Classical Mechanics
Phy 235, Lecture 22.

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KLM.
Not only for transportation of people.

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Announcements

- No recitations and office hours this week.
- The due date of the Phy 235 term paper is Wednesday November 26 at noon:
  - You need to submit the draft and the final version in pdf format to the dropbox link that is posted on the PHY 235 web. Use the following naming convention: **FirstDraftPhy235XXYYYYYYYY.pdf** and **FinalPaperPhy235XXYYYYYYYY.pdf** where XX is your last name and YYYYYYYY is your student id number.
  - You need to discuss the draft with the writing center fellows, I need to receive a confirmation from them that they have discussed the draft with you, and I need to be able to see that you addressed their comments on your draft.
- Homework set # 10 is due on Friday December 5 at noon.

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Comments on Exam 3

- Problem 1:
  - Homework problem 2 of Homework Set # 7.
  - Average score: 20.6/25.
- Problem 2:
  - Discussed in class without the assumption that $m_1 = m_2$ (lecture 15 on 10/27).
  - Average score: 12.0/25.
- Problem 3:
  - Discussed in class (lecture 17 on 11/3).
  - Average score: 15.6/25.
- Problem 4:
  - Fly business class on KLM on a transcontinental flight..
  - Average score: 13.1/25.

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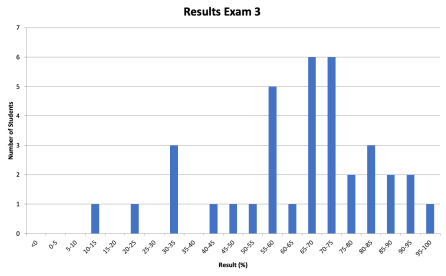
Exam # 3.



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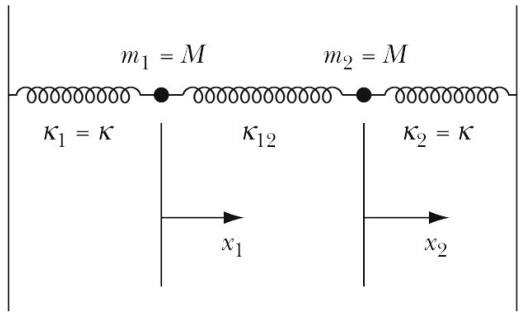
A quick review. Two Coupled Harmonic Oscillators.



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Two Coupled Harmonic Oscillators.

Two approaches:

- Approach 1:
  - Write down the coupled equations of motion.
  - Try trial functions for x_1 and x_2 with the same frequency.
  - The two frequency will have different amplitudes.
- Approach 2:
  - Cary out a coordinate transformation to decouple the coupled equations.
  - Solve each decoupled equation.
  - Each solution may have a different frequency.
  - Use the solutions of the decoupled equations and the "inverse" coordinate transformation to find the solution.

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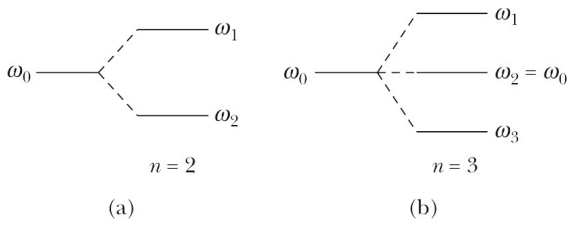
The frequencies compared to the natural frequencies.



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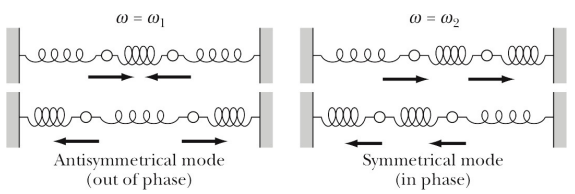
Two Coupled Harmonic Oscillators. Two modes.



