
Classical Mechanics

Phy 235, Lecture 13.

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My fall break.



Course Comments

- Review of the material covered on Exam 2 is scheduled for Monday (10/20).
- The next homework set is due on Friday 10/17. Extra credit homework # 2 is due on Friday 10/14.
- Details about the term paper can be found on the web at:
<http://teacher.pas.rochester.edu/PHY235/CourseInformation/TermPaper.htm>
- Note:
 - 42 days until your paper is due!!!
 - Before that: 1) you need to submit a proposal and 2) you need to review your draft with the writing fellows. Now is the time to schedule your meeting!!!

Two-Body Central-Force Motion.

- For two-body central force problems, we found:
 - The problem can be reduced to a one-body problem.
 - Angular momentum is a conserved quantity.
 - Kepler's second law is a direct consequence of conservation of angular momentum.
 - Since the Lagrangian does not depend explicitly on time, energy is conserved.

$$\begin{aligned} E &= T + U = \frac{1}{2} \mu (\dot{r}^2 + r^2 \dot{\theta}^2) + U(r) = \frac{1}{2} \mu \left(\dot{r}^2 + r^2 \left(\frac{l}{\mu r^2} \right)^2 \right) + U(r) = \\ &= \frac{1}{2} \mu \dot{r}^2 + \frac{1}{2} \frac{l^2}{\mu r^2} + U(r) \end{aligned}$$

↑
Modification to the potential energy.

Orbital motion: orbital properties.

- Properties of the orbits can be found by solving the following integrals:

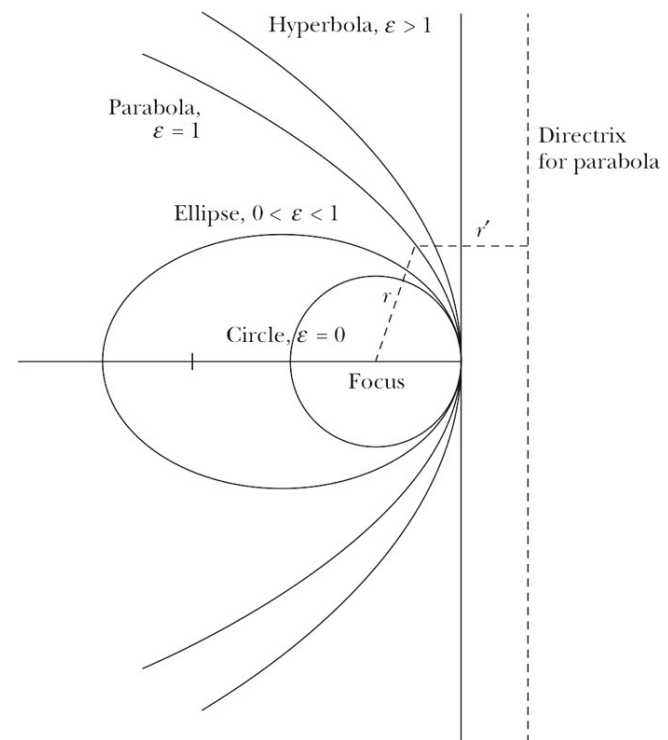
$$t = \int dt = \pm \int \frac{1}{\sqrt{\frac{2}{\mu}(E - U(r)) - \frac{\ell^2}{\mu^2 r^2}}} dr$$

$$\theta(r) = \int \frac{\dot{\theta}}{\dot{r}} dr = \pm \int \frac{\ell}{r^2 \sqrt{2\mu \left(E - U(r) - \frac{\ell^2}{2\mu r^2} \right)}} dr$$

Orbital motion: orbital properties (shape).

