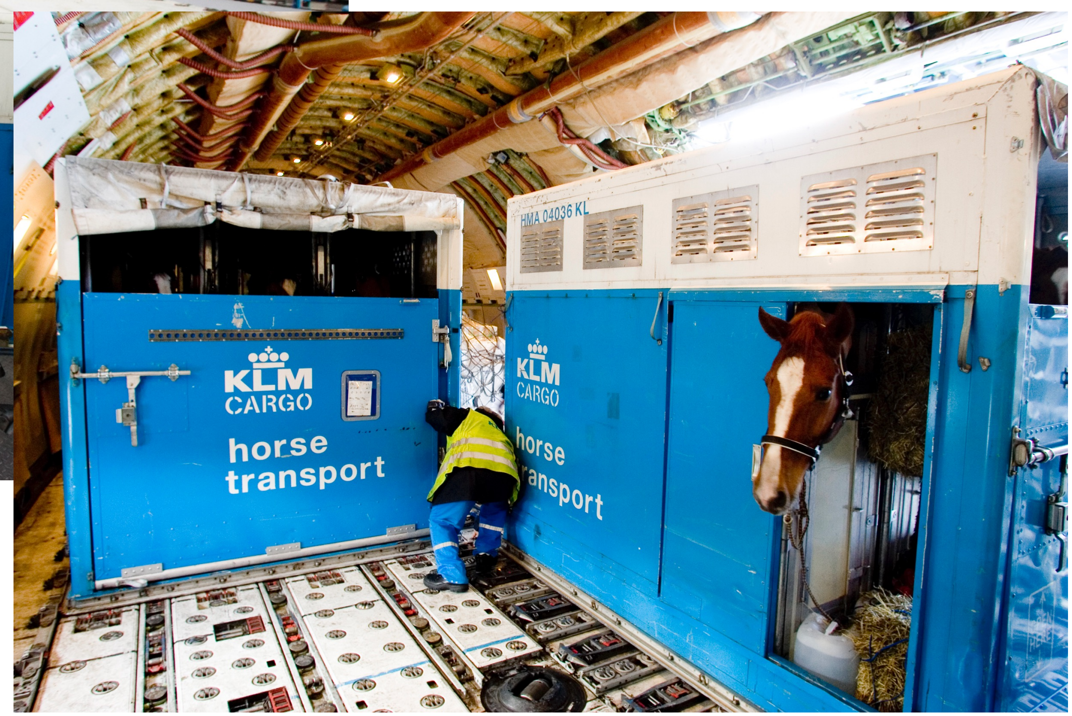
Classical Mechanics
Phy 235, Lecture 22.

Frank L. H. Wolfs
Department of Physics and Astronomy
University of Rochester

KLM.
Not only for transportation of people.



