Classical Mechanics
Phy 235, Lecture 12.

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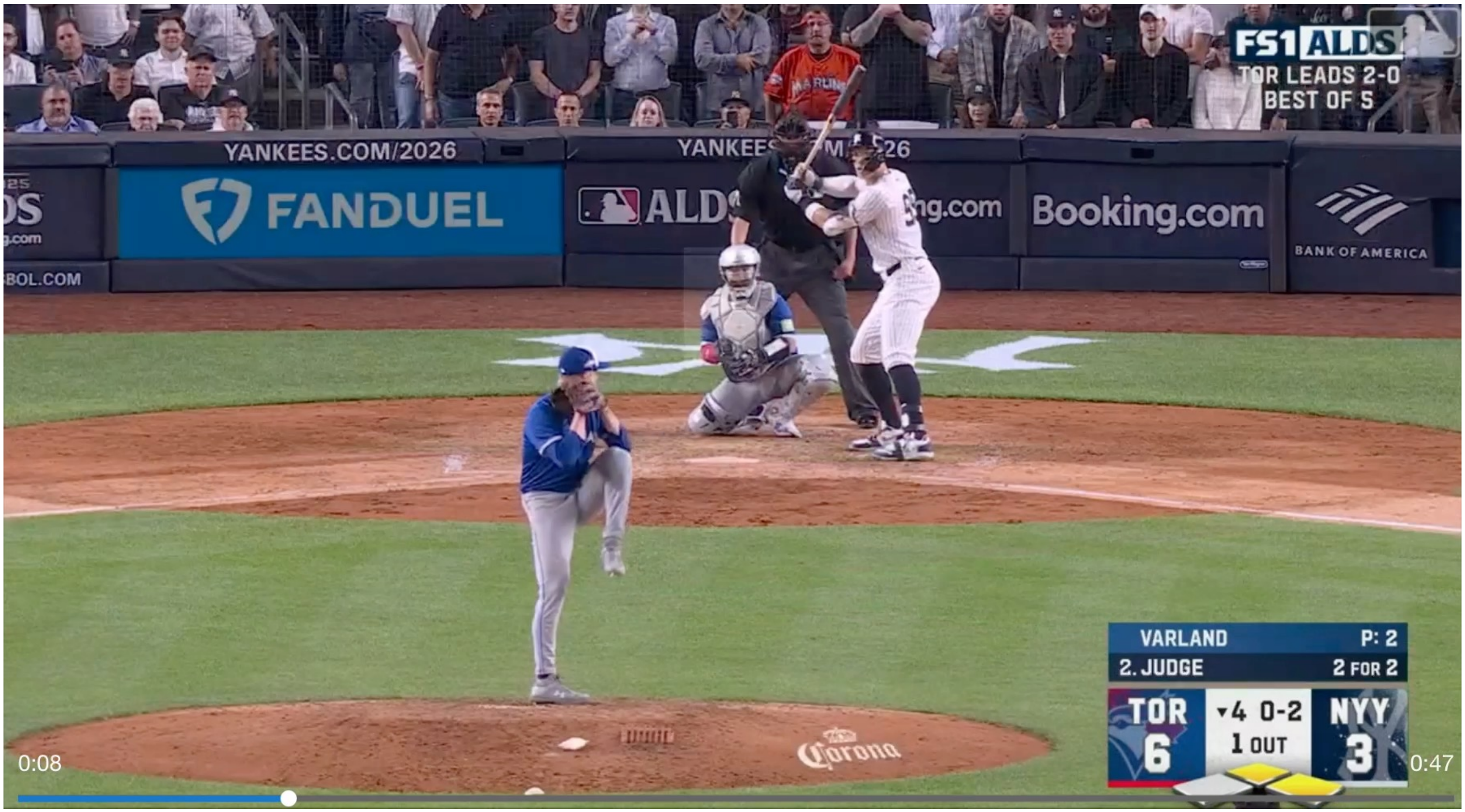
A Boeing 747 of the KLM, landing at Schiphol.

But today we did not need to use the KLM as a backup.



Course Comments

- The next exam will take place on Thursday October 23.
  - Material covered: Chapters 5 – 7.
  - Material will be reviewed on Monday October 20.
- You must submit a written proposal of the topic to be covered in your term paper by Friday October 24, at noon. Details about the term paper can be found on the web at:

http://teacher.pas.rochester.edu/PHY235/CourseInformation/TermPaper.htm

Overview.

- Central force problems are important problems in physics.
- In Chapter 8 we focus on the motion associated with central forces.
- Today we will solve the problem of orbital motion.
- What is different today compared to your introductory course?
  - We do not assume that one object is more massive than the other object.
  - We do not assume that the more massive object is at rest.
  - We determine the general properties of the orbit and not restrict ourselves to circular motion.

