Classical Mechanics
Phy 235, Lecture 24.

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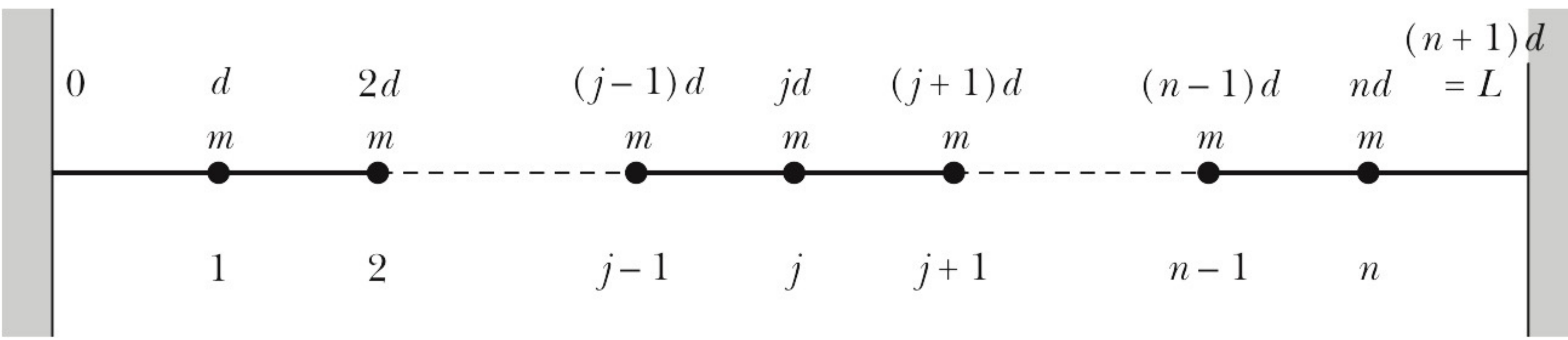
PH-AKA arriving in Montreal.

Friday December 5.
An important evening: pakjes avond.

Announcements

- Homework set # 10 is due on Friday December 5 at noon.
- On December 1, the University opened the end-of-semester course survey.
  - The survey closes on December 11.
  - I encourage all of you to complete this survey. Your opinion matters!
  - If by December 11, 95% of the PHY 235 students have completed the survey, I add 5% to everyone's final exam score.
- There will be no recitations next week, but office hours will be continue as scheduled to help you with final-exam related questions.
- The final exam will take place on Sunday December 14 between 12.30 pm and 3.30 pm in Dewey 1101.

Chapter 13:
from the loaded string to a “real” string.



Chapter 13: from the loaded string to a "real" string.

- Consider the following:
  - Increase the number of masses n to infinity.
  - Decrease the distance d to zero such that $(n + 1)d = L$.
  - Decrease the mass m to zero such that m/d = constant.
- In this case, the displacement q can be written as:

$$q_j(t) = \sum_s \beta_s \sin\left(s\pi \frac{x}{L}\right) e^{i\omega_s t} = q(x,t)$$

- The eigen frequencies are

$$\omega_s = \frac{s\pi}{L}\sqrt{\frac{\tau}{\rho}}$$

Energy in a string.

- Kinetic energy:

$$T = \int_0^L dT = \frac{L}{4}\rho \sum_s \left(\omega_s \nu_s \cos(\omega_s t) + \omega_s \mu_s \sin(\omega_s t)\right)^2$$

