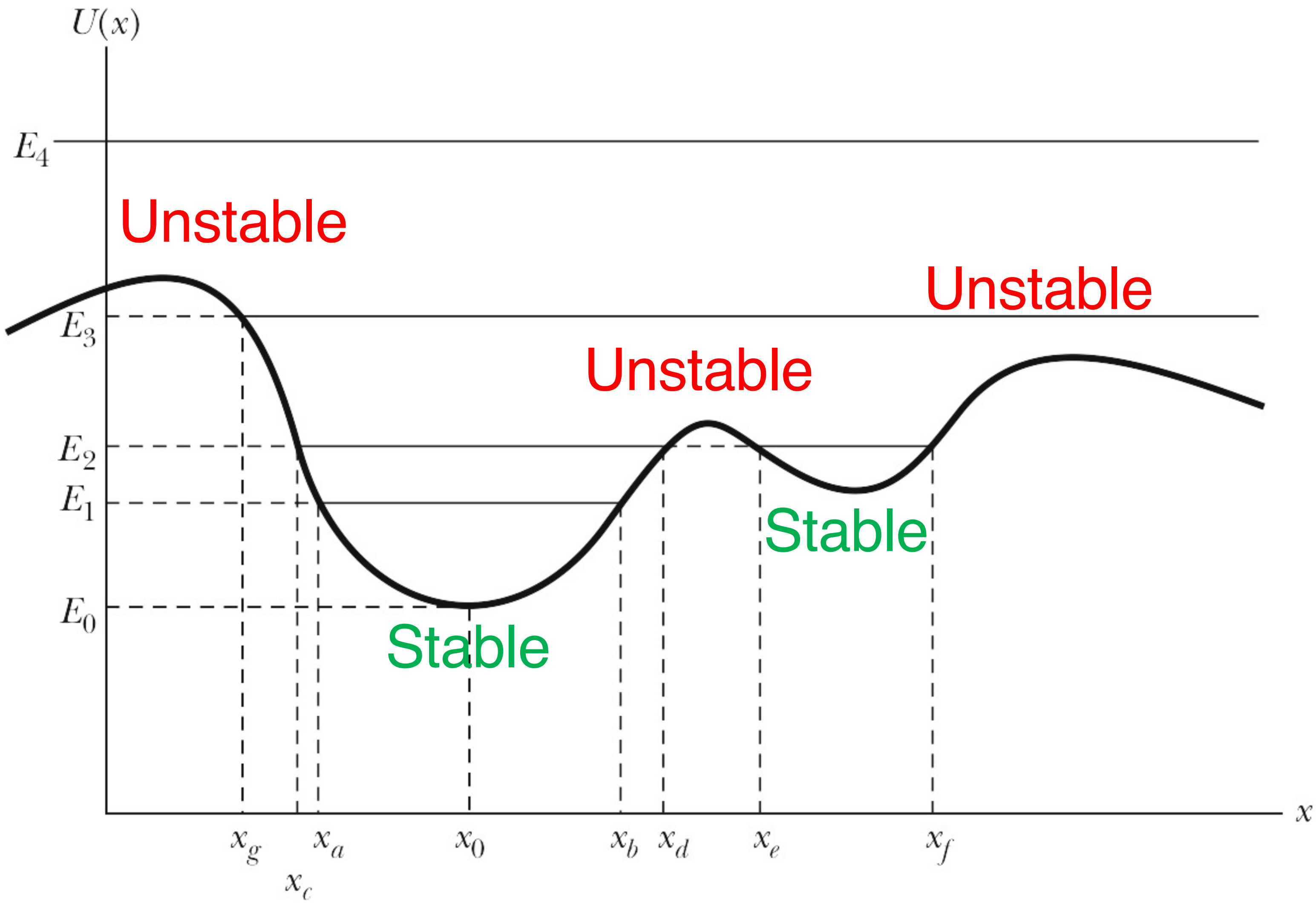
$$\vec{F} = -\vec{\nabla} U$$

$$\frac{\partial U}{\partial t} = 0$$

- The relation $\vec{F} = -\vec{\nabla} U$ allows us to quickly check is a force is conservative since

$$\vec{\nabla} \times \vec{F} = -\vec{\nabla} \times \vec{\nabla} U = 0$$

Predicting motion based on *U*.



Problem 2.37

A particle of mass m has a speed $v = \alpha/x$, where x is its displacement. Find the force $F(x)$ responsible for this motion.

Chapter 3.

Harmonic motion.

- Harmonic motion:
  - Motion around a position of stable equilibrium.
  - Simple harmonic motion:
    - At small distances around the equilibrium position, the force is approximately equal to $-kx$.
    - The total energy of the system is constant. The kinetic and potential energy will be time dependent.
  - Damped and driven harmonic motion:
    - Damped harmonic motion occurs when friction or drag forces are acting on the system. Energy is dissipated and the system will gradually come to rest.
    - Driven harmonic motion adds a driving force in order to compensate for damping losses.

Solving Second-Order Differential Equations.

- General form:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

- If you find two linearly independent solutions, every other solution will be a linear combination of these two solutions.
- The general solution has two constants, defined by the initial conditions.

