Classical Mechanics
Phy 235, Lecture 21.

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Course Announcements.

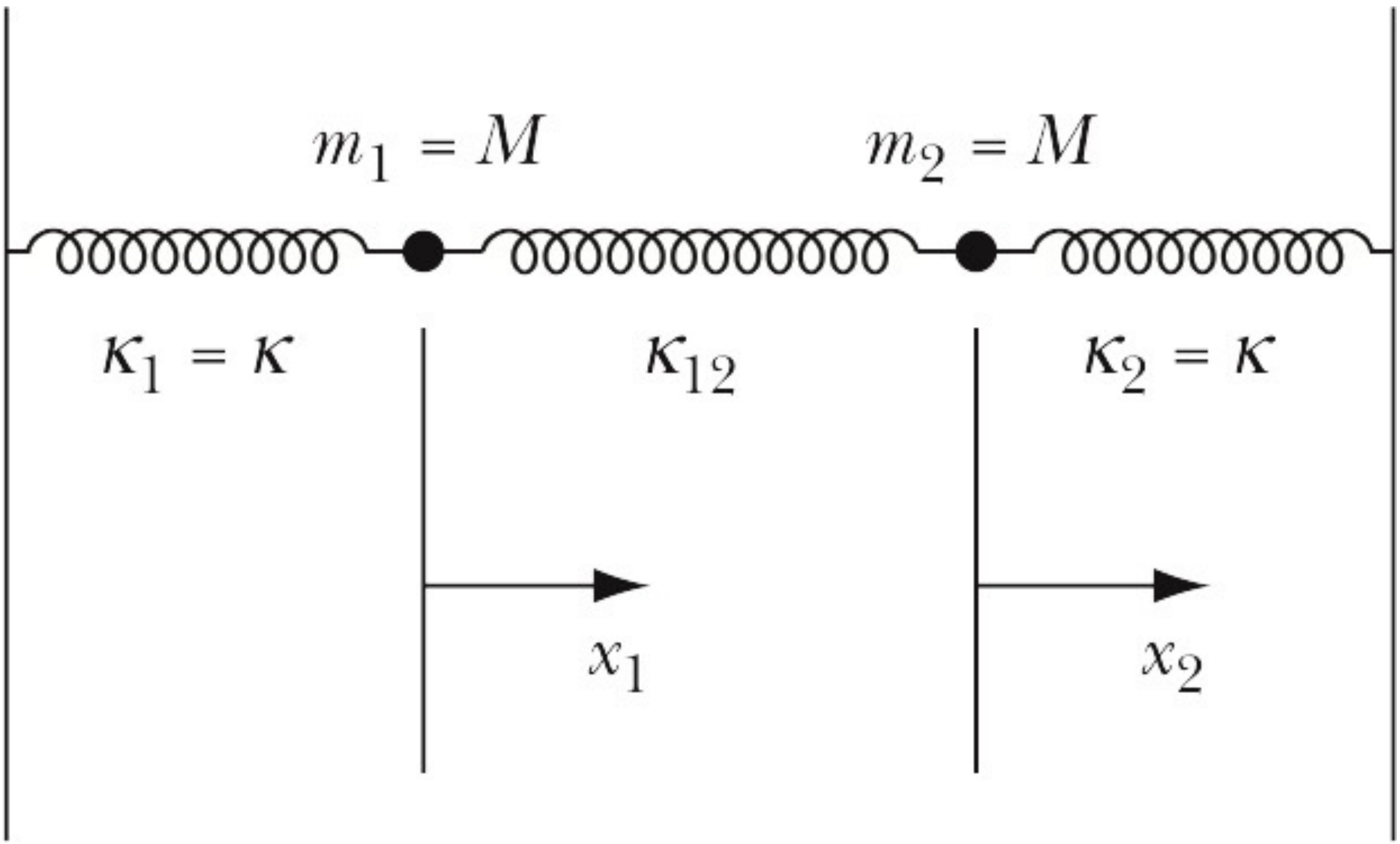
- There will be no office hours and recitations next week (the week of Thanksgiving) but there will be lecture on Monday November 24.
- The term paper is due on Wednesday November 26. You need to submit your final paper and the draft discussed with the writing fellows. I expect to receive a confirmation from the writing fellows that they discussed the original draft with you and their observations.