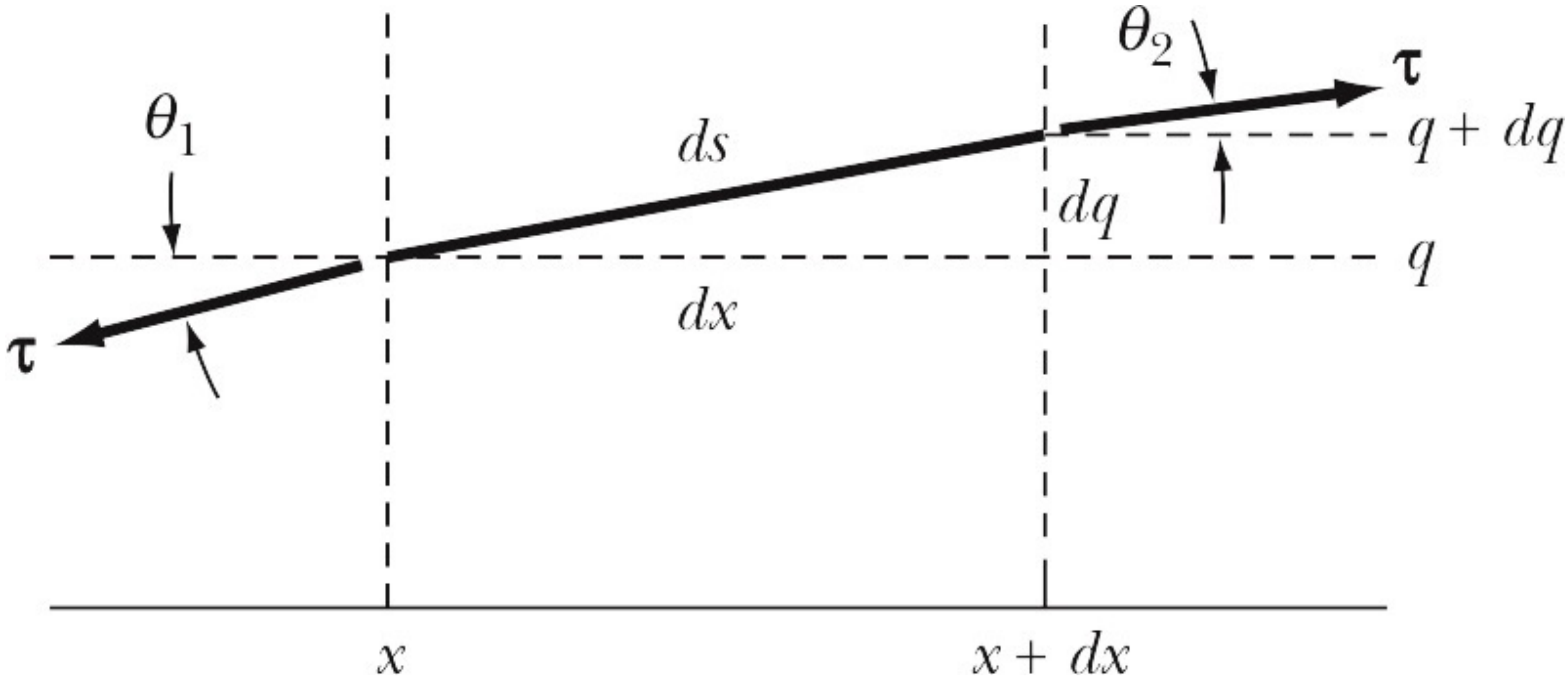
- Potential energy:

$$U = \frac{\tau}{2}\int_0^L \left(\frac{\partial q}{\partial x}\right)^2 dx = \frac{\rho L}{4}\sum_s \omega_s^2 \left(\mu_s \cos(\omega_s t) - \nu_s \sin(\omega_s t)\right)^2$$

- Total energy:

$$E = T + U = \frac{\rho L}{4}\sum_s \omega_s^2 (\mu_s^2 + \nu_s^2) = \text{constant}$$

The wave equation.



The wave equation.

- The ideal wave equation:

$$\frac{\partial^2 q}{\partial x^2} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

- The “real” wave equation:

$$\frac{\partial^2 q}{\partial x^2} - \frac{D}{\tau}\frac{\partial q}{\partial t} + \frac{F(x,t)}{\tau} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2}$$

2 Minute 27 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 27 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Problem 13.11.

- When a particular driving force is applied to a string, it is observed that the string's vibration is purely in the n^{th} harmonic. Find the driving force.

$$f_s(t) = \int_0^L F(x,t) \sin \frac{s\pi x}{b} \, dx = 0 \quad \text{for } s \neq n$$

$$= \int_0^L F(x,t) \sin \frac{s\pi x}{b} \, dx \neq 0 \quad \text{for } s = n$$

Solving the ideal wave equation.

- The ideal wave equation:

$$\frac{\partial^2 q}{\partial x^2} = \frac{\rho}{\tau}\frac{\partial^2 q}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 q}{\partial t^2}$$

- No dissipation: energy is conserved.
- Use **separation of variables** to solve the wave equation:

$$q(x,t) = \psi(x)\chi(t)$$

- This results on two differential equations:

$$\frac{v^2}{\psi}\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{\chi}\frac{\partial^2 \chi}{\partial t^2} = \text{constant} = \omega^2$$

Solving the ideal wave equation.

$$\frac{\partial^2\psi}{\partial x^2} - \frac{\omega^2}{\nu^2}\psi = 0$$

$$\frac{\partial^2\chi}{\partial t^2} - \omega^2\chi = 0$$

Solutions:

$$\psi(x) = e^{\pm i\left(\frac{\omega}{\nu}\right)x}$$

$$\chi(t) = e^{\pm i\omega t}$$

ENOUGH FOR TODAY?