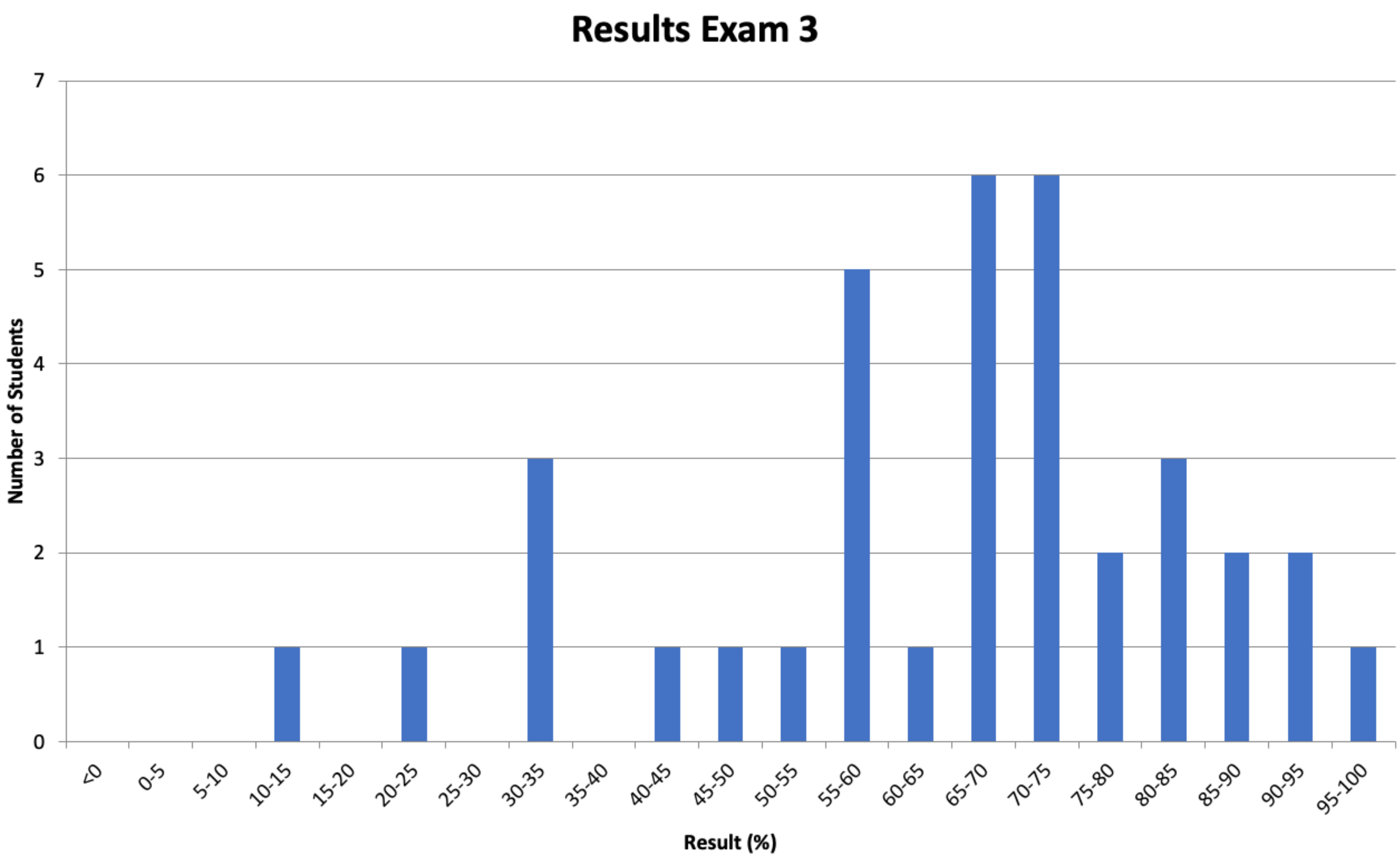
Announcements

- No recitations and office hours this week.
- The due date of the Phy 235 term paper is Wednesday November 26 at noon:
  - You need to submit the draft and the final version in pdf format to the dropbox link that is posted on the PHY 235 web. Use the following naming convention: **FirstDraftPhy235XXYYYYYYYY.pdf** and **FinalPaperPhy235XXYYYYYYYY.pdf** where XX is your last name and YYYYYYYY is your student id number.
  - You need to discuss the draft with the writing center fellows, I need to receive a confirmation from them that they have discussed the draft with you, and I need to be able to see that you addressed their comments on your draft.
- Homework set # 10 is due on Friday December 5 at noon.

