Classical Mechanics
Phy 235, Lecture 23.

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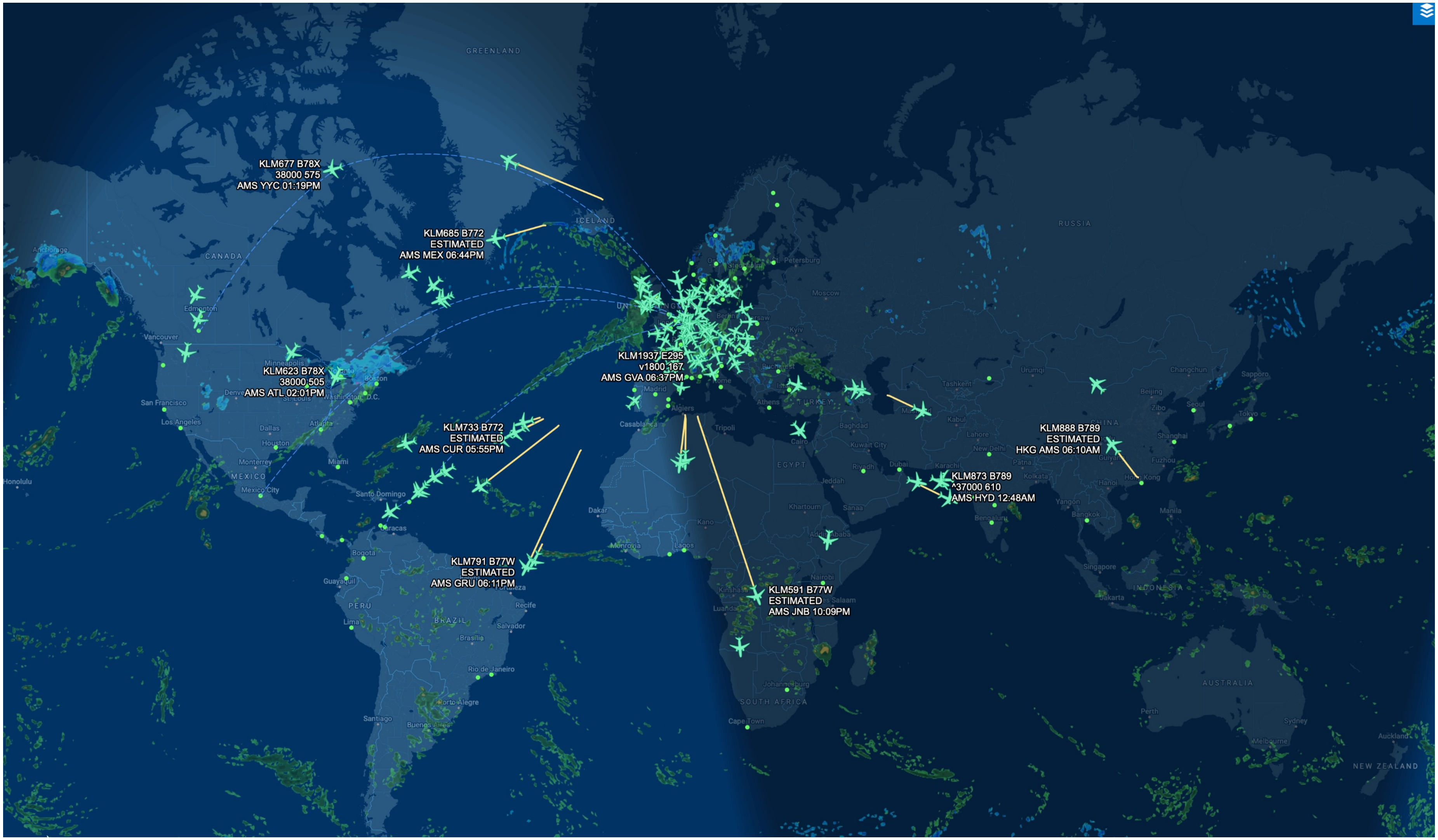
A beautiful landing at SFO.

A small country; a huge reach.
KLM planes flying at 12.33 pm on 11/30.



Final remarks about Chapter 12. Solving coupled oscillator problems.

- Follow these steps in order to solve most coupled oscillator problems:
  - Choose generalized coordinates.
  - Determine the A and m tensors.
  - Determine the eigen frequency and the eigen vectors.
  - Determine the scale factors required to match the initial conditions.
  - Determine the normal coordinates.

