Classical Mechanics
Phy 235, Lecture 17.

Frank L. H. Wolfs
Department of Physics and Astronomy
University of Rochester

1

Amazing …… classical mechanics at work.

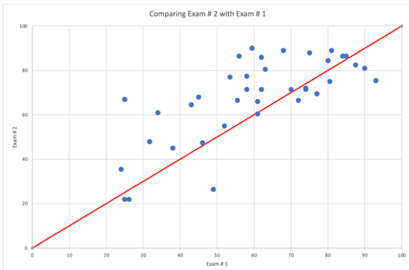
2

Course Announcements.

- Exam # 3 will take place on Tuesday November 18, 8.00 am – 9.20 am in B&L 109.
- The material covered on Exam # 3 includes Chapters 8 – 10.
- Any requests for regrades for specific problems on Exam # 2 should be made by Wednesday November 5 (end of lecture). I will need the following:
  - Your blue book(s).
  - A written explanation why you feel you deserve more points.

3

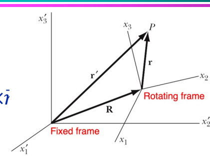
Give a physicist some numbers, he will analyze them!



4

A quick summary.

Results from Wednesday:



$$(d\vec{r})_{fixed} = d\vec{\theta}\times\vec{r}$$

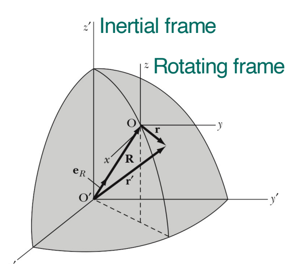
$$\vec{v}_f = \left(\frac{d\vec{r}'}{dt}\right)_{fixed} = \left(\frac{d\vec{R}}{dt}\right)_{fixed} + \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\vec{r} = \vec{V} + \vec{v}_r + \vec{\omega}\times\vec{r}$$

$$\vec{a}_{fixed} = \vec{a}_{rot} + \left(\frac{d\vec{V}}{dt}\right)_{fixed} + 2\vec{\omega}\times\vec{v}_r + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

5

The centripetal force.

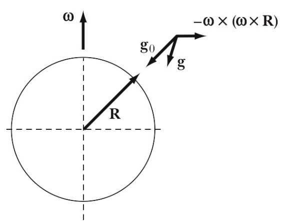
- The Earth is not a good inertial reference frame.
- The biggest "non-inertial" effect is due to the daily rotation around its axis.
- We use a rotating *xyz* frame, fixed on the surface of the Earth, and a fixed inertial reference frame *x'y'z'* whose origin is located at the center of the Earth.



6

The centripetal force.

- Consider a pendulum at rest in our reference frame on the surface of the Earth.
- In the inertial reference frame we would measure a gravitational acceleration g_0.
- In the rotating reference frame (our laboratory on Earth) we measure an acceleration g.



7

Force on a pendulum.

- The effective force on our pendulum is

$$\vec{F}_{eff} = m\vec{a}_f - m\dot{\vec{\omega}}\times\vec{r} - 2m\vec{\omega}\times\vec{v}_r - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

- Since the pendulum is at rest in our rotation reference frame, the velocity $v_r = 0$ m/s.
- The gravitational acceleration we measure in our Earth frame is equal to

$$\vec{g} = \vec{g}_0 - \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

Correction due to rotation.
Note: correction is position dependent.

8

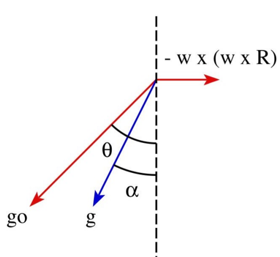
Force on a pendulum.

- The effect of the rotation of the Earth is a change in the equilibrium angle of the pendulum.
- The change in angle is equal to

$$\Delta\theta = \theta - \alpha$$

where

$$\alpha = \operatorname{atan}\left(\left(1 - \frac{\omega^2 R}{g_0}\right)\tan\theta\right)$$

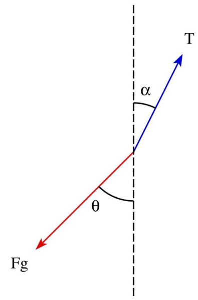


9

Force on a pendulum. As viewed in the fixed reference frame.