Chapter 12.
Coupled Oscillations

- Coupled oscillators:
  - Oscillators that are connected in such a way that energy can be transferred between them.
  - The motion of coupled oscillators is complex and in general not periodic.
  - We can always find a coordinate frame in which each oscillator oscillated with a well-defined frequency.

Coupled motion.

Two Coupled Harmonic Oscillators.



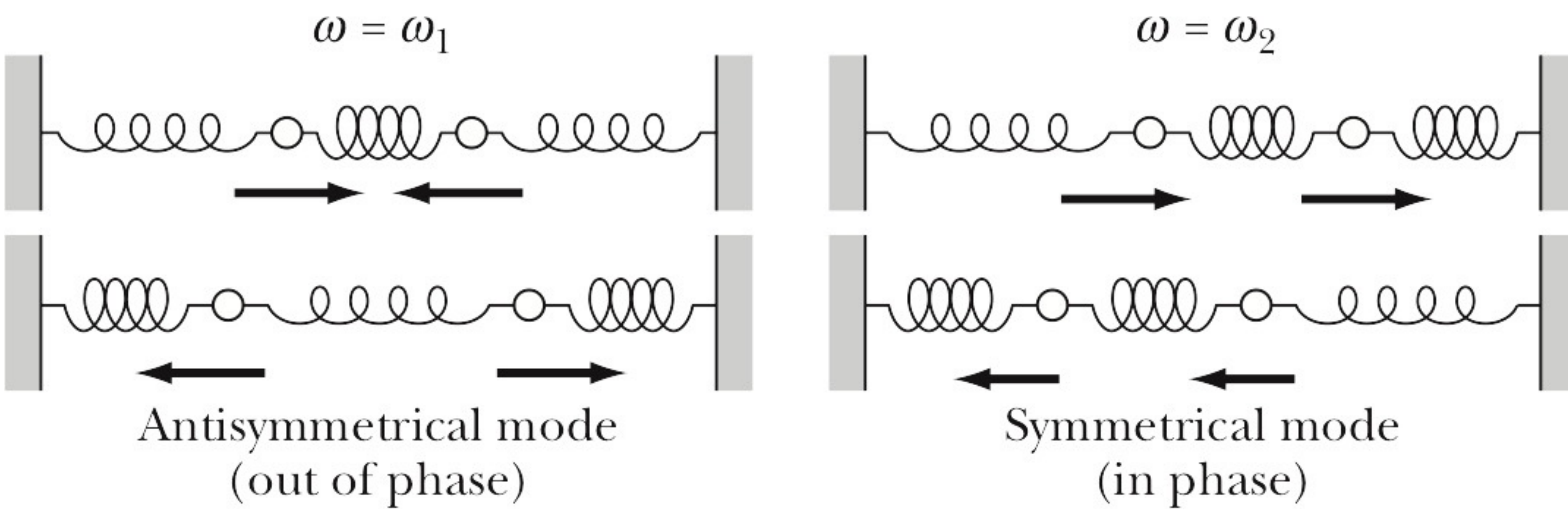
Two Coupled Harmonic Oscillators.

- Two approaches:

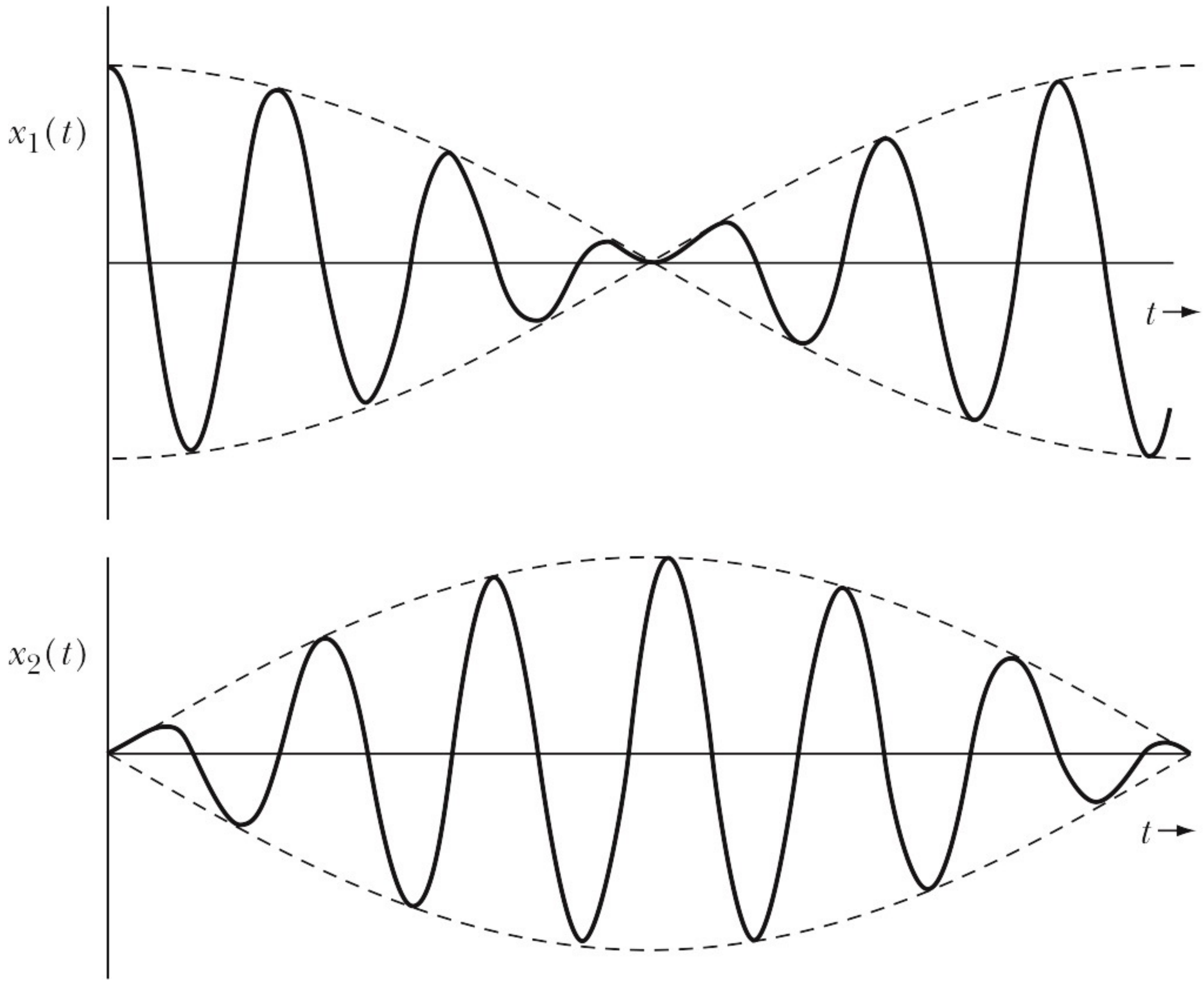
- Approach 1:
  - Write down the coupled equations of motion.
  - Try trial functions for x_1 and x_2 with the same frequency.
  - The two frequency will have different amplitudes.

- Approach 2:
  - Cary out a coordinate transformation to decouple the coupled equations.
  - Solve each decoupled equation.
  - Each solution may have a different frequency.
  - Use the solutions of the decoupled equations and the “inverse” coordinate transformation to find the solution.

Two Coupled Harmonic Oscillators. Two modes.