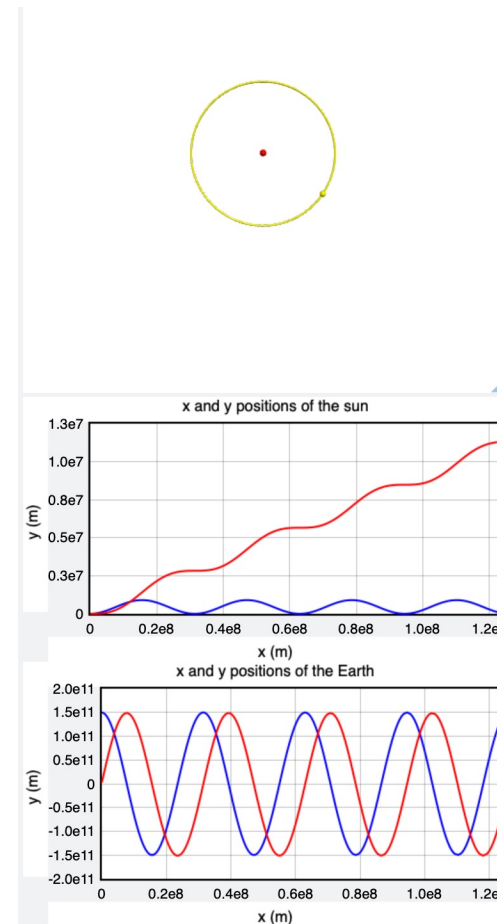
- Properties:

- $E > 0$: Hyperbola
- $E = 0$: Parabola
- $V_{min} < E < 0$: Ellipse
- $E = V_{min}$: Circle



Simulations are a good way to study orbital motion.

- The following links point towards two interesting simulations of orbital motion using glowscript.
- Sun-Earth system:
 - <https://www.glowscript.org/#/user/wolfs/folder/Public/program/Phy235-OrbitalMotion>
- Double star system:
 - <https://www.glowscript.org/#/user/wolfs/folder/Public/program/Phy235-DoubleStarSystem>



Orbital motion: orbital properties (period).

- To find the period of the orbit we use:

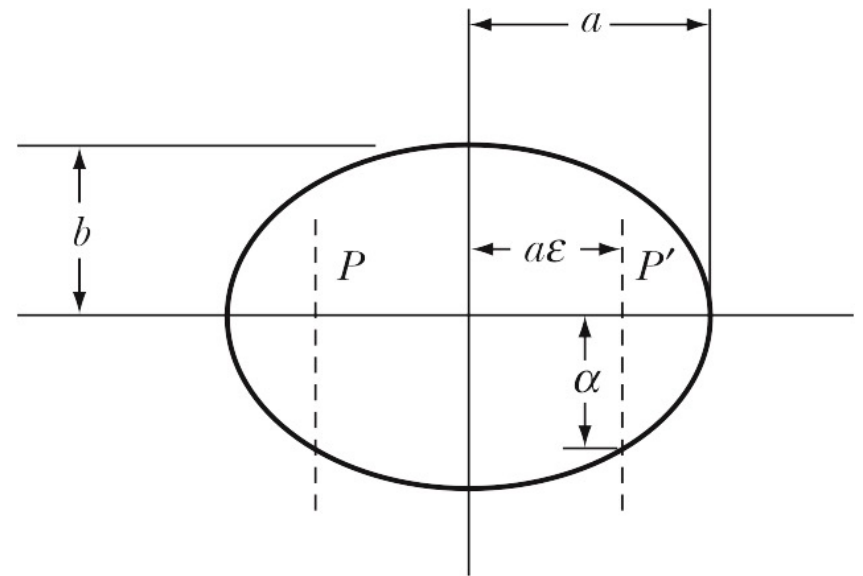
- The known area velocity:

$$dt = \left(\frac{2\mu}{\ell}\right) dA.$$

- The known area of an ellipse: πab .