Comments on Exam 3

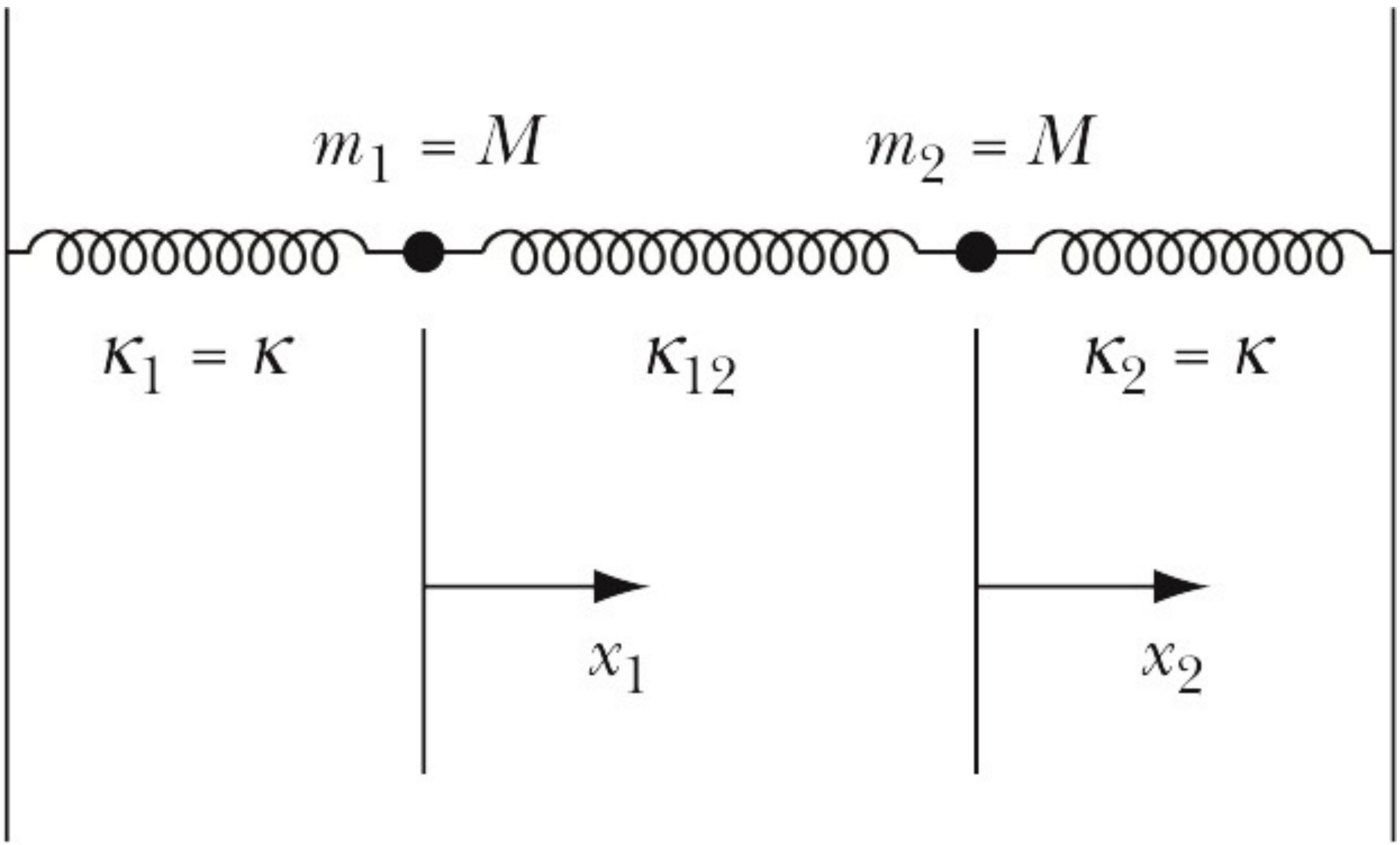
- Problem 1:
  - Homework problem 2 of Homework Set # 7.
  - Average score: 20.6/25.
- Problem 2:
  - Discussed in class without the assumption that $m_1 = m_2$ (lecture 15 on 10/27).
  - Average score: 12.0/25.
- Problem 3:
  - Discussed in class (lecture 17 on 11/3).
  - Average score: 15.6/25.
- Problem 4:
  - Fly business class on KLM on a transcontinental flight..
  - Average score: 13.1/25.

Exam # 3.



Results Exam 3
Number of Students
0
1
2
3
4
5
6
7
<0
0-5
5-10
10-15
15-20
20-25
25-30
30-35
35-40
40-45
45-50
50-55
55-60
60-65
65-70
70-75
75-80
80-85
85-90
90-95
95-100
Result (%)

A quick review.
Two Coupled Harmonic Oscillators.