- **Homogeneous equation:**
  - *f(x)* is equal to 0.
  - Simple harmonic motion when $a = 0$.
- **Inhomogeneous equation:**
  - *f(x)* is not equal to 0.

Homogeneous Equation

- Consider a damping force –*bv* and a restoring force –*kx*. The equation of motion for such system is: *ma* = –*bv* – *kx*.
- This provides us with the homogeneous equation:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$$

- Try the following solution: $x = e^{rt}$.
- This is a valid solution if $r^2 + 2\beta r + \omega_0^2 = 0$. This requires:

$$r = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

- Three different scenarios:
  - $\beta^2 > \omega_0^2$: over damping. Two values of *r*.
  - $\beta^2 = \omega_0^2$: critical damping. One value of *r*. Second solution is $te^{-\beta t}$.
  - $\beta^2 < \omega_0^2$: under damping. Two values of *r*; r is a complex number

Inhomogeneous Equation

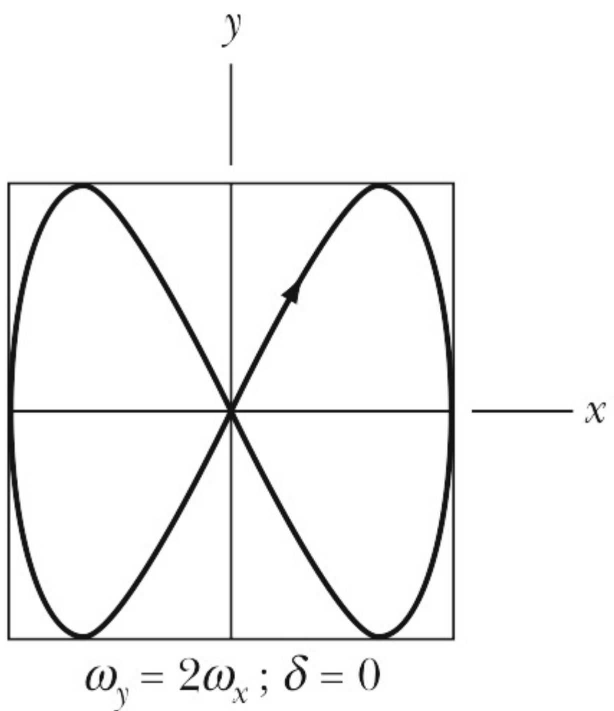
- Consider a damping force $-bv$, a restoring force $-kx$, and a driving force $f(t)$. The equation of motion for such system is: $ma = -bv - kx + f(t)$.
- The equation of motion can be rewritten as:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = f(t)$$

- Suppose:
  - v is a solution of the inhomogeneous equation (this is called the **particular solution**).
  - u is the general solution of the homogeneous equation (this is called the **complementary solution**).

then $u + v$ is the general solution of the inhomogeneous equation.

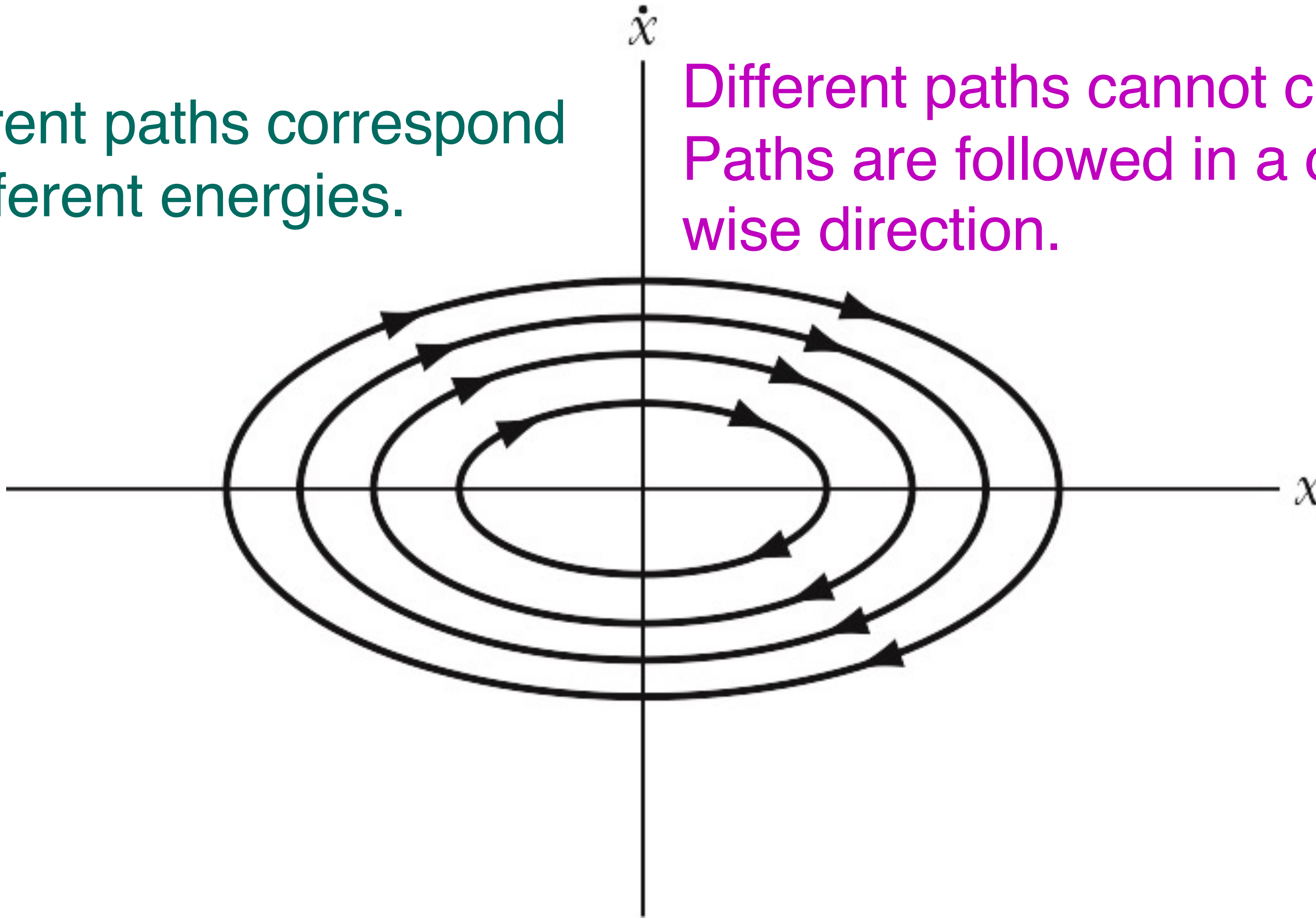
Visualizing Harmonic Motion. *y* vs *x* for different restoring forces.