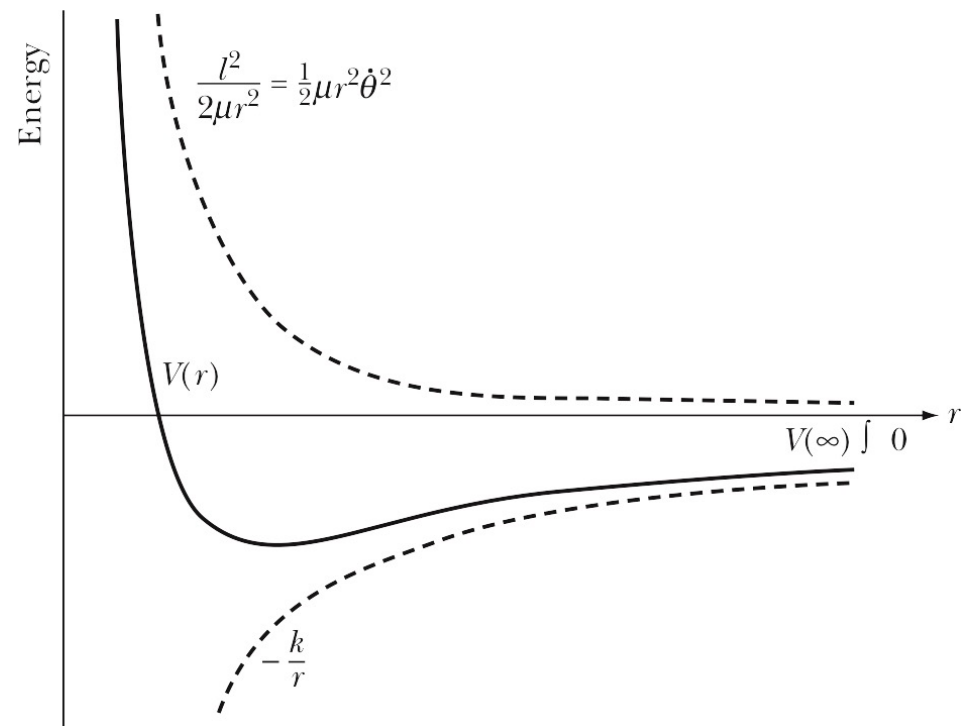
- We can now determine the orbital period:

$$\tau = \int dt = \frac{2\mu}{\ell} \int dA = \frac{2\mu}{\ell} (\pi ab) = \frac{2\mu}{\ell} \left(\pi \frac{k}{2|E|} \frac{\ell}{\sqrt{2\mu|E|}} \right) = \pi k \sqrt{\frac{\mu}{2}} |E|^{-3/2}$$



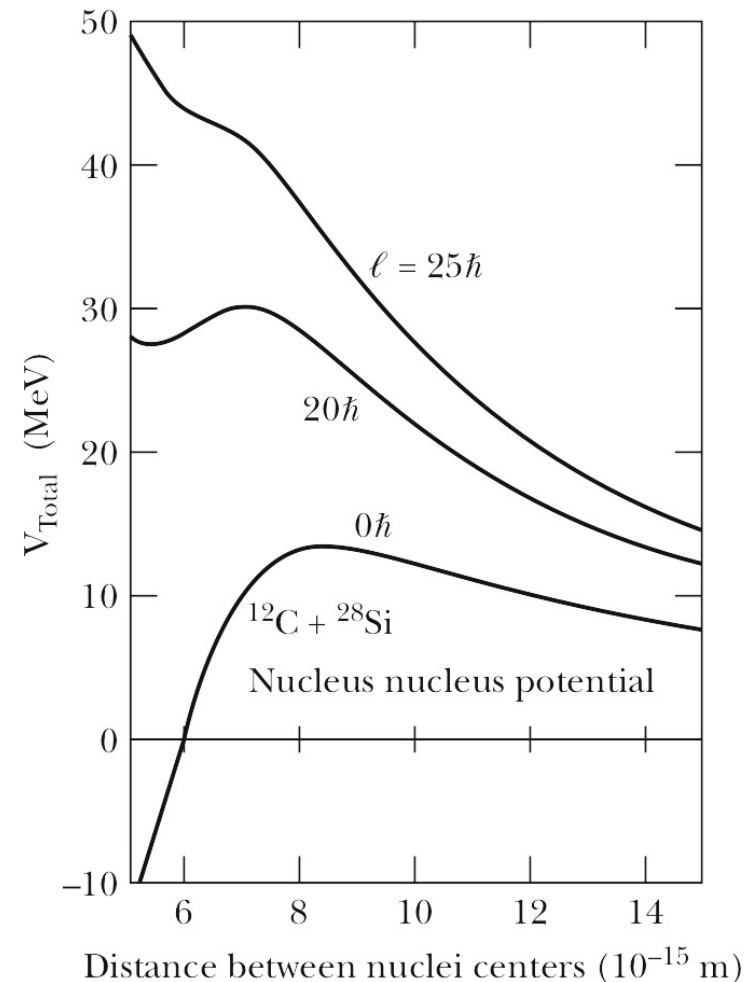
The “effective” potential.