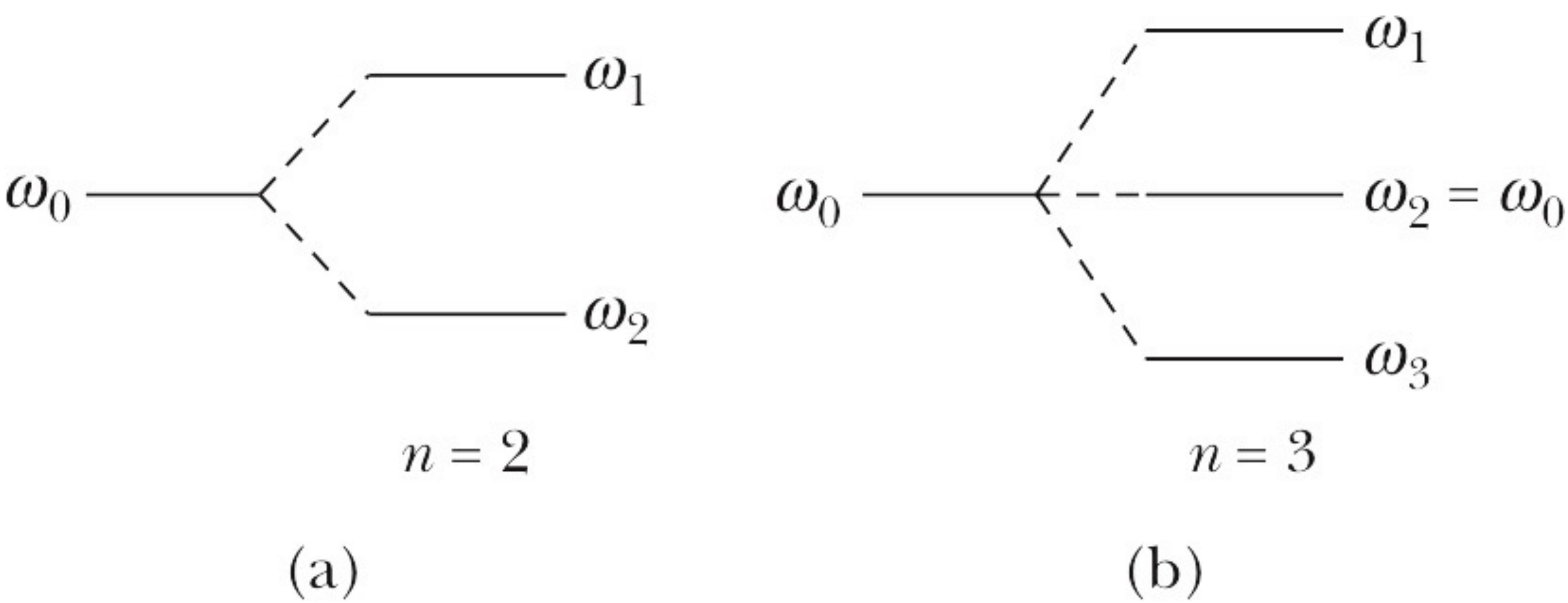


Two Coupled Harmonic Oscillators.

