Classical Mechanics
Phy 235, Lecture 25.

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Course Announcements.

- There will be regular office hours this week to help you with questions about the final exam.

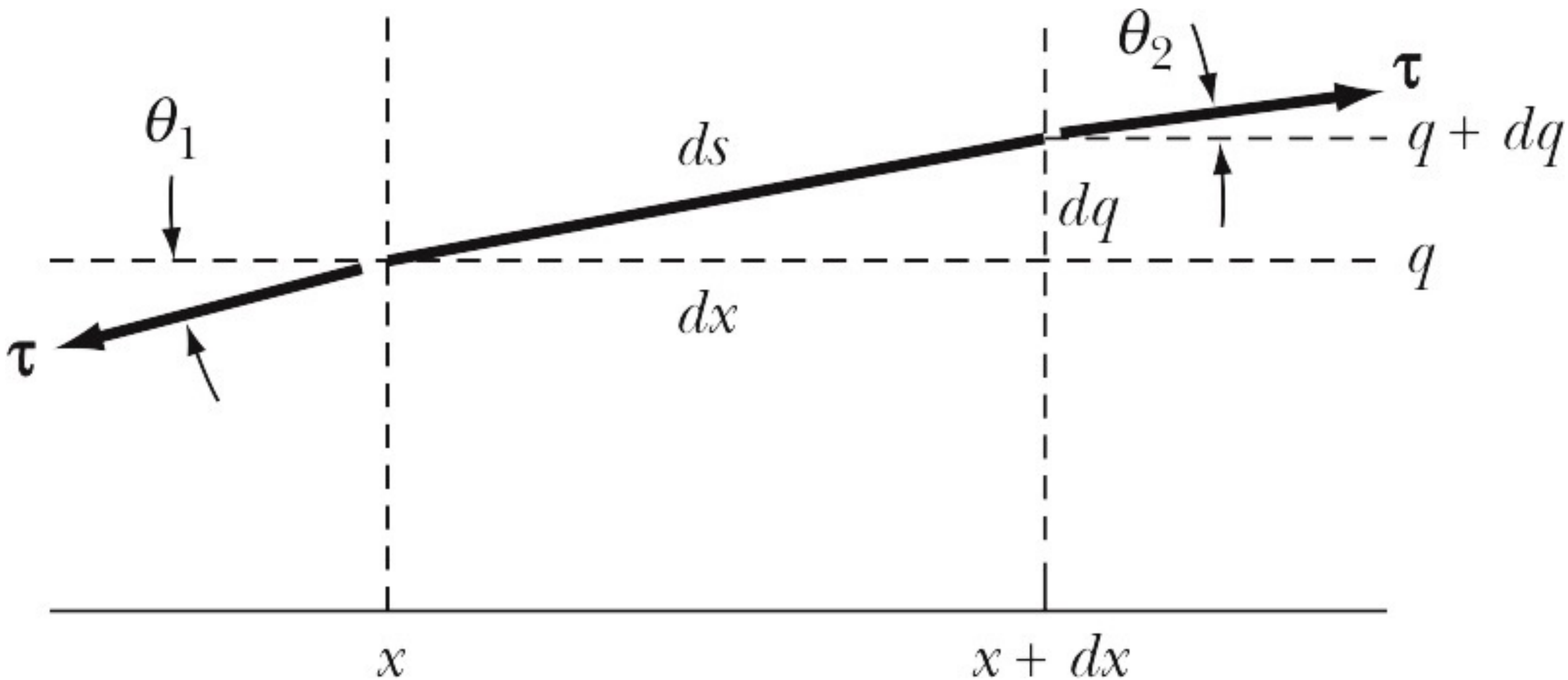
- Any questions related to the grading of exam # 3 need to be submitted to Prof. Wolfs by the end of class today. Drop your exam and a note describing why you feel you deserve more points.

- The final exam will cover Chapter 1 – 13. The exam will take place on Sunday December 14, 12.30 pm – 3.30 pm in Dewey 1101.

Some words of wisdom

- In Spring 2019, I was on a walking trip with my family in Spain, covering 500 miles between St. Jean in France and Santiago de Compostela in Spain in 33 days. I made many observations during these 33 days, some of which may apply to you. After departing St. Jean on April 4, I observed that those who only focused each day on the final destination, wanting to get there first, or in the shortest possible walking time, were most likely to drop out. Those who walked slowly, including myself, and enjoyed every blooming flower, every new view emerging around every corner, and took the time to look back and be amazed by how far we had come, reached the finish line, even when on certain days it was difficult to keep moving one foot in front of the other. Soon all of you are heading in a new direction, taking a new road to a new destination, and my advice to you is to enjoy the road just as much as your destination. Do not focus only on your destination or how long it will take to reach your destination, but make sure you enjoy the road to your destination as much as reaching your destination.