Course Information.

- If you did not pick up Exam # 3 during lecture last week, you can pick it up during recitations this week.
- Any requests for regrades for specific problems on Exam # 3 should be made by Monday December 8 (end of lecture). I will need the following:
  - Your blue book(s).
  - A written explanation why you feel you deserve more points.
- I have graded about 20% of the term papers. I will return each paper as soon as I have graded it, but I will not complete this until sometime next week.

Problem 12.21.

Three oscillators of equal mass *m* are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1\left(x_1{}^2 + x_3{}^2\right) + \kappa_2 x_2{}^2 + \kappa_3\left(x_1 x_2 + x_2 x_3\right)\right]$$

where

$$\kappa_3 = \sqrt{2\kappa_1\kappa_2}$$

Find the eigen frequencies by solving the secular equation. What is the physical interpretation of the zero-frequency mode?

General steps to solve this type of problems.

- Find {A}.

$$\{\mathbf{A}\} = \begin{bmatrix} \kappa_1 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 \end{bmatrix}$$

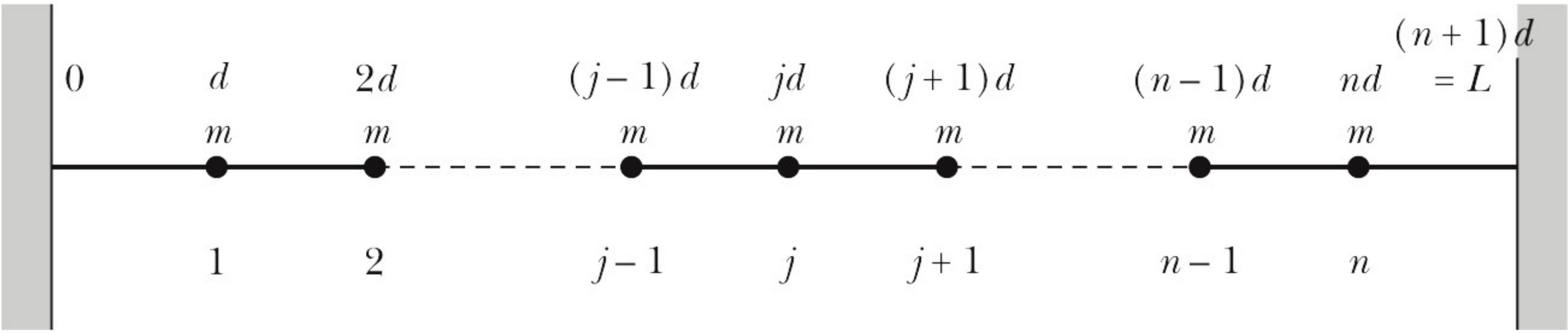
- Solve secular determinant.

$$\begin{vmatrix} \kappa_1 - m\omega^2 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 - m\omega^2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 - m\omega^2 \end{vmatrix} = 0$$