- The effective potential is composed of the real potential and the centrifugal potential energy.
- Observations:
 - The effective potential may show a dip that indicates that for certain energies, the orbit is bound.
 - For small distances, the effective force becomes repulsive.



The “effective” potential for nuclear reactions.

- The effective potential for nuclear reactions includes the nuclear potential, the Coulomb potential, and the centrifugal potential.
- The dynamics of nuclear interactions can be understood if we look at the effective potential:
 - If $\ell = 0$, nuclear fusion can occur if $E > 15$ MeV.
 - For large ℓ , the centrifugal barrier prevents nuclear fusion.





2 Minute 36 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 36 second intermission.
- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.



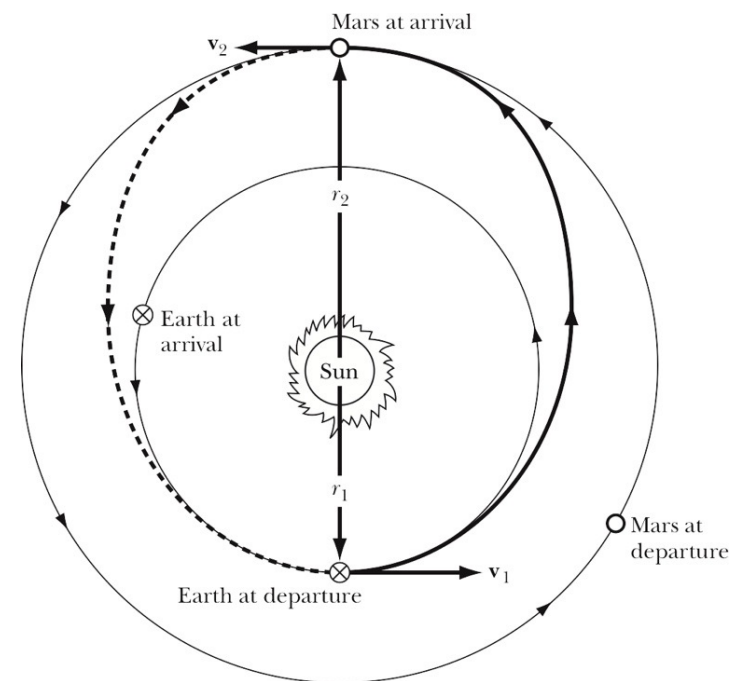
Problem 8.22

- Discuss the motion of a particle moving in an attractive central-force field described by $F(r) = -k/r^3$. Sketch some of the orbits for different values of the total energy.

Orbital motion.

Travelling to Mars.

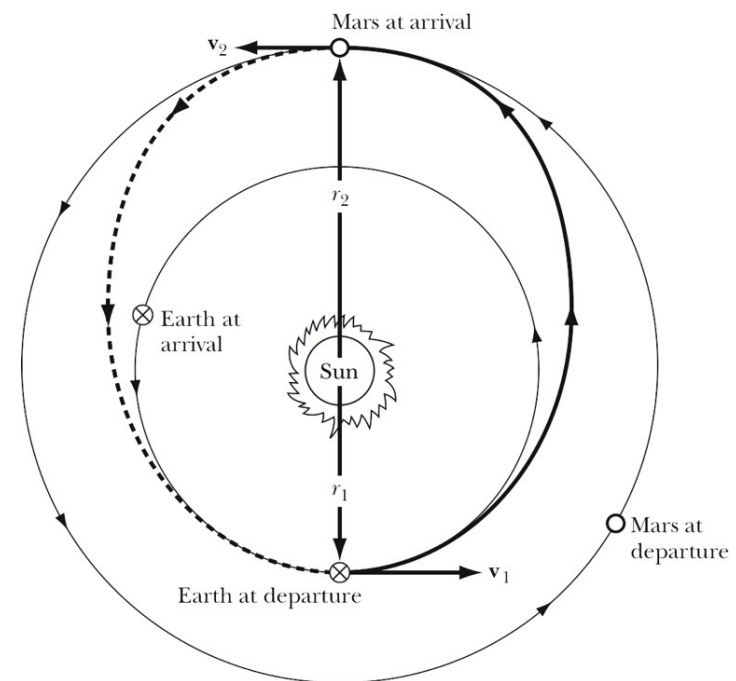
- By changing the energy of a spaceship, we change the shape of its orbit.
- When we launch a spaceship from Earth, we can increase (or decrease) its total energy and change its orbit from circular to elliptical.
- If the proper conditions exist, we can use this to travel to Mars.