The wave equation.



The wave equation.

- The ideal wave equation:

$$\frac{\partial^2 q}{\partial x^2} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

- The “real” wave equation:

$$\frac{\partial^2 q}{\partial x^2} - \frac{D}{\tau}\frac{\partial q}{\partial t} + \frac{F(x,t)}{\tau} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

Solving the ideal wave equation.

- The ideal wave equation:

$$\frac{\partial^2 q}{\partial x^2} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

- No dissipation: energy is conserved.
- Use **separation of variables** to solve the wave equation:

$$q(x,t) = \psi(x)\chi(t)$$

- This results on two differential equations:

$$\frac{v^2}{\psi}\frac{\partial^2 \psi}{\partial x^2} = \omega^2 \quad \Leftrightarrow \quad \frac{\partial^2 \psi}{\partial x^2} - \frac{\omega^2}{v^2}\psi = 0$$

$$\frac{1}{\chi}\frac{\partial^2 \chi}{\partial t^2} = \omega^2 \quad \Leftrightarrow \quad \frac{\partial^2 \chi}{\partial t^2} - \omega^2 \chi = 0$$

4 Minute 07 Second Intermission.

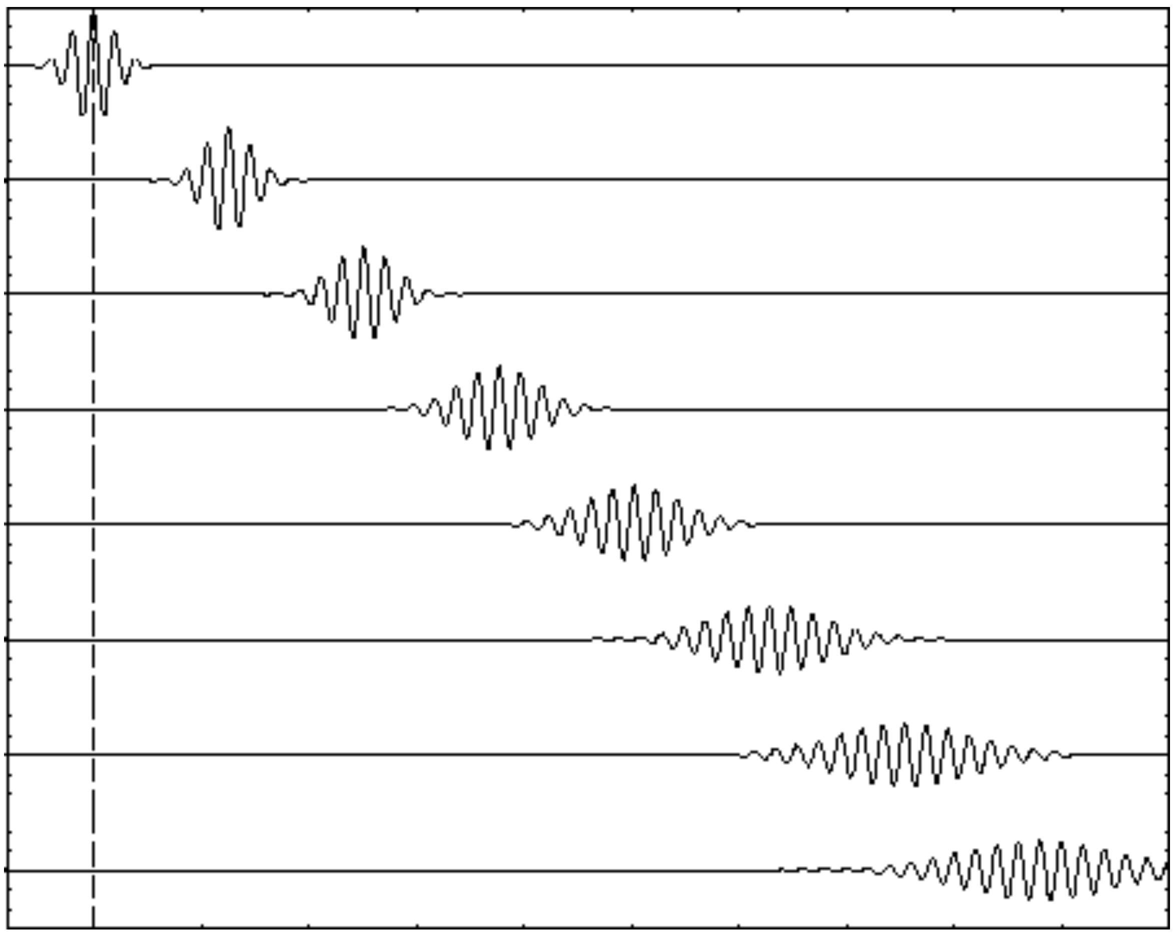
- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 07 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Wave velocities