- We could have viewed this problem also from our fixed reference frame.
- In this reference frame, the mass is rotating, and there must thus be a net force with magnitude mv^2/r acting on it:

$$F_r = m\frac{v^2}{R\sin\theta} = m\omega^2 R\sin\theta$$



Frank L. H. Wolfs — Department of Physics and Astronomy, University of Rochester, Slide 10

10

Force on a pendulum. As viewed in the fixed reference frame.

- The net force is provided by the horizontal components of the tension T and the gravitational force:

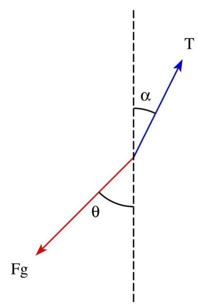
$$mg_0\sin\theta - T\sin\alpha = m\omega^2 R\sin\theta$$

- In the vertical direction, the net force must be 0:

$$T\cos\alpha = mg\cos\theta$$

- The angle α can now be determined:

$$\tan\alpha = \frac{T\sin\alpha}{T\cos\alpha} = \left(1 - \frac{\omega^2 R}{g_0}\right)\tan\theta$$

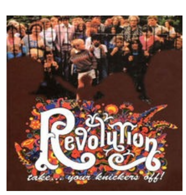


Frank L. H. Wolfs — Department of Physics and Astronomy, University of Rochester, Slide 11

11

3 Minute 23 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 23 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



Frank L. H. Wolfs — Department of Physics and Astronomy, University of Rochester, Slide 12

12

One more comment on Chapter 10. The effective force.

Coriolis force.

$$\bar{F}_{eff} = m\bar{a}_f - m\dot{\bar{\omega}} \times \bar{r} - 2m\bar{\omega} \times \bar{v}_r - m\bar{\omega} \times \{\bar{\omega} \times \bar{r}\}$$

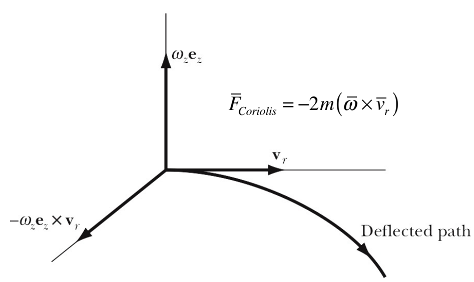
Centripetal force.

- If $v_r = 0$, the centripetal force dominates.
- If $v_r > 0$, the Coriolis force may dominate. Where do we draw the line?
  - On the equator: $|\bar{\omega} \times (\bar{\omega} \times \bar{r})| = r\omega^2 = 3.4 \times 10^{-2}$ m/s^2
  - If we look at motion perpendicular to the angular velocity, the Coriolis acceleration will be: $2|\bar{\omega} \times \bar{v}_r| = 2\omega v_r$
  - Both corrections will have the same magnitude when the velocity is 830 km/h (520 mph).

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Slide 13

13

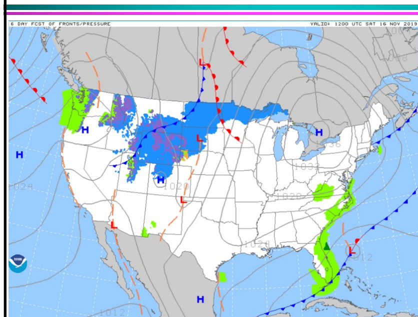
Effects of the Coriolis Force.



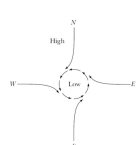
Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Slide 14

14

Air flowing West to East: deflected South. Air flowing East to West: deflect North.



Note: only applies to the Northern hemisphere.



Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Slide 15

15

Problem 10.20.

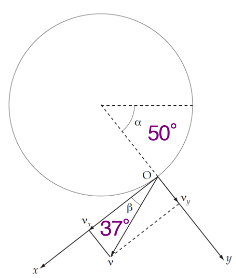
- Calculate the effective gravitational acceleration g at the Earth's surface at the pole and at the equator. Take into account the difference in the equatorial (6378 km) and polar (6357) radius as well as the centrifugal force.

Frank L. H. Wolfs — Department of Physics and Astronomy, University of Rochester, Slide 16

16

Problem 10.18.

A British warship fires a projectile due south near the Falkland Islands during World War I at latitude 50° South. If the shells are fired at 37° elevation with a speed of 800 m/s, by how much do the shells miss their target and in what direction?



Frank L. H. Wolfs — Department of Physics and Astronomy, University of Rochester, Slide 17

17

ENOUGH FOR TODAY?

Frank L. H. Wolfs — Department of Physics and Astronomy, University of Rochester, Slide 18

18