Visualizing Harmonic Motion. Phase Diagrams.

Different paths correspond to different energies.

Different paths cannot cross. Paths are followed in a clock-wise direction.



Problem 3.12.

A simple pendulum of mass m is suspended from a fixed point by a weightless rod of length l. Obtain the equation of motion for small angles.

Discuss the motion when it takes place in a viscous medium with a retarding force $2m\sqrt{gl}\dot{\theta}$.

Chapter 4.

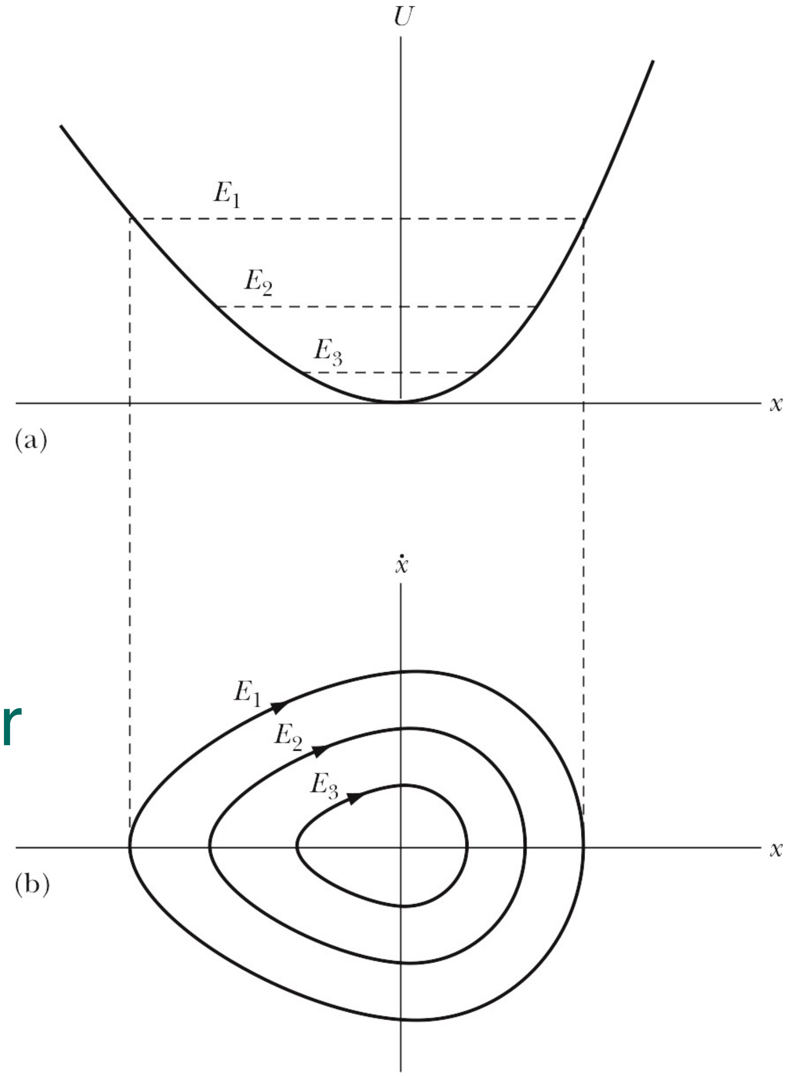
Non-linear oscillations.