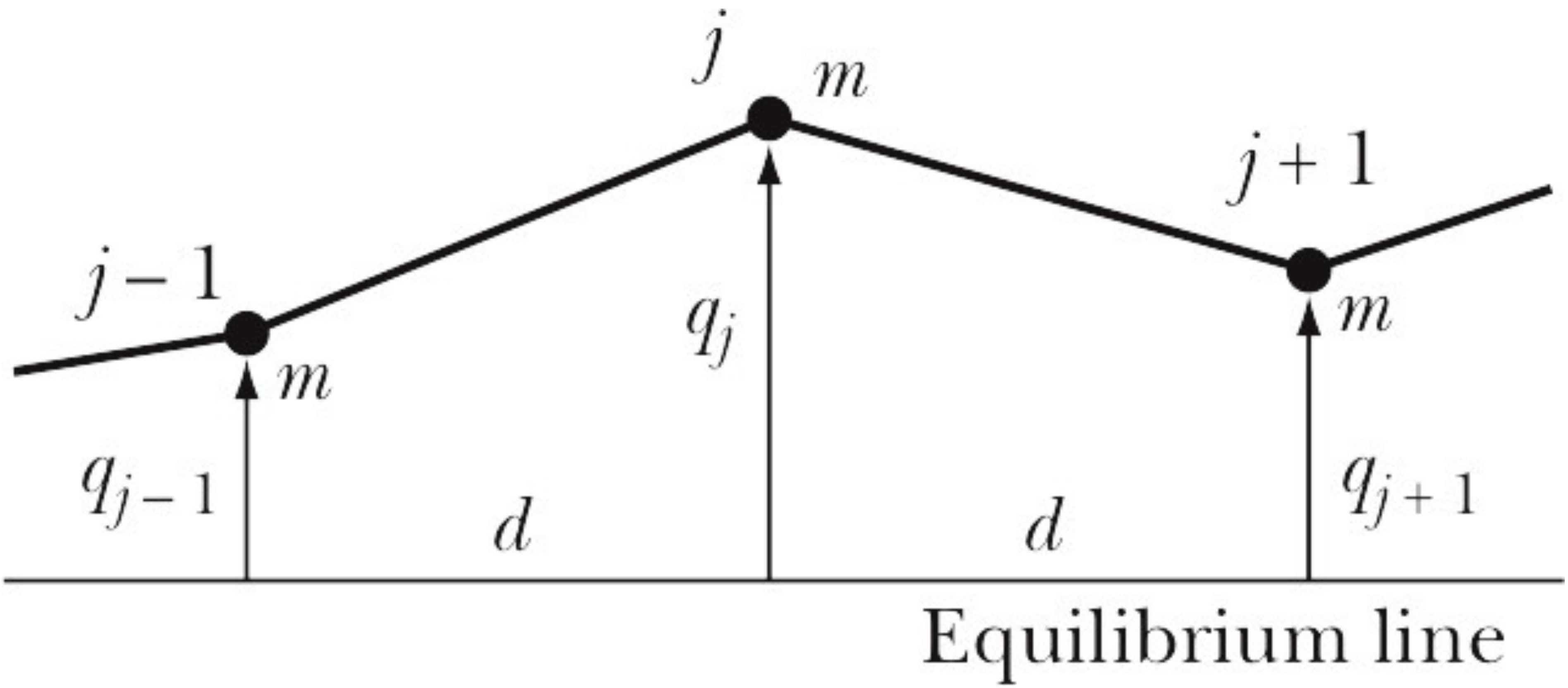
- Find {m}.

$$\{\mathbf{m}\} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

The loaded string.



The loaded string.
Motion in the vertical direction.



The loaded string.

- General solution:

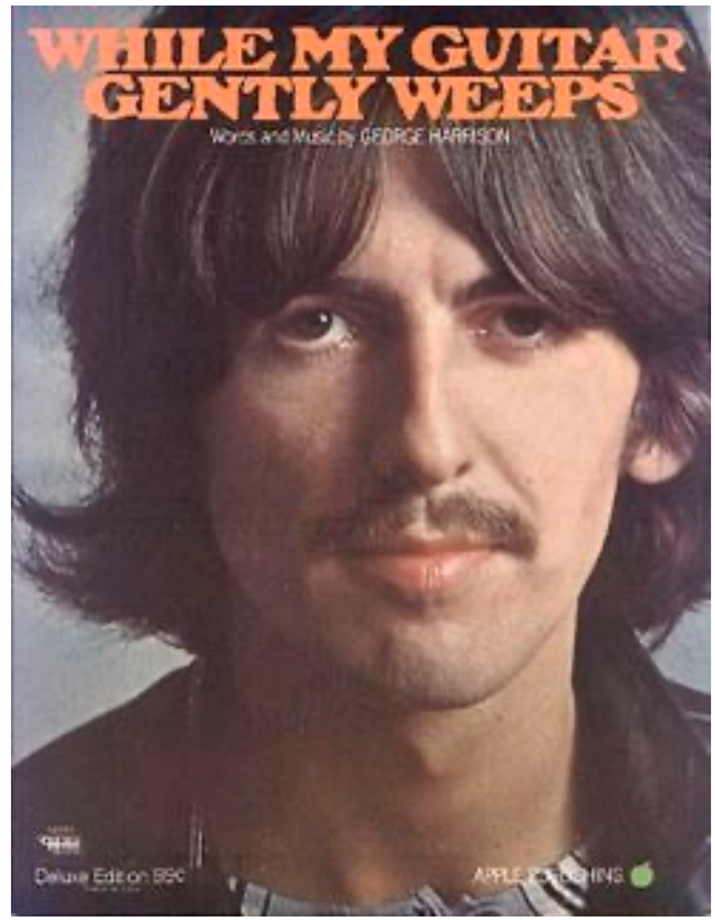
$$q_j(t) = \sum_s \beta_s \sin\left(j \frac{s\pi}{(n+1)} \right) e^{i\omega_s t}$$

- The frequency is given by

$$\omega_s = 2\sqrt{\frac{\tau}{md}} \sin\left(\frac{s\pi}{2(n+1)} \right)$$

4 Minute 47 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 47 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Chapter 13: Waves.