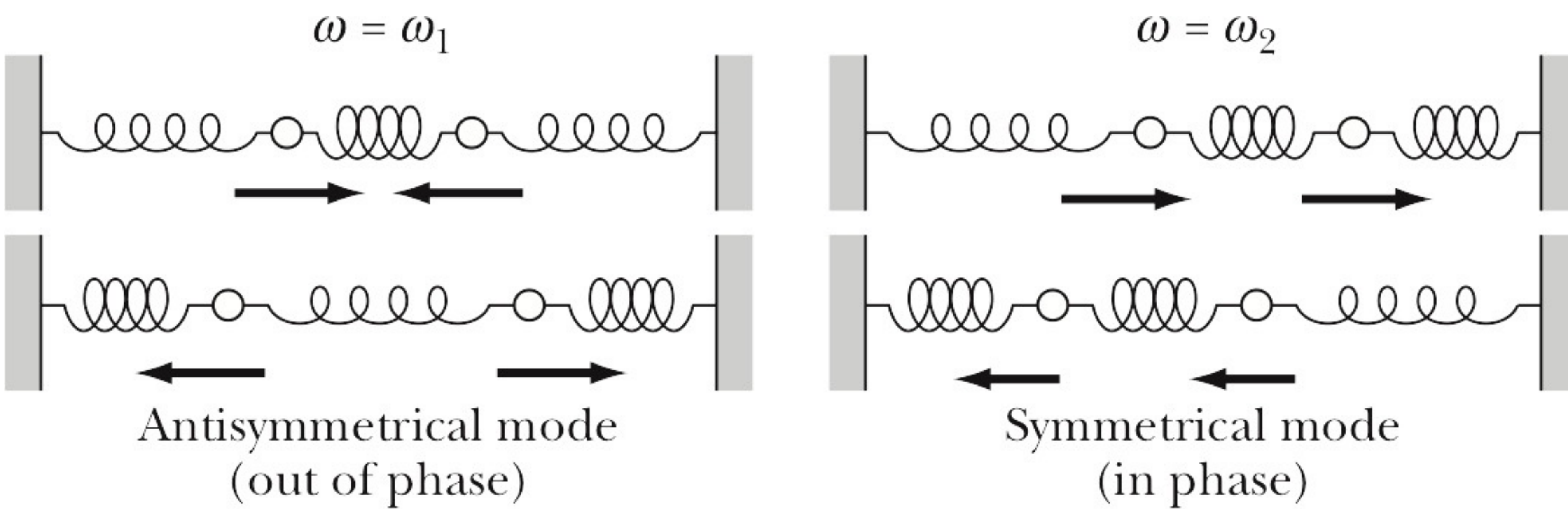
Two approaches:

- Approach 1:
  - Write down the coupled equations of motion.
  - Try trial functions for x_1 and x_2 with the same frequency.
  - The two frequency will have different amplitudes.
- Approach 2:
  - Cary out a coordinate transformation to decouple the coupled equations.
  - Solve each decoupled equation.
  - Each solution may have a different frequency.
  - Use the solutions of the decoupled equations and the "inverse" coordinate transformation to find the solution.

The frequencies compared to the natural frequencies.