Orbital motion.

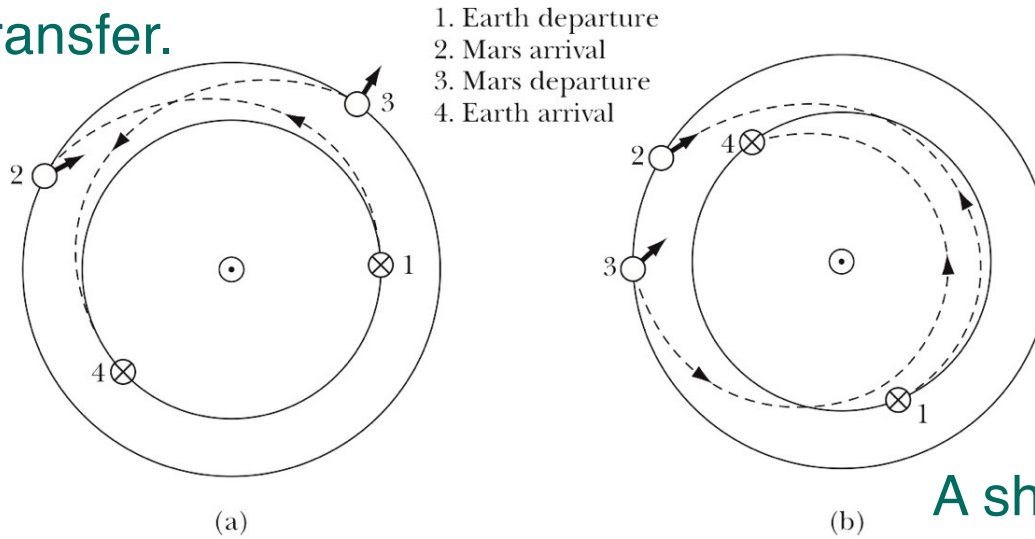
Travelling to Mars.

- This mode of travelling to Mars minimizes the amount of energy required.
- It is called the **Hohmann transfer**.
- Comments:
 - This transfer minimizes energy.
 - It does not minimize time.
 - This mode of transport only works if the conditions are correct (Mars needs to be in the proper location).



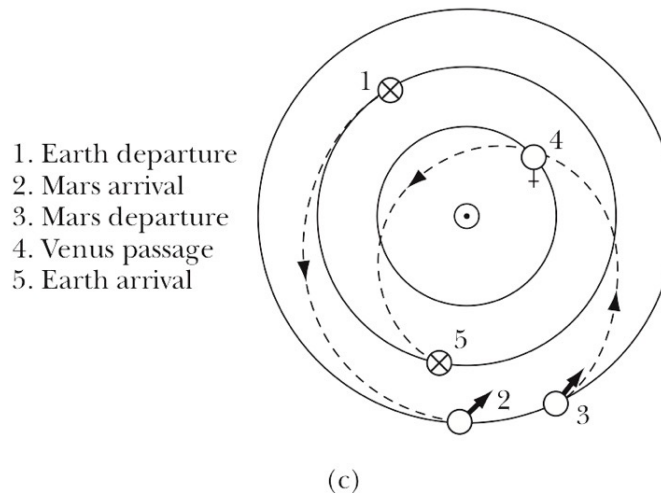
Other ways to get to Mars.