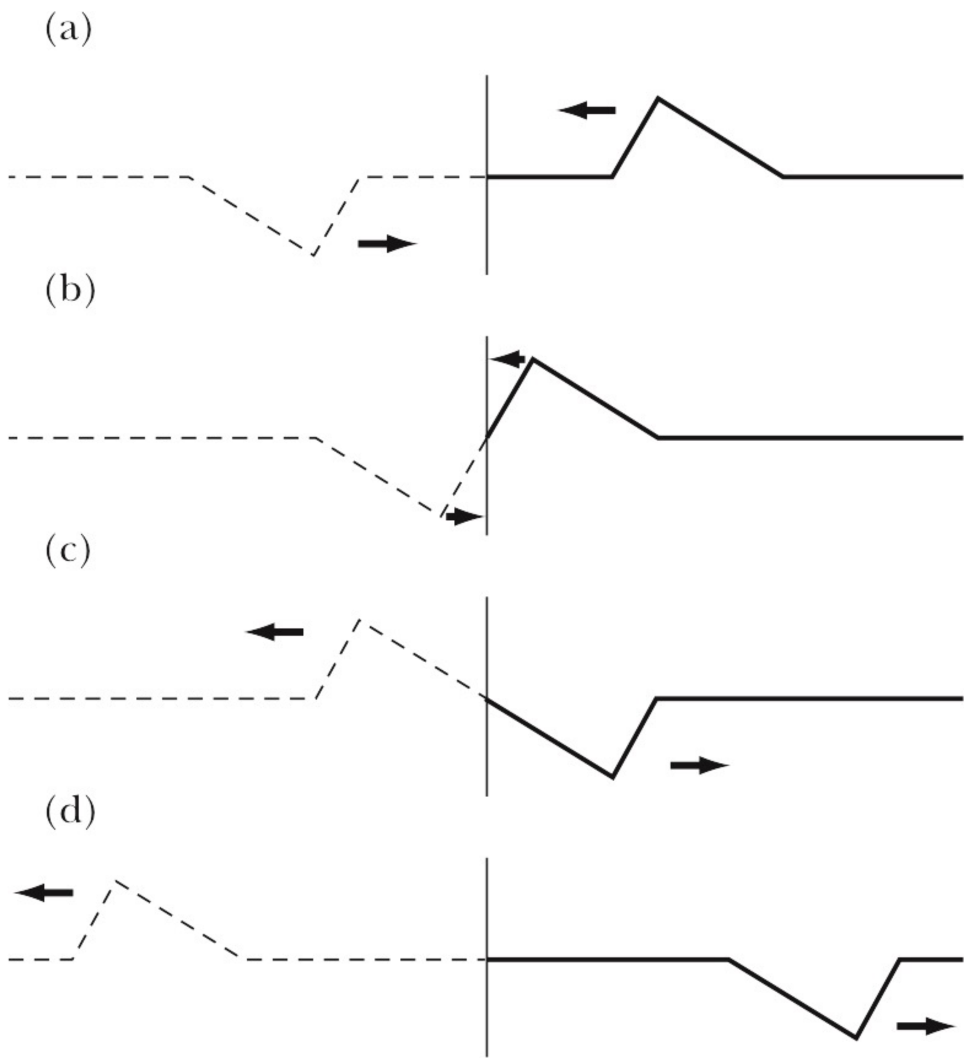
- Wave velocity:
  - Velocity that keeps the amplitude constant: v.
- Phase velocity:
  - Velocity that keeps the phase constant: V.
- The velocities depend on the wave number. When this is the case, the medium is called a **dispersive medium**.

Dispersion.
Limits information transmission.



http://jick.net/skept/GWP/

Wave Propagation: reflection at fixed end.



Other wave features.

- Two wave travelling in opposite directions can create a **standing wave**:

- For the loaded string:
  - There is a minimum wave length: $\lambda_n = \frac{2L}{n}$
  - There is a maximum wave number: $k_n = \frac{2\pi}{\lambda_n} = \frac{\pi}{d}$
  - There is a maximum frequency: $\omega_n = 2\sqrt{\frac{\tau}{md}}\backslash\text{sin}\left(\frac{k_n d}{2}\right) = 2\sqrt{\frac{\tau}{md}}$

Complex wavenumbers.