- Linear differential equations:
  - Terms are proportional to acceleration, velocity, and position:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

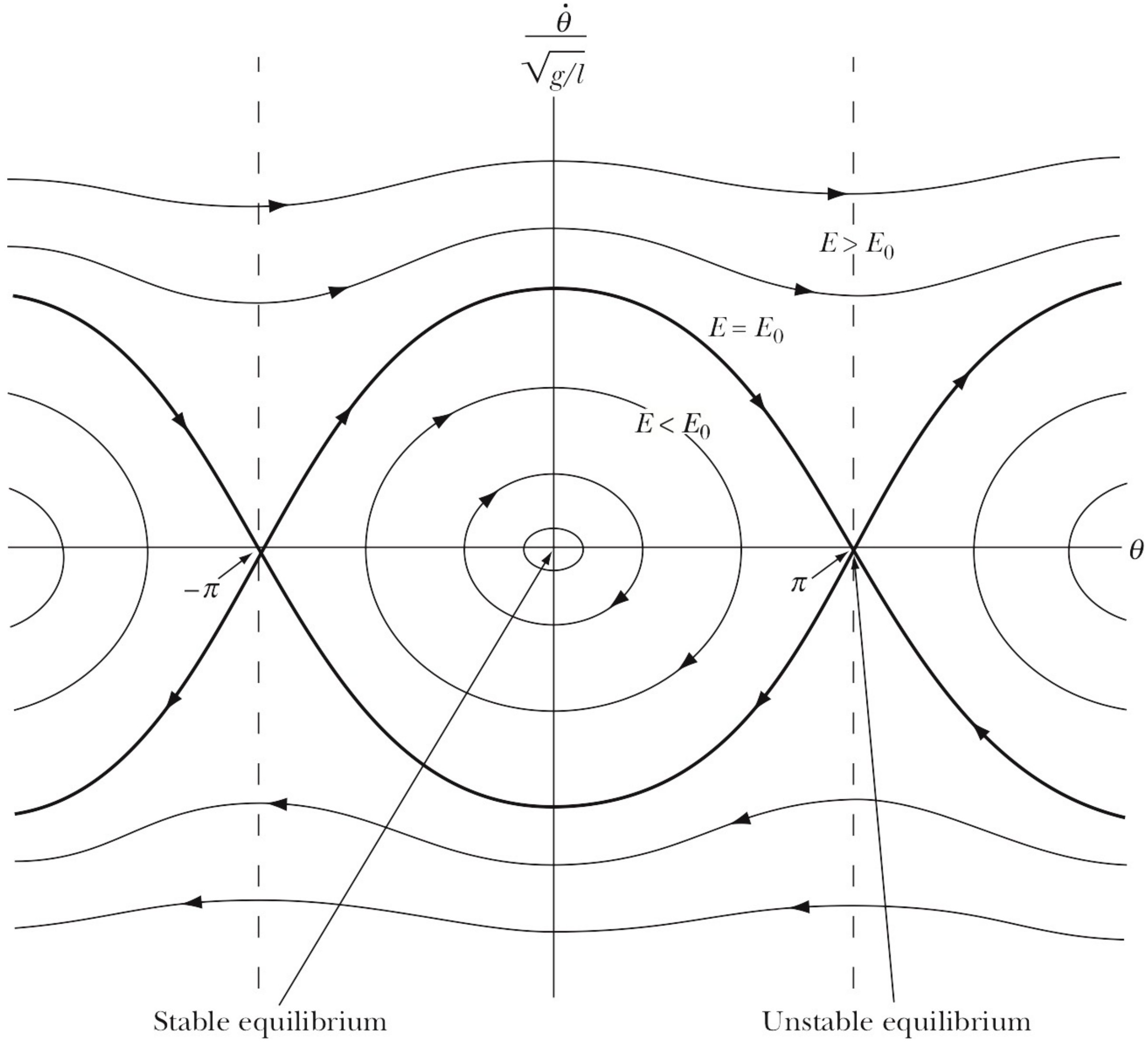
- Non-linear differential equations:
  - Include terms that non-linear in term of acceleration, velocity, and position.
- Non-linear terms are divided in two groups:
  - Symmetric around the equilibrium position. This requires terms proportional to εr^3. If $\varepsilon>0$: soft system. If $\varepsilon<0$: hard system.
  - Asymmetric around the equilibrium position. This requires terms proportional to r^2.

Phase Diagrams.
Asymmetric for asymmetric potentials.