Hohmann transfer.



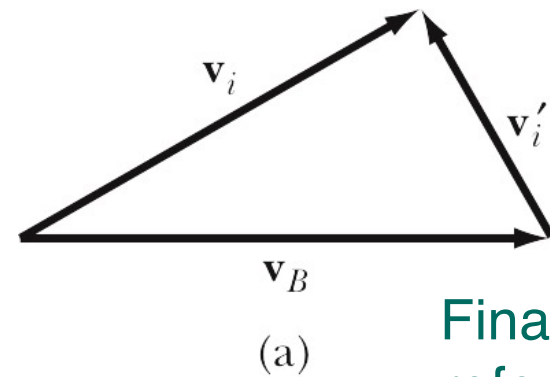
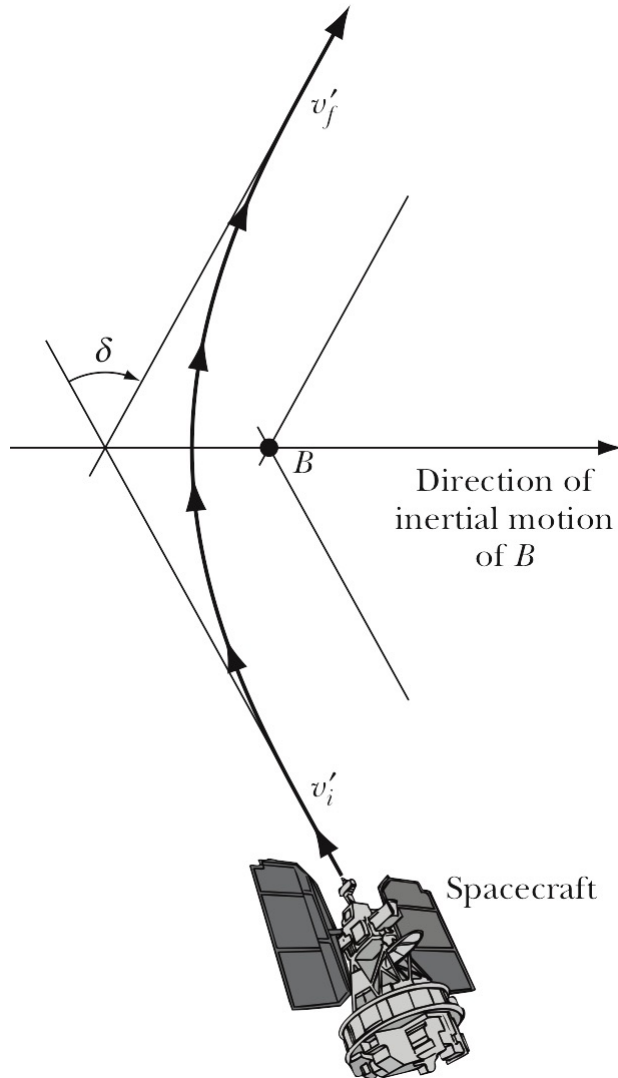
A shorter mission:

- More fuel required.
- Closer to the sun

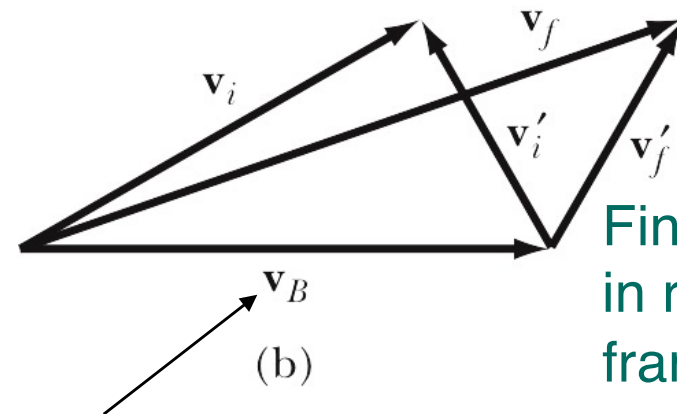


Reduce fuel consumption if Venus is in the proper position.