Higher frequencies can be supported if the wavenumber becomes complex:

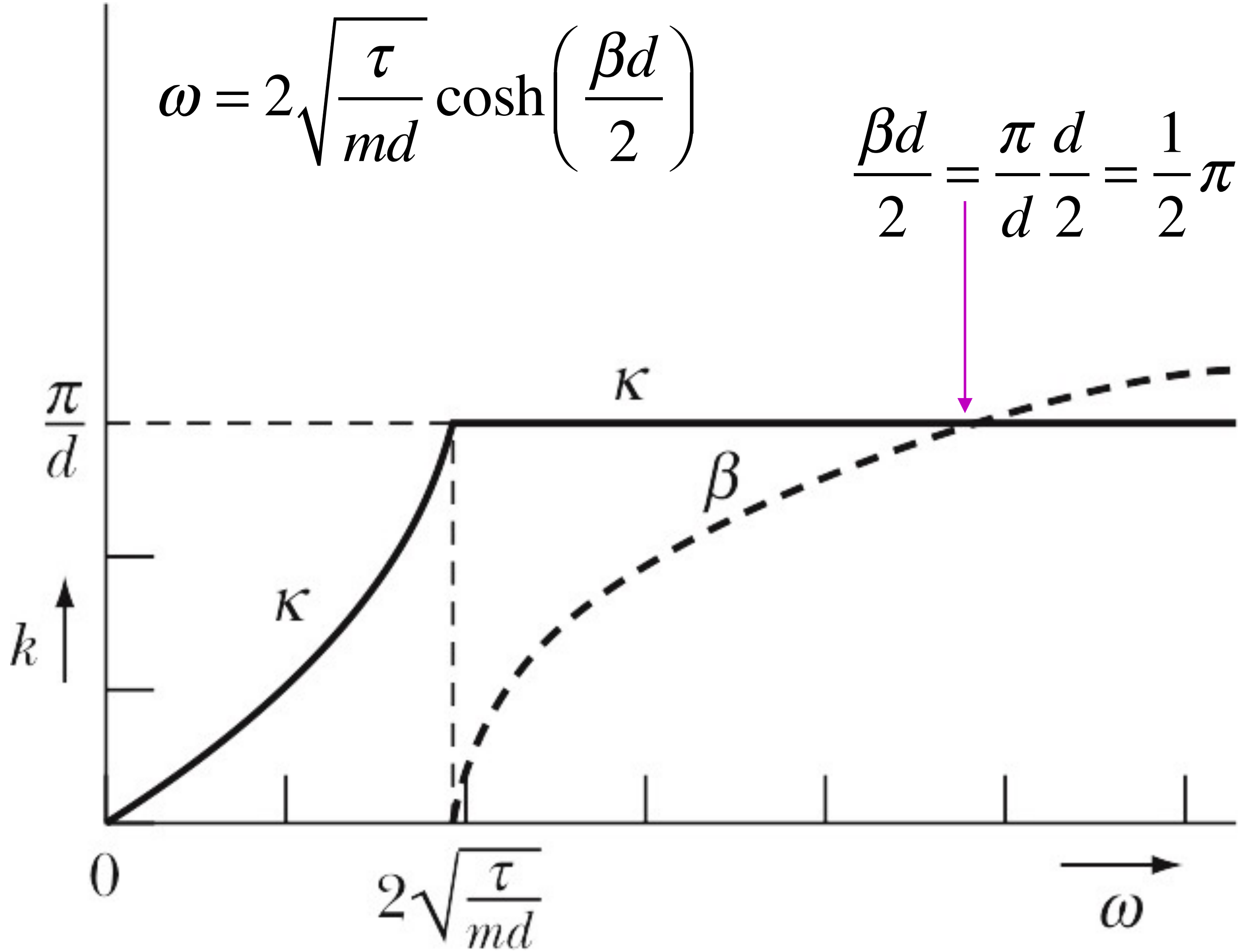
$$\begin{aligned}
\omega &= 2\sqrt{\frac{\tau}{md}}\sin\left(\frac{d}{2}(\kappa - i\beta\right) = \\
&= 2\sqrt{\frac{\tau}{md}}\left\{\sin\left(\frac{d}{2}\kappa\right)\cos\left(\frac{i\beta d}{2}\right) - \cos\left(\frac{d}{2}\kappa\right)\sin\left(\frac{i\beta d}{2}\right)\right\} = \\
&= 2\sqrt{\frac{\tau}{md}}\left\{\sin\left(\frac{d}{2}\kappa\right)\cosh\left(\frac{\beta d}{2}\right) - i\cos\left(\frac{d}{2}\kappa\right)\sinh\left(\frac{\beta d}{2}\right)\right\}
\end{aligned}$$

Note: the equation in my notes is missing κ in the last two steps.