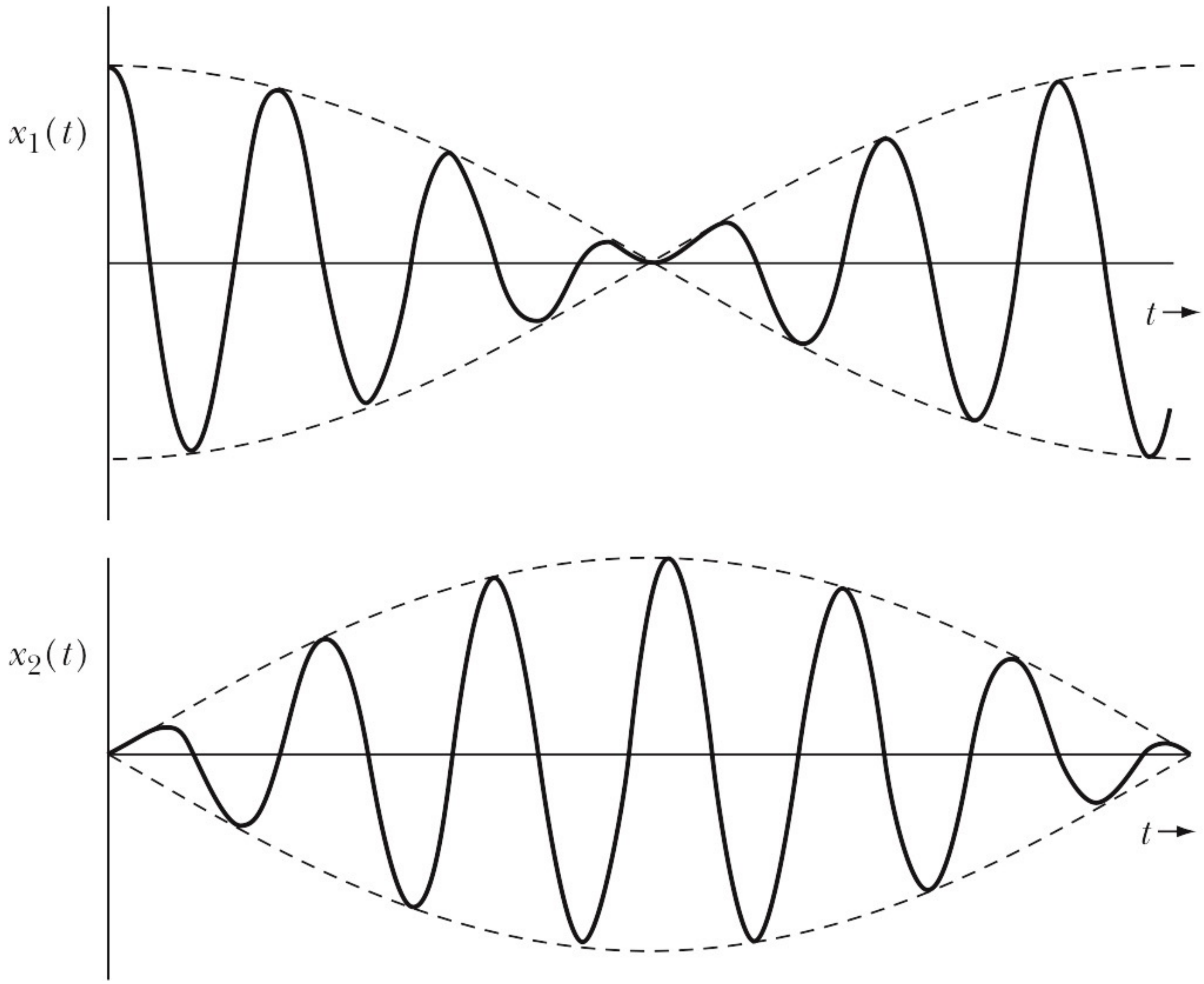


Two Coupled Harmonic Oscillators. Two modes.



Weak Coupling.

$\omega_1 \approx \omega_2$



N Coupled Oscillators

- We will have n coupled equations (A and m are the amplitude and mass tensors):