Fuel consumption can be reduced: gravity assists.



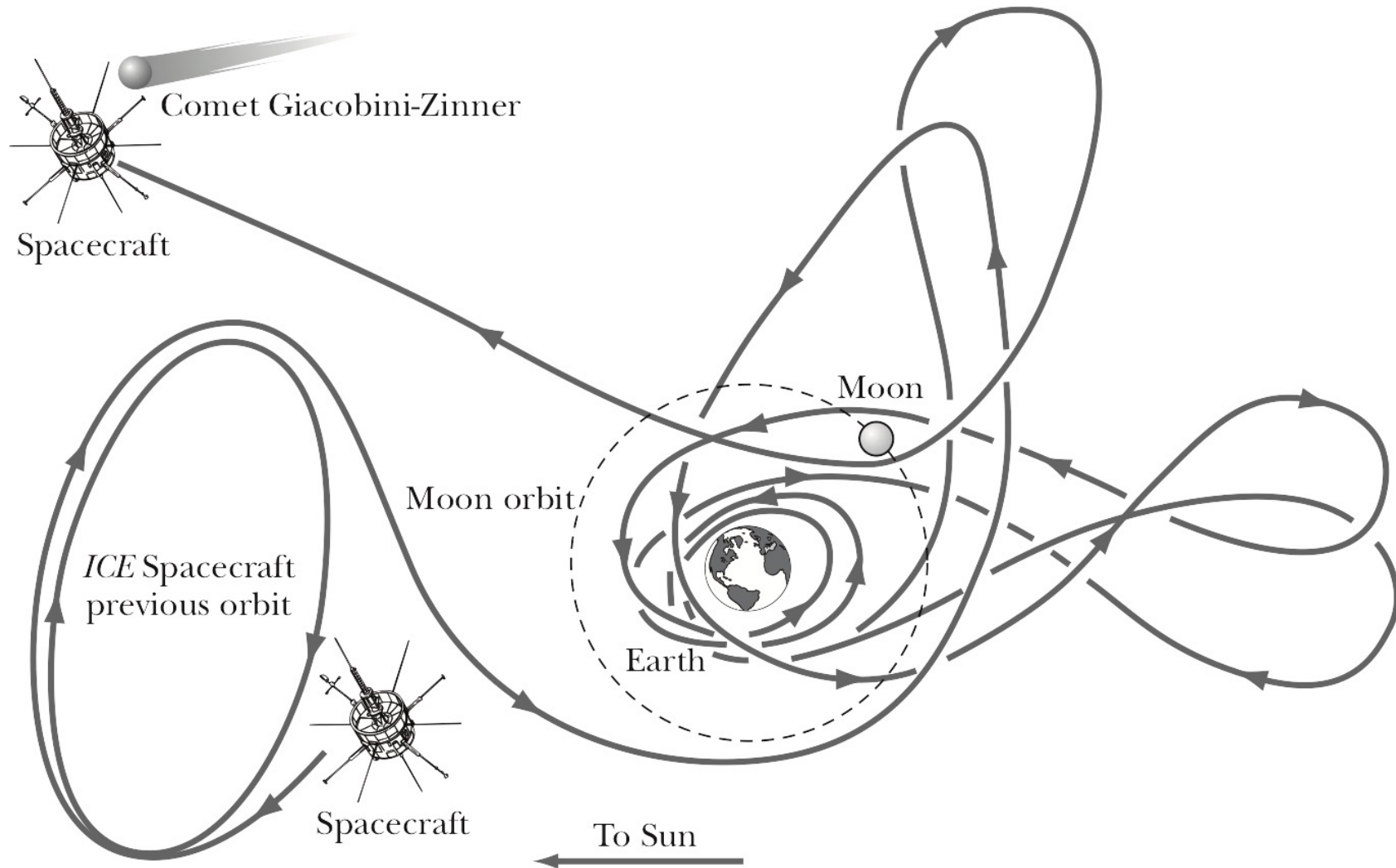
Final velocity in our reference frame.