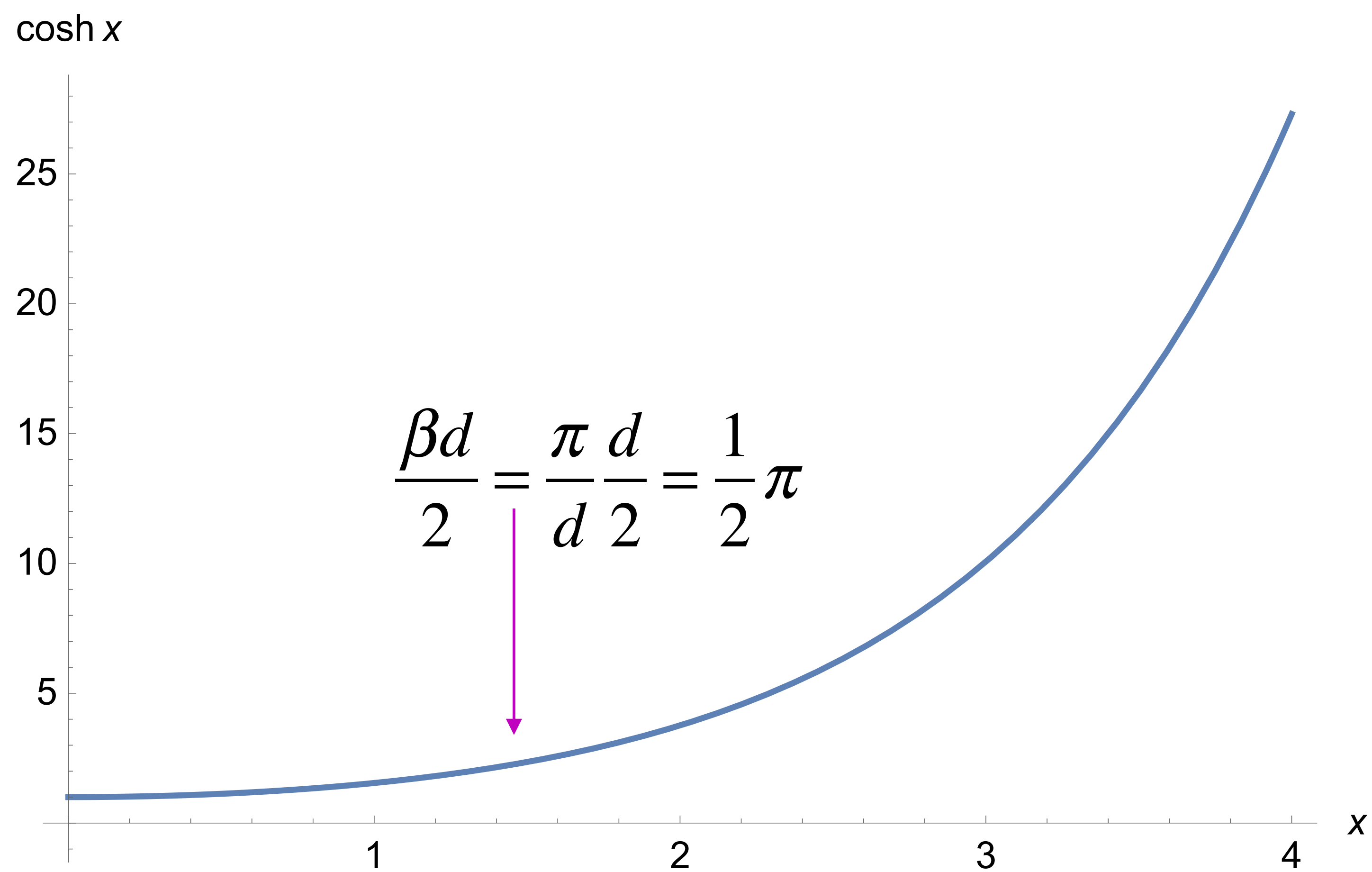
Solutions at high frequencies.

$$\omega = 2\sqrt{\frac{\tau}{md}}\cosh\left(\frac{\beta d}{2}\right)$$

$$\frac{\beta d}{2} = \frac{\pi}{d}\frac{d}{2} = \frac{1}{2}\pi$$



cosh(x) function.



ENOUGH FOR TODAY?