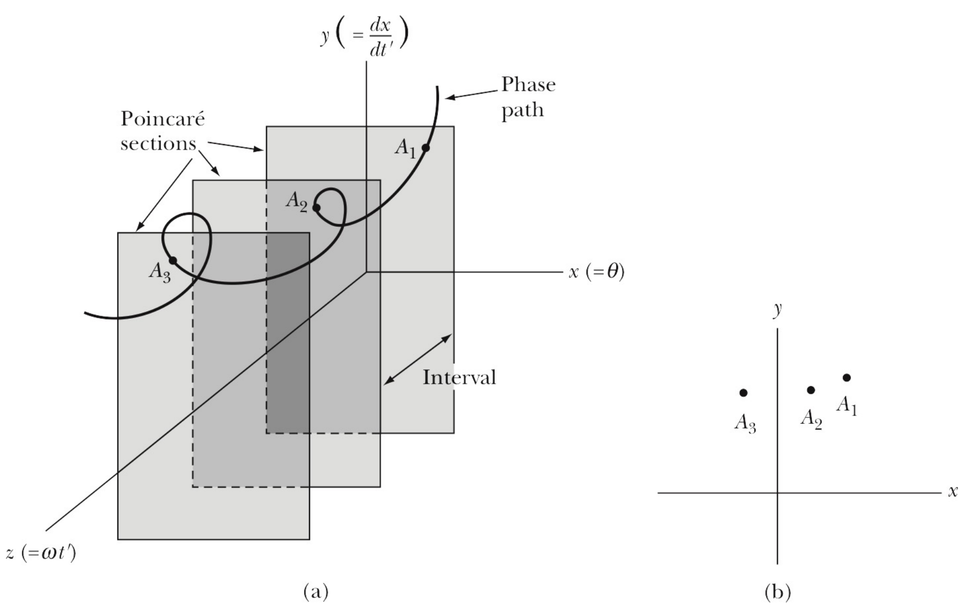


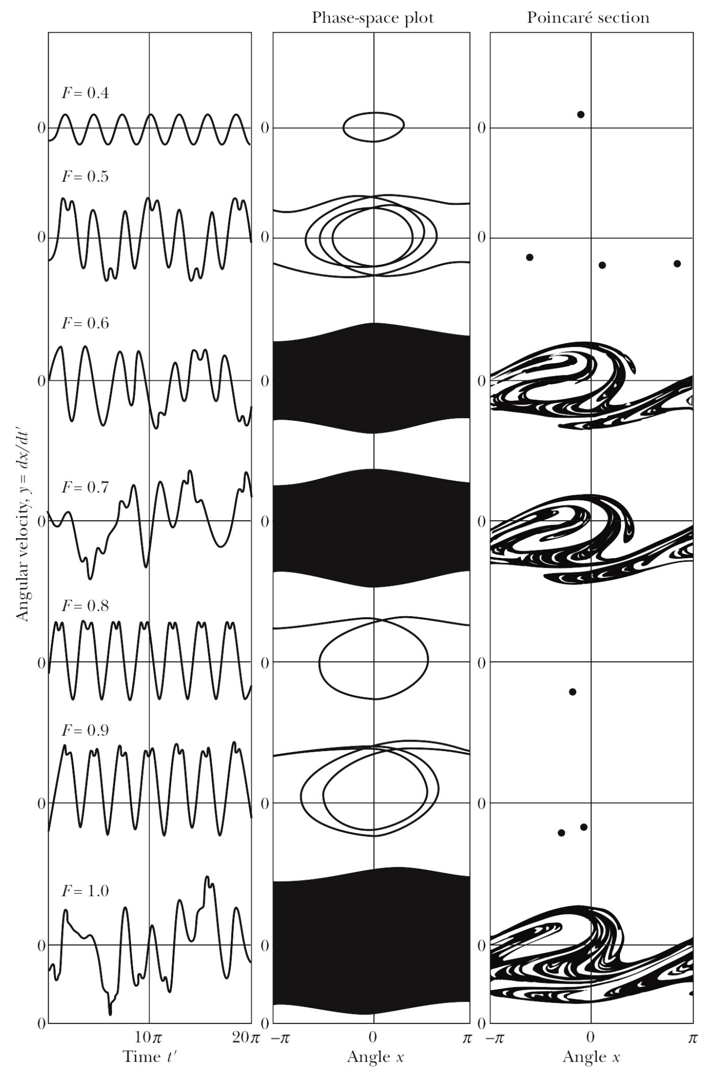
Closed contours for motion around stable equilibrium positions.

Phase diagram for the plane pendulum.



Visualizing chaos.
Poincare plots.





SHM

Periodic

Chaotic

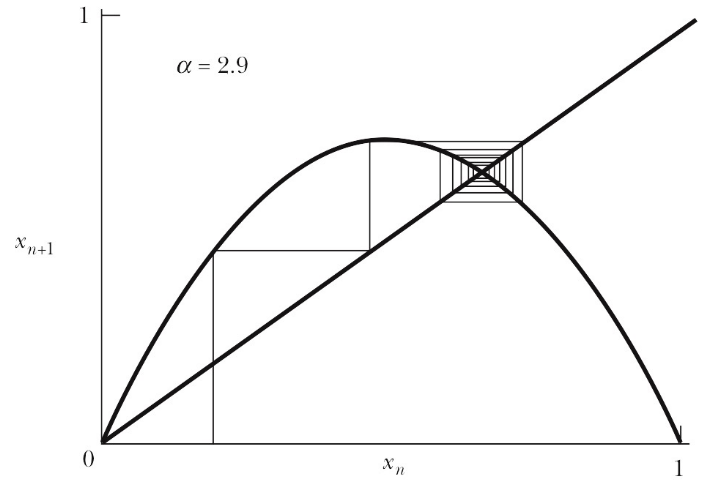
Chaotic

"SHM"