Final velocity in reference frame B.

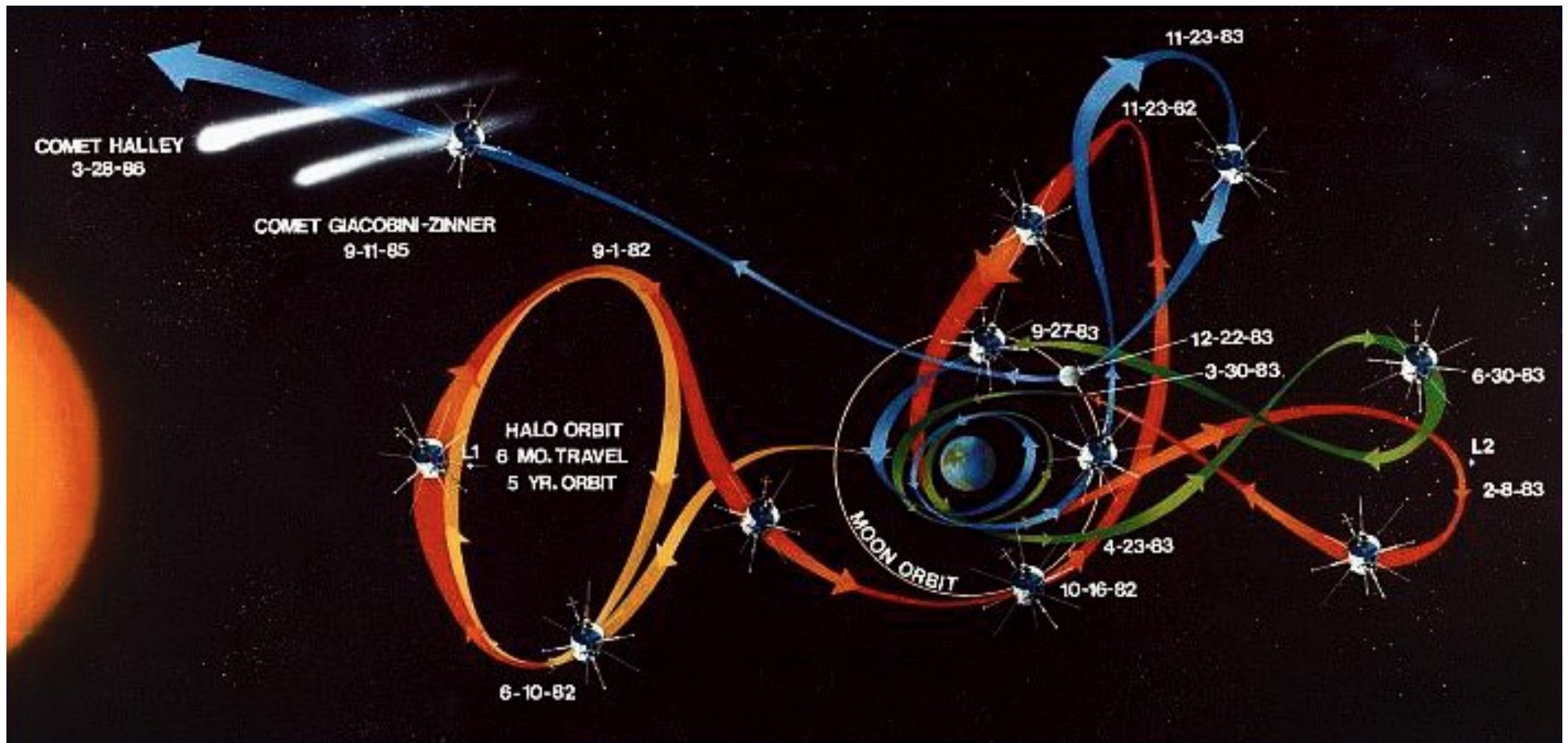
Velocity of B in our reference frame.

Gravity assists



How long did it take?

A great application of numerical calculations!



Launched in 1978.

ENOUGH FOR TODAY?