$$\sum_k \left(A_{kj} - \omega^2 m_{kj} \right) a_k = 0$$

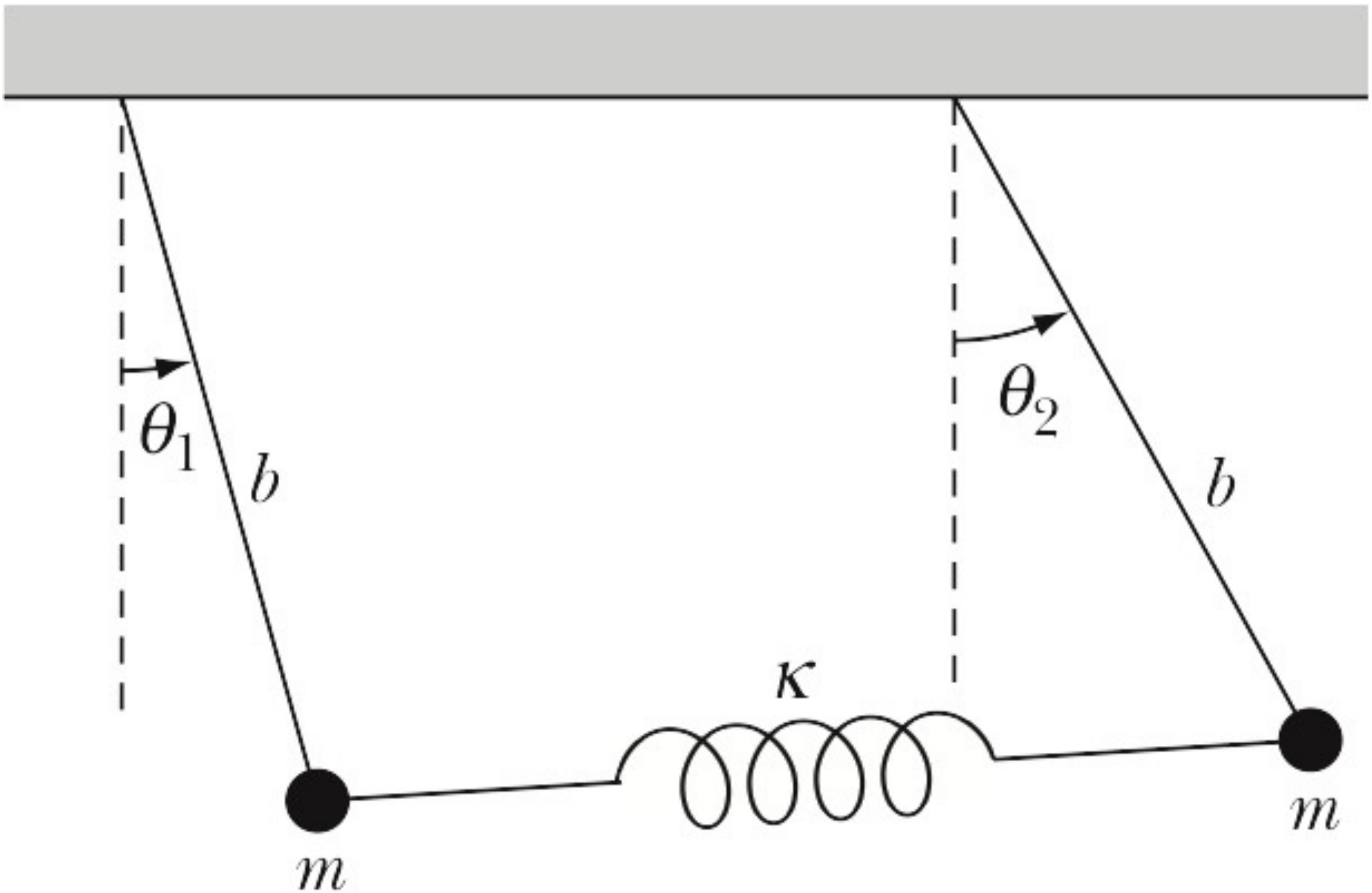
- This set of equation will have non-trivial solutions if

$$\begin{vmatrix} A_{11} - \omega^2 m_{11} & A_{12} - \omega^2 m_{12} & A_{13} - \omega^2 m_{13} & \ldots\ldots \\ A_{12} - \omega^2 m_{12} & A_{22} - \omega^2 m_{22} & A_{23} - \omega^2 m_{23} & \ldots\ldots \\ A_{13} - \omega^2 m_{13} & A_{32} - \omega^2 m_{32} & A_{33} - \omega^2 m_{33} & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{vmatrix} = 0$$

Steps

- Follow these steps in order to solve most coupled oscillator problems:
  - Choose generalized coordinates.
  - Determine the A and m tensors.
  - Determine the eigen frequency and the eigen vectors.
  - Determine the scale factors required to match the initial conditions.
  - Determine the normal coordinates.

Example 12.4.