Chapter 8.
Central-Force Motion.

- Many important problems in physics involve the motion of two bodies with a central force acting between them.
- Assume the potential depends on the position between the two objects.
- The Lagrangian can be written in terms of the coordinates of the two masses:

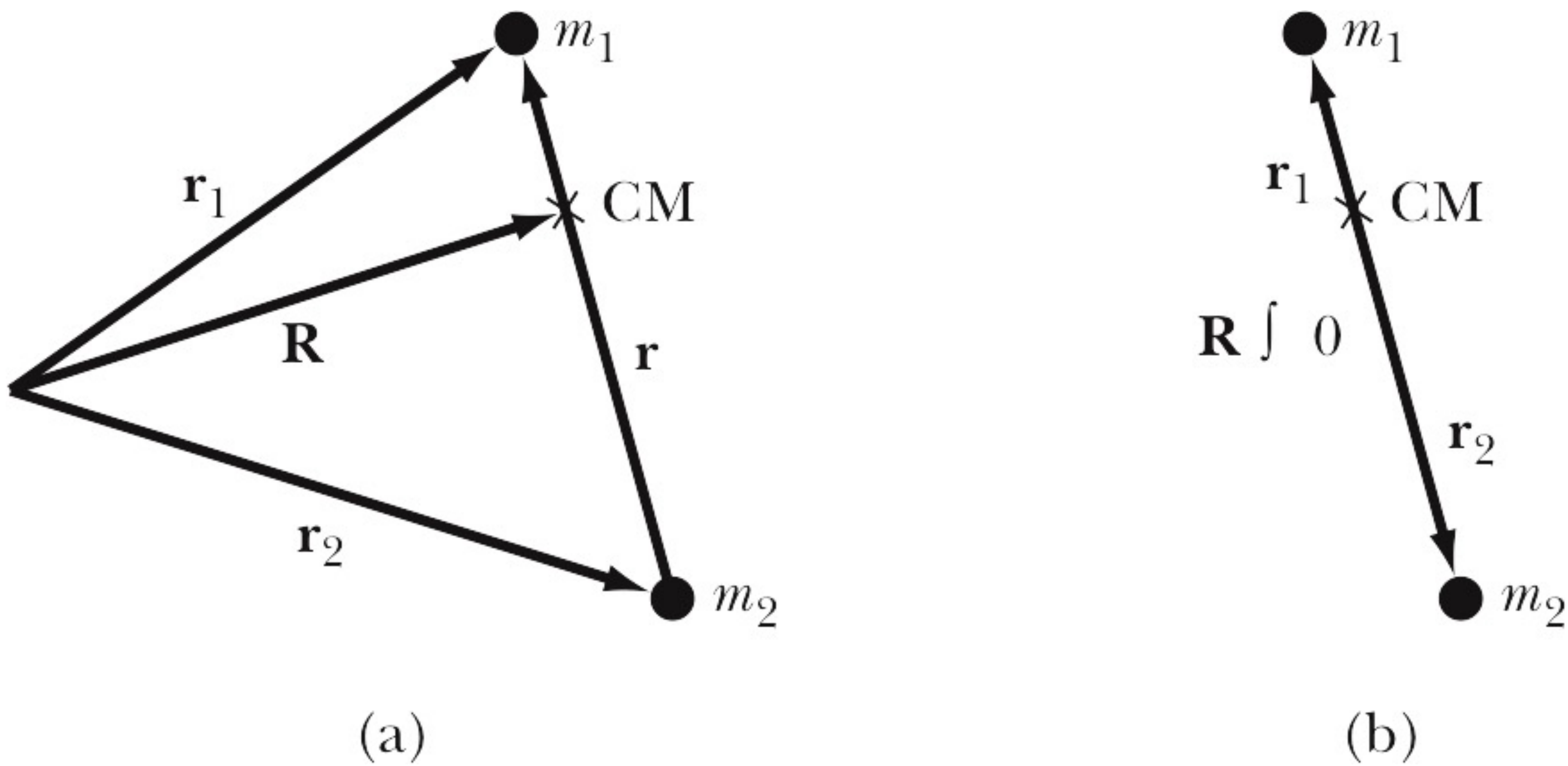
$$L = \frac{1}{2} m_1 \left|\dot{\vec{r}}_1\right|^2 + \frac{1}{2} m_2 |\dot{\vec{r}}_2|^2 - U(\vec{r}_1 - \vec{r}_2)$$

- In terms of their relative position:

$$L = \frac{1}{2} \mu \left|\dot{\vec{r}}\right|^2 - U(\vec{r})$$

- Note: the two-body problem has been reduced to a one-body problem.

Changing a 2-body problem into a 1-body problem.



Conservation of angular momentum.
Spherical symmetry: *U* only depends on *r*.