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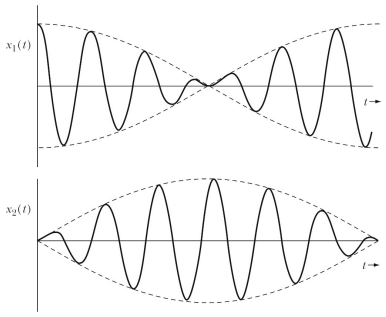
Weak Coupling.
$\omega_1 \approx \omega_2$



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N Coupled Oscillators

- We will have n coupled equations (A and m are the amplitude and mass tensors):

$$\sum_k \left(A_{kj} - \omega^2 m_{kj}\right) a_k = 0$$

- This set of equation will have non-trivial solutions if

$$\begin{vmatrix} A_{11}-\omega^2 m_{11} & A_{12}-\omega^2 m_{12} & A_{13}-\omega^2 m_{13} & \cdots\cdots \\ A_{12}-\omega^2 m_{12} & A_{22}-\omega^2 m_{22} & A_{23}-\omega^2 m_{23} & \cdots\cdots \\ A_{13}-\omega^2 m_{13} & A_{32}-\omega^2 m_{32} & A_{33}-\omega^2 m_{33} & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \end{vmatrix} = 0$$

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Steps

- Follow these steps in order to solve most coupled oscillator problems:
  - Choose generalized coordinates.
  - Determine the A and m tensors.
  - Determine the eigen frequency and the eigen vectors.
  - Determine the scale factors required to match the initial conditions.
  - Determine the normal coordinates.

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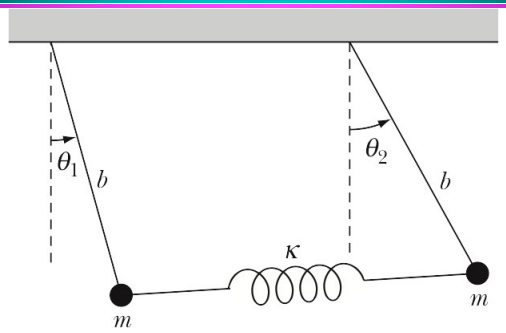
Example 12.4.



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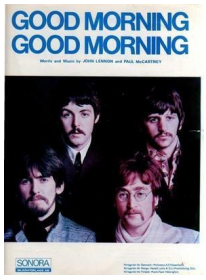
2 Minute 41 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 41 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



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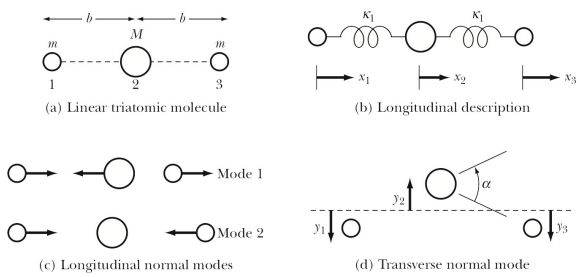
Molecular Vibrations



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Problem 12.21.

Three oscillators of equal mass m are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1\left(x_1^{\,2} + x_3^{\,2}\right) + \kappa_2 x_2^{\,2} + \kappa_3\left(x_1 x_2 + x_2 x_3\right)\right]$$

where

$$\kappa_3 = \sqrt{2\kappa_1\kappa_2}$$

Find the eigen frequencies by solving the secular equation. What is the physical interpretation of the zero-frequency mode?

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General steps to solve this type of problems.

- Find {A}.
- Solve secular determinant.

$$\{\mathbf{A}\} = \begin{bmatrix} \kappa_1 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 \end{bmatrix} \qquad \begin{vmatrix} \kappa_1 - m\omega^2 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 - m\omega^2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 - m\omega^2 \end{vmatrix} = 0$$

- Find {m}.

$$\{\mathbf{m}\} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

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I wish you call a happy and safe thanksgiving holiday.

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ENOUGH FOR TODAY?

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