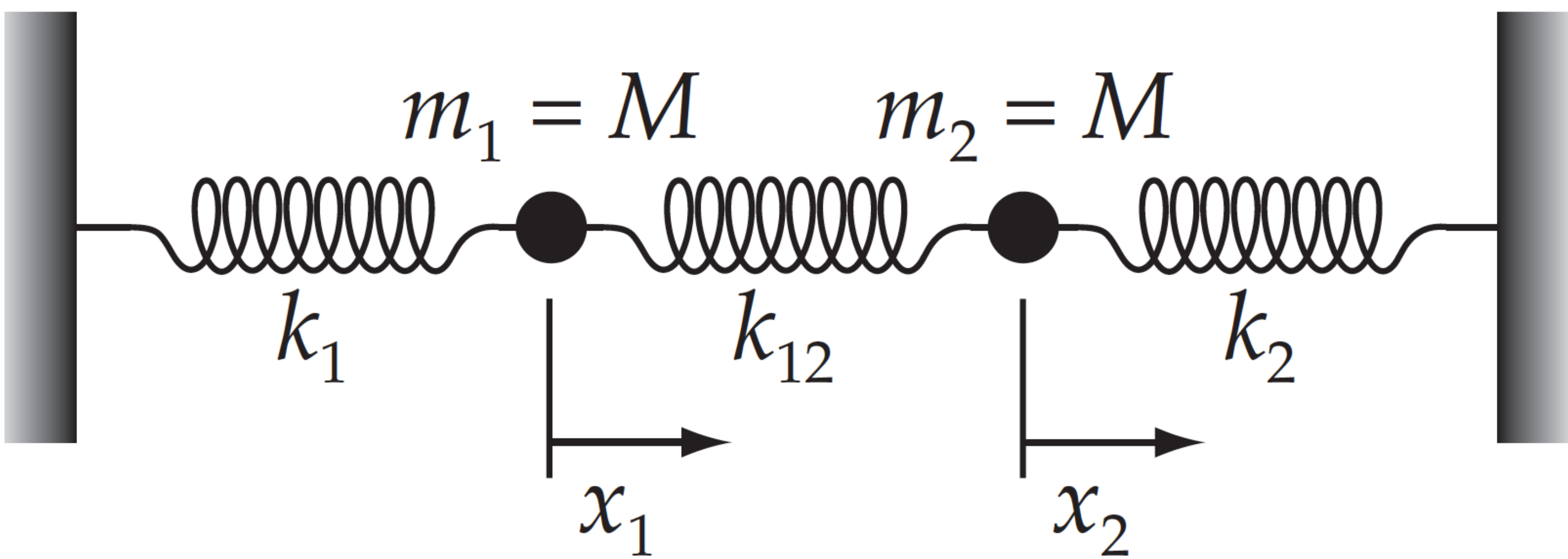


Weak Coupling



Problem 12.1.

Reconsider the problem of two coupled oscillators discussion in Section 12.2 in the event that the three springs all have different force constants. Find the two characteristic frequencies, and compare the magnitudes with the natural frequencies of the two oscillators in the absence of coupling.



2 Minute 59 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 59 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Problem 12.3.

Two identical harmonic oscillators (with masses M and natural frequencies ω_0) are coupled such that by adding to the system a mass m, common to both oscillators, the equations of motion become

$$\ddot{x}_1 + \frac{m}{M}\ddot{x}_2 + \omega_0^2 x_1 = 0$$
$$\ddot{x}_2 + \frac{m}{M}\ddot{x}_1 + \omega_0^2 x_2 = 0$$

Solve this pair of coupled equations and obtain the frequencies of the normal modes of the system.