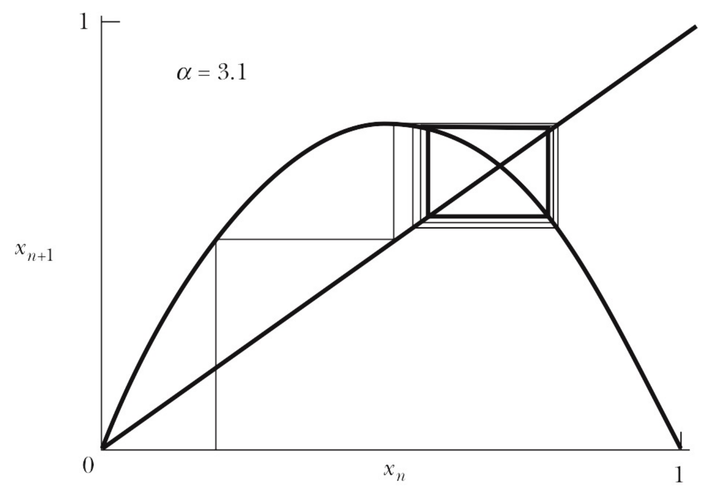
Periodic

Chaotic

Logistic equations.
Creating chaos with maps.

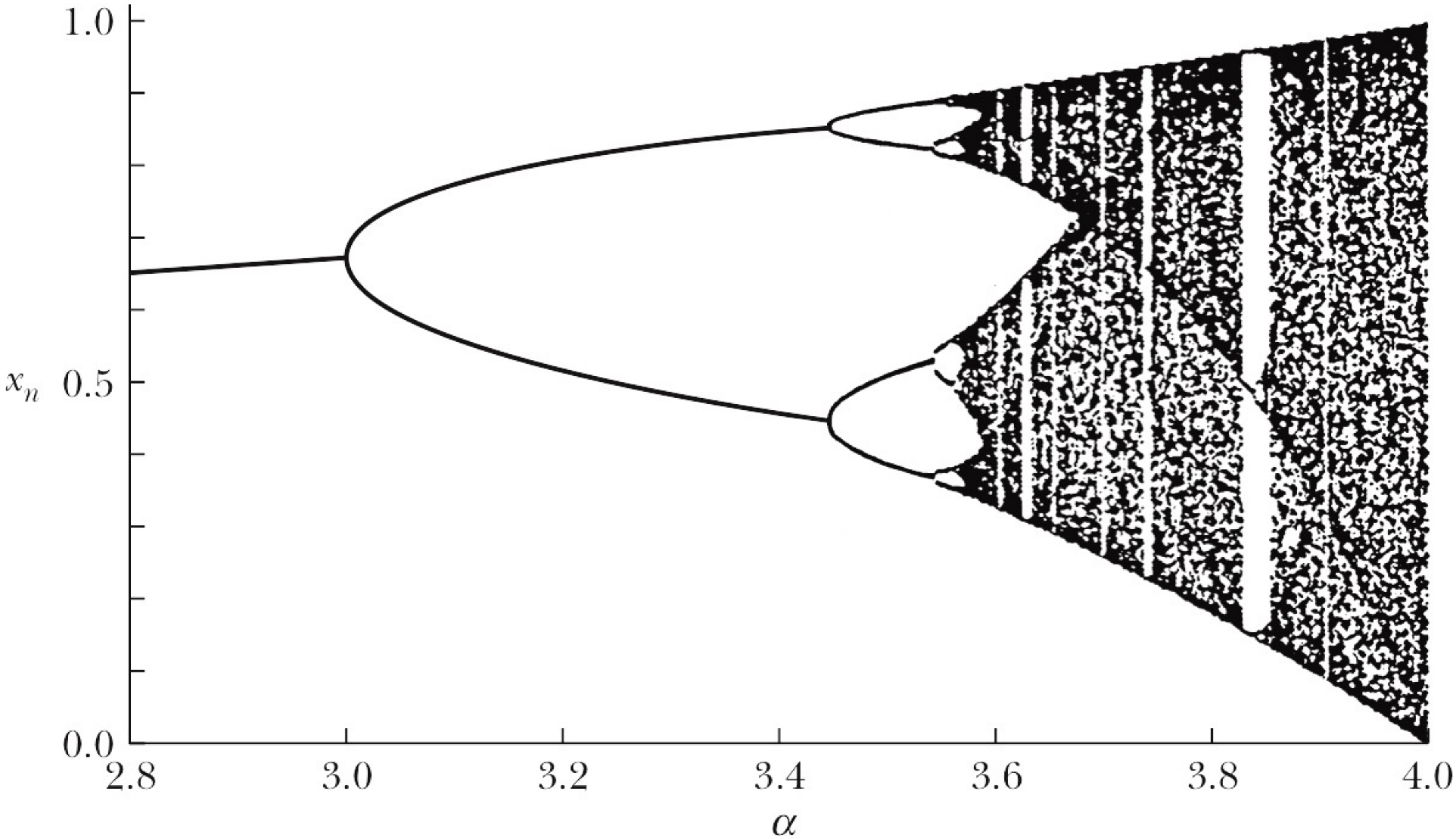


(a)



(b)

Visualizing chaos.
Bifurcation diagrams.