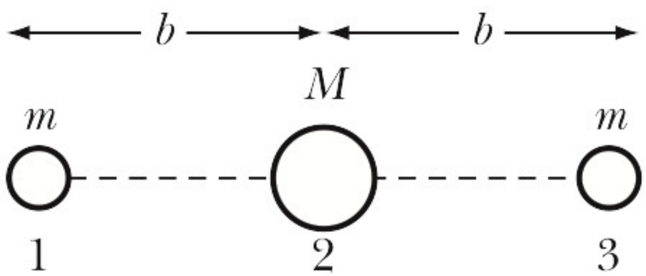


2 Minute 41 Second Intermission.

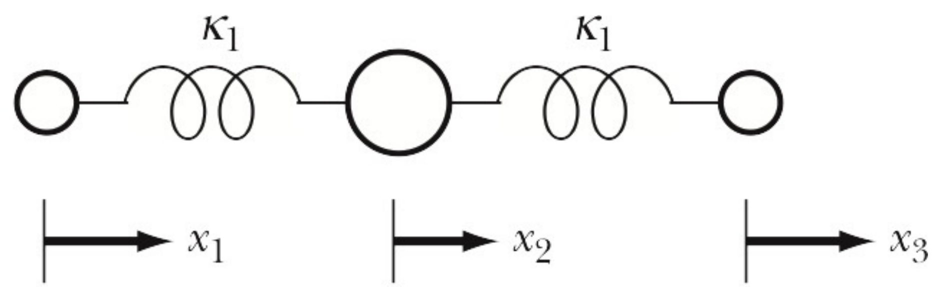
- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 41 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

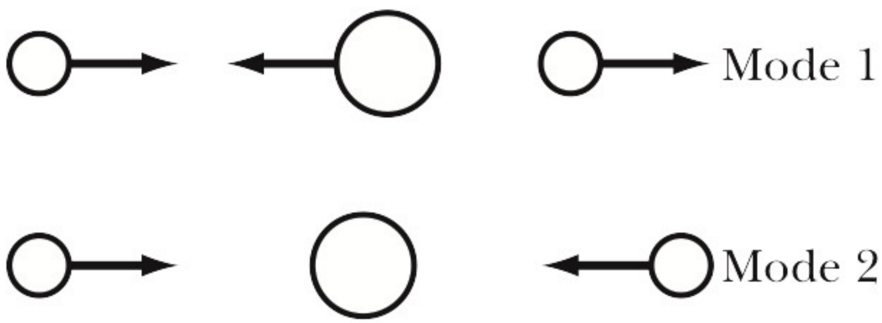
Molecular Vibrations



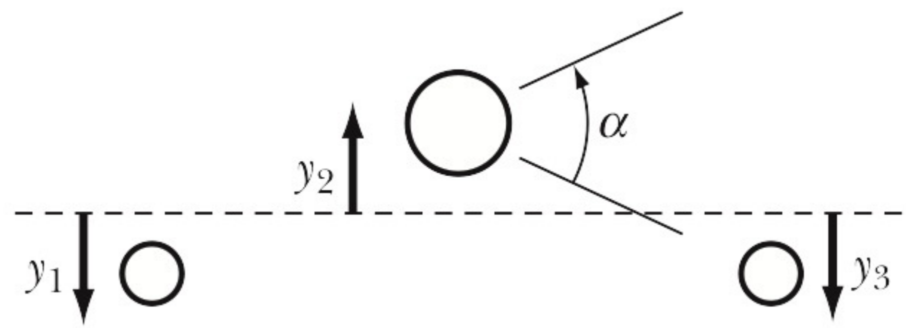
(a) Linear triatomic molecule



(b) Longitudinal description



(c) Longitudinal normal modes



(d) Transverse normal mode

Problem 12.21.

Three oscillators of equal mass m are coupled such that the potential energy of the system is given by

$$U = \frac{1}{2}\left[\kappa_1\left(x_1{}^2 + x_3{}^2\right) + \kappa_2 x_2{}^2 + \kappa_3\left(x_1 x_2 + x_2 x_3\right)\right]$$

where

$$\kappa_3 = \sqrt{2\kappa_1\kappa_2}$$

Find the eigen frequencies by solving the secular equation. What is the physical interpretation of the zero-frequency mode?

General steps to solve this type of problems.

- Find {A}.

$$\{\mathbf{A}\} = \begin{bmatrix} \kappa_1 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 \end{bmatrix}$$

- Find {m}.

$$\{\mathbf{m}\} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

- Solve secular determinant.

$$\begin{vmatrix} \kappa_1 - m\omega^2 & \frac{1}{2}\kappa_3 & 0 \\ \frac{1}{2}\kappa_3 & \kappa_2 - m\omega^2 & \frac{1}{2}\kappa_3 \\ 0 & \frac{1}{2}\kappa_3 & \kappa_1 - m\omega^2 \end{vmatrix} = 0$$

I wish you call a happy and safe thanksgiving holiday.

ENOUGH FOR TODAY?