N Coupled Oscillators

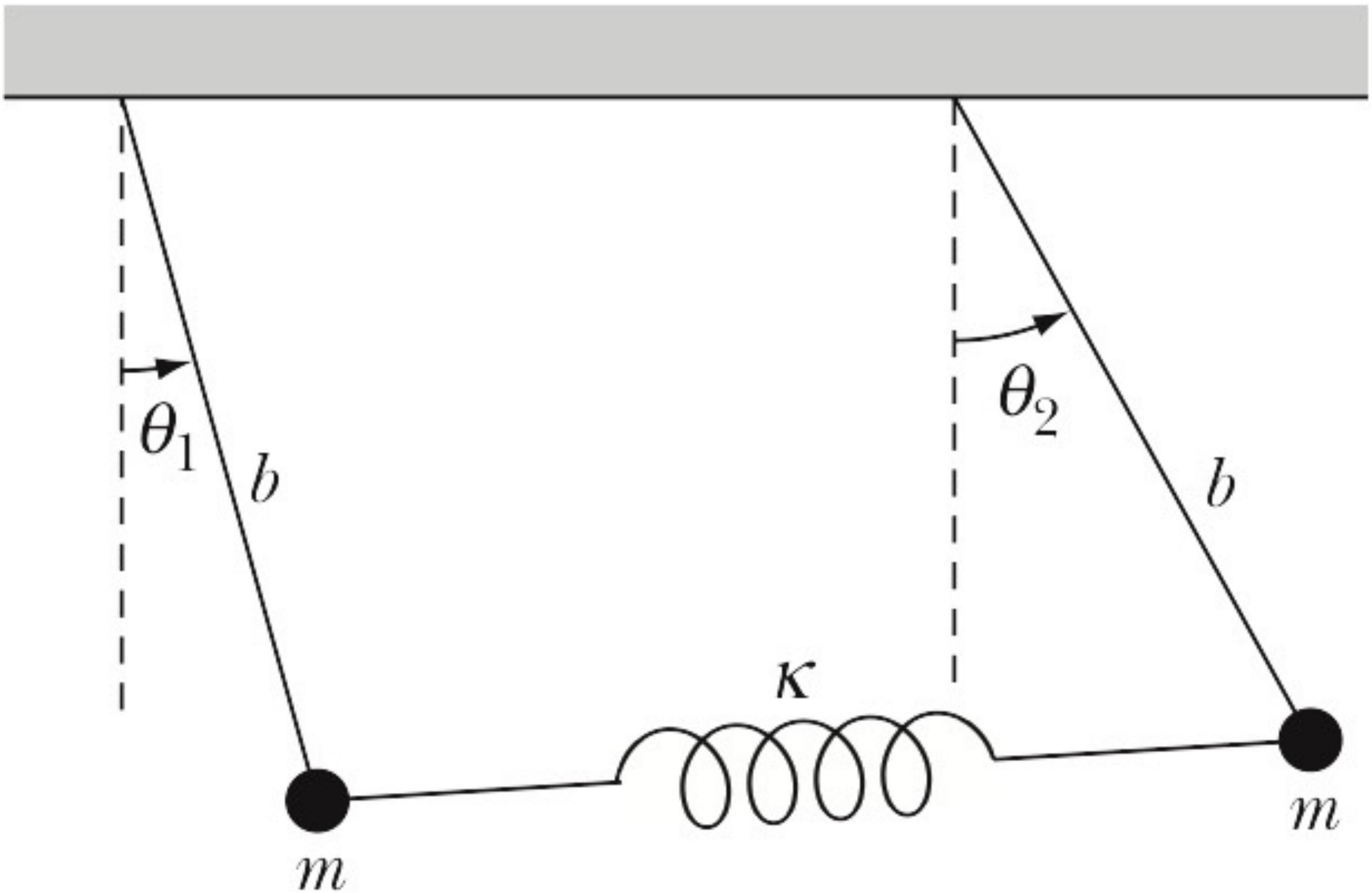
- We will have n coupled equations (A and m are the amplitude and mass tensors):

$$\sum_k \left(A_{kj} - \omega^2 m_{kj}\right) a_k = 0$$

- This set of equation will have non-trivial solutions if

$$\begin{vmatrix} A_{11} - \omega^2 m_{11} & A_{12} - \omega^2 m_{12} & .... \\ A_{12} - \omega^2 m_{12} & A_{22} - \omega^2 m_{22} & .... \\ .... & .... & .... \end{vmatrix} = 0$$

Example 12.4.



Steps

- Follow these steps in order to solve most coupled oscillator problems:
  - Choose generalized coordinates.
  - Determine the A and m tensors.
  - Determine the eigen frequency and the eigen vectors.
  - Determine the scale factors required to match the initial conditions.
  - Determine the normal coordinates.

ENOUGH FOR TODAY?