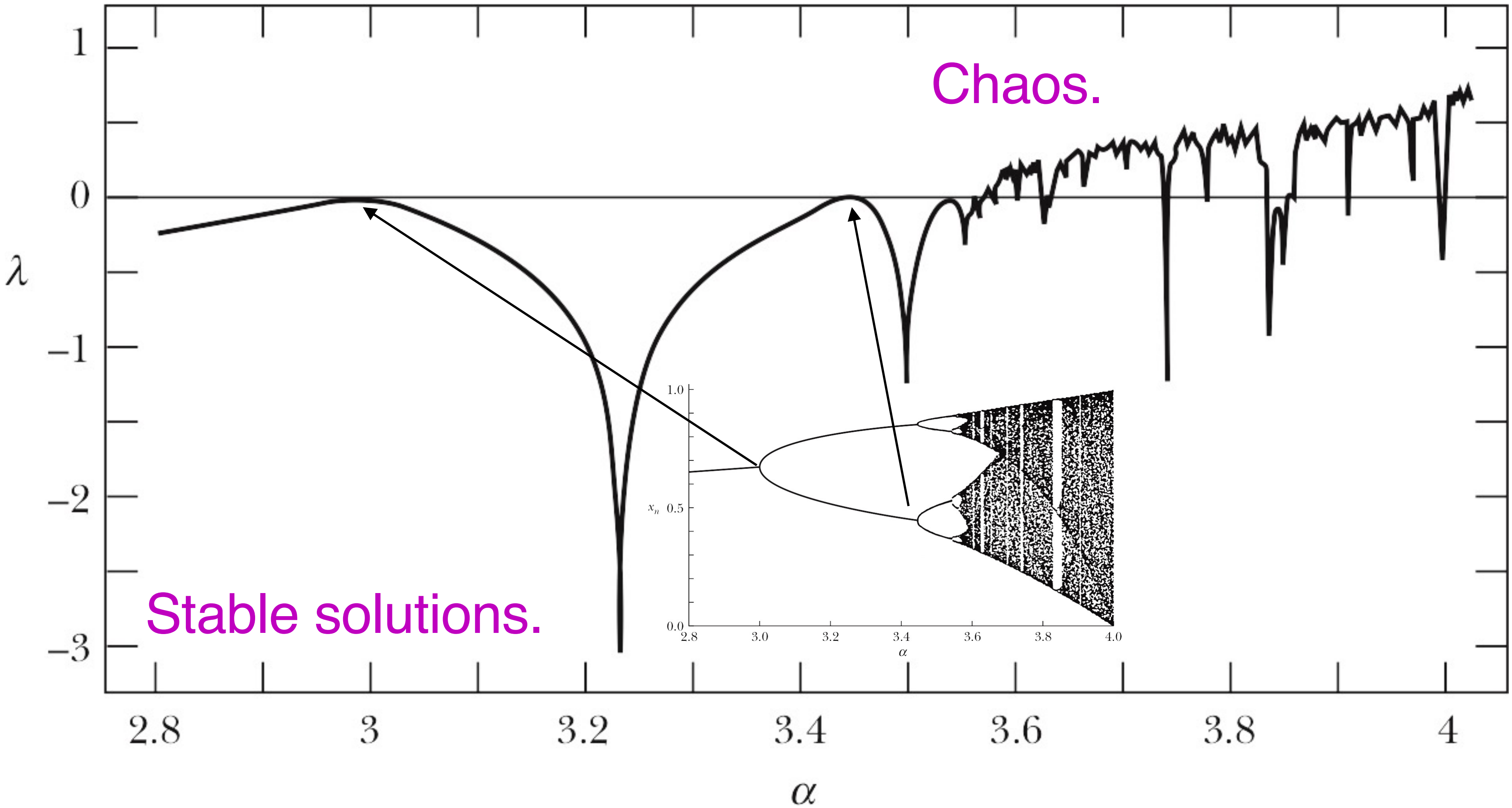
The Lyapunov exponent λ.

- The development of chaos can be studied by examining the Lyapunov exponent λ:

$$\lambda = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n-1}\left|\frac{df(\alpha, x_i)}{dx}\right|\right)$$

- This exponent is a measure of the difference between solutions when we make a small change in the initial conditions:
  - If $\lambda < 0$: stable solutions.
  - If $\lambda = 0$: doubling of the number of solutions
  - If $\lambda > 0$: chaos.

Visualizing chaos.
The Lyapunov exponent λ.