- Consider the following:
  - Increase the number of masses *n* to infinity.
  - Decrease the distance *d* to zero such that $(n + 1)d = L$.
  - Decrease the mass *m* to zero such that m/d = constant.
- In this case, the displacement *q* can be written as:

$$q_j(t) = \sum_s \beta_s \sin\left(s\pi \frac{x}{L}\right) e^{i\omega_s t} = q(x,t)$$

Wave solution.

- If the displacement and velocity at $t = 0$ are known, the constants in the expression for q can be determined.

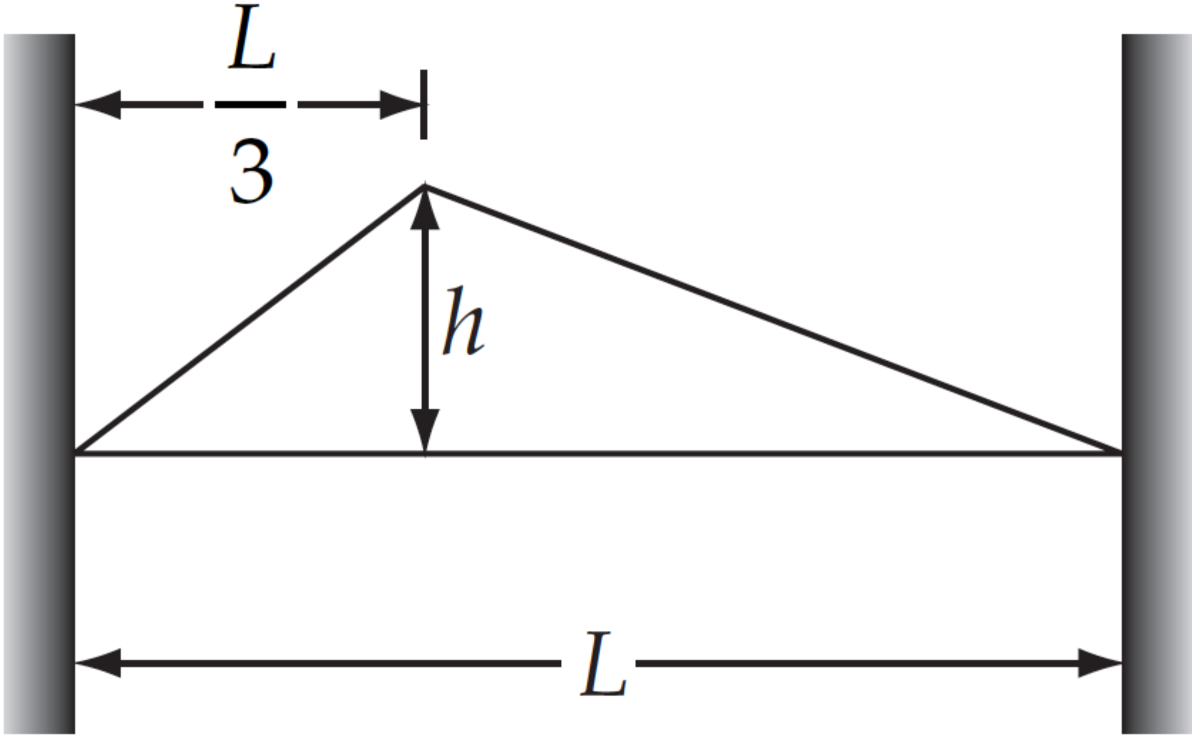
- In order to find these constants, we multiply each side by $\sin(r\pi x/L)$ and integrate x between 0 and L. We use the following fact:

$$\int_0^L \sin\left(s\pi\frac{x}{L}\right)\sin\left(r\pi\frac{x}{L}\right)dx =$$

$$= \frac{1}{2}\int_0^L \left\{\cos\left(\left(s-r\right)\pi\frac{x}{L}\right) - \cos\left(\left(s+r\right)\pi\frac{x}{L}\right)\right\}dx = \frac{L}{2}\delta_{rs}$$

Problem 13.2

- Rework the problem in Example 13.1 in the event that the plucked point is a distance $L/3$ from one end. Comment on the nature of the allowed modes.



ENOUGH FOR TODAY?