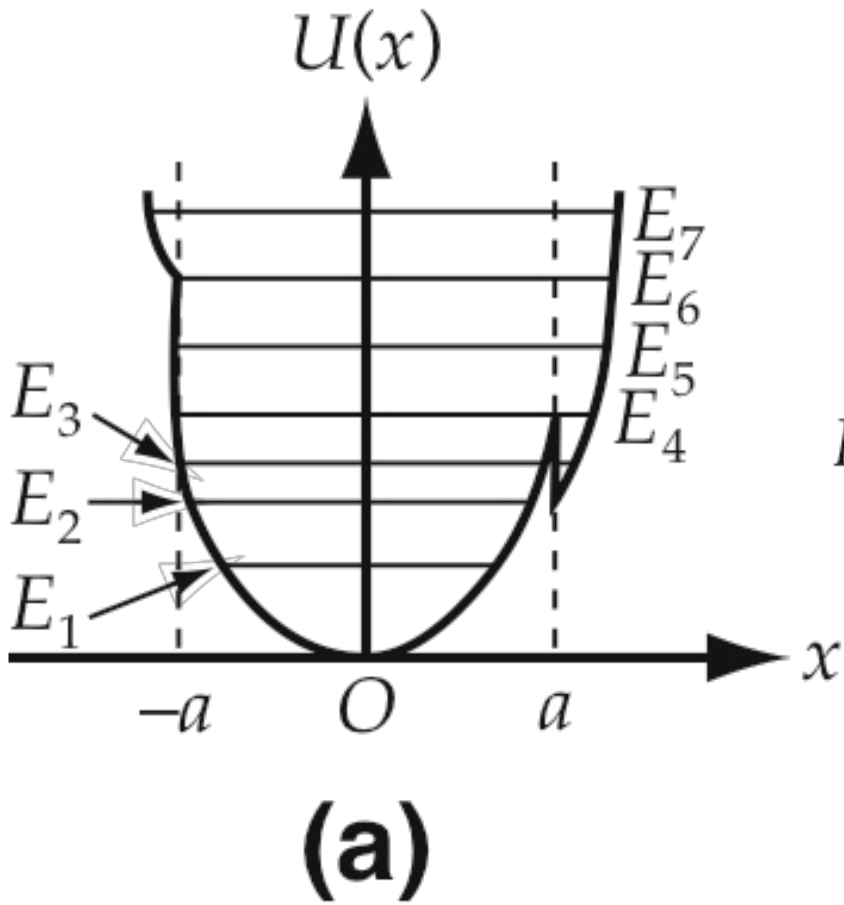
No need to always solve differential equations. Problem 4.9.

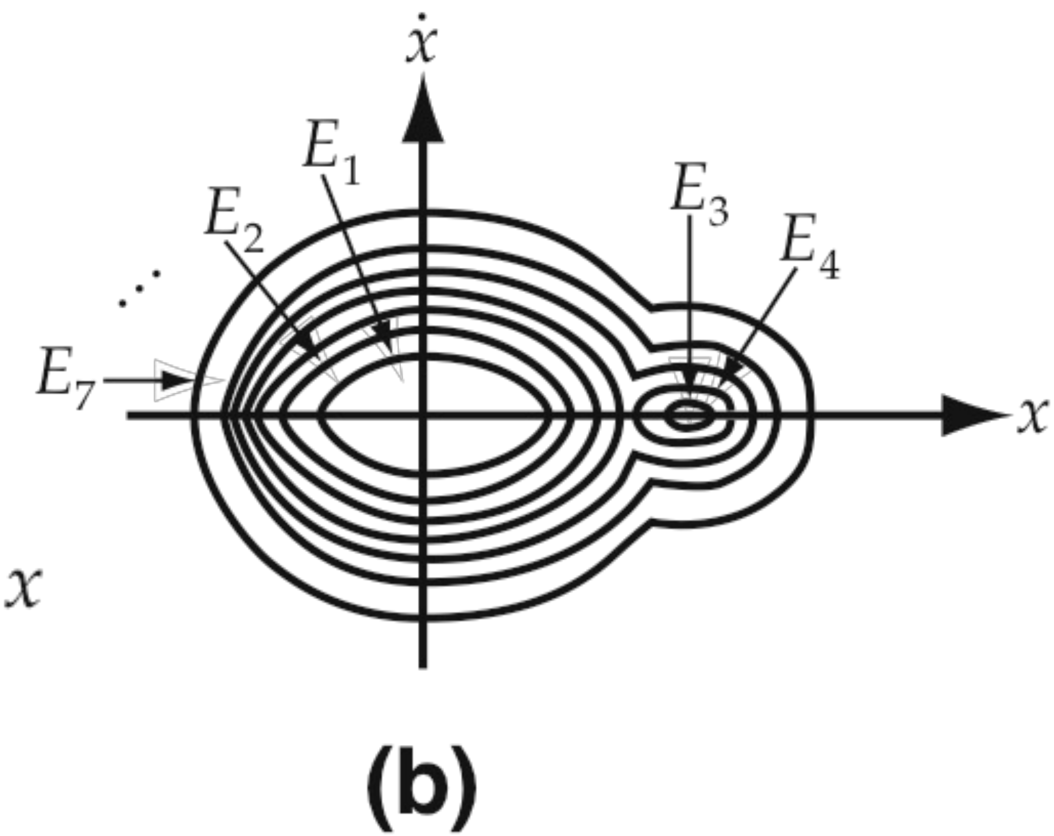
Investigate the motion of an undamped particle, subject to a force of the form:

$$F(x) = \begin{cases} -kx & |x| < a \\ -(k+\delta)x + \delta a & |x| > a \end{cases}$$

k and δ are positive constants.

Solution





ENOUGH FOR TODAY?