Starting from the Lagrangian:

$$L = \frac{1}{2}\mu\left|\dot{\vec{r}}\right|^2 - U(\vec{r}) = \frac{1}{2}\mu(\dot{r}^2+r^2\dot{\theta}^2) - U(\vec{r})$$

we define the generalized momenta:

$$p_r = \frac{\partial L}{\partial \dot{r}}\mu\dot{r}$$

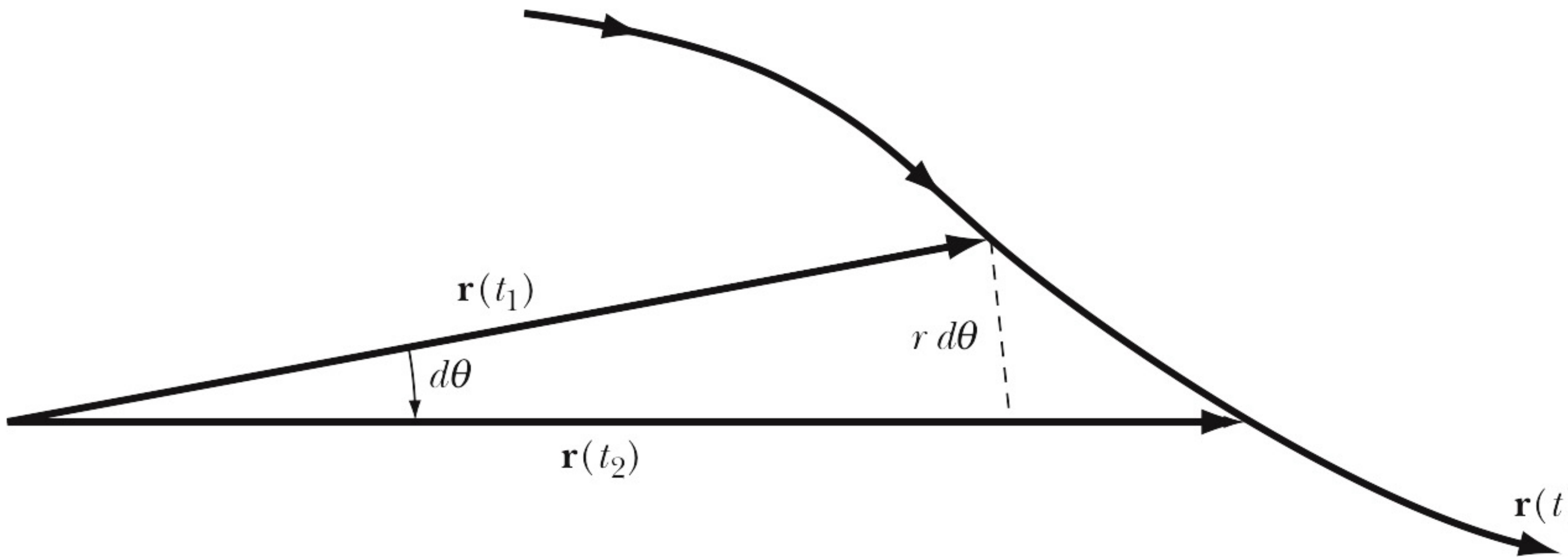
$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \mu r^2\dot{\theta}$$

The time derivatives of the generalized momenta are:

$$\dot{p}_r = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = \frac{\partial L}{\partial r} = \mu r\dot{\theta}^2 - \frac{\partial U}{\partial r}$$

$$\dot{p}_\theta = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} = 0$$

Areal Velocity.



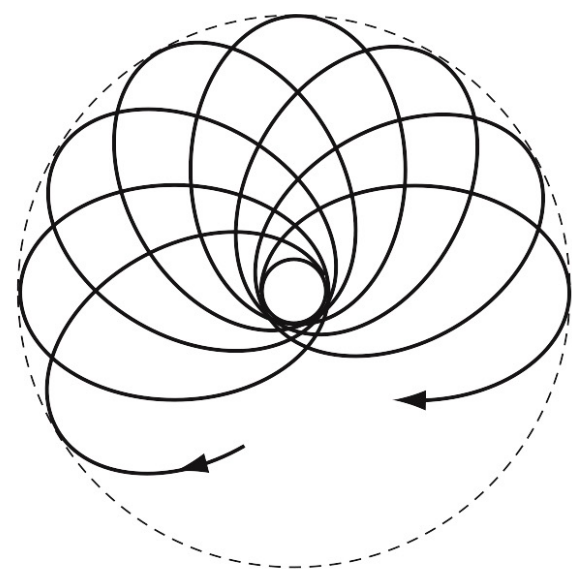
Orbital Motion.

- Equation of motion:

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{\mu}(E-U) - \frac{\ell^2}{\mu^2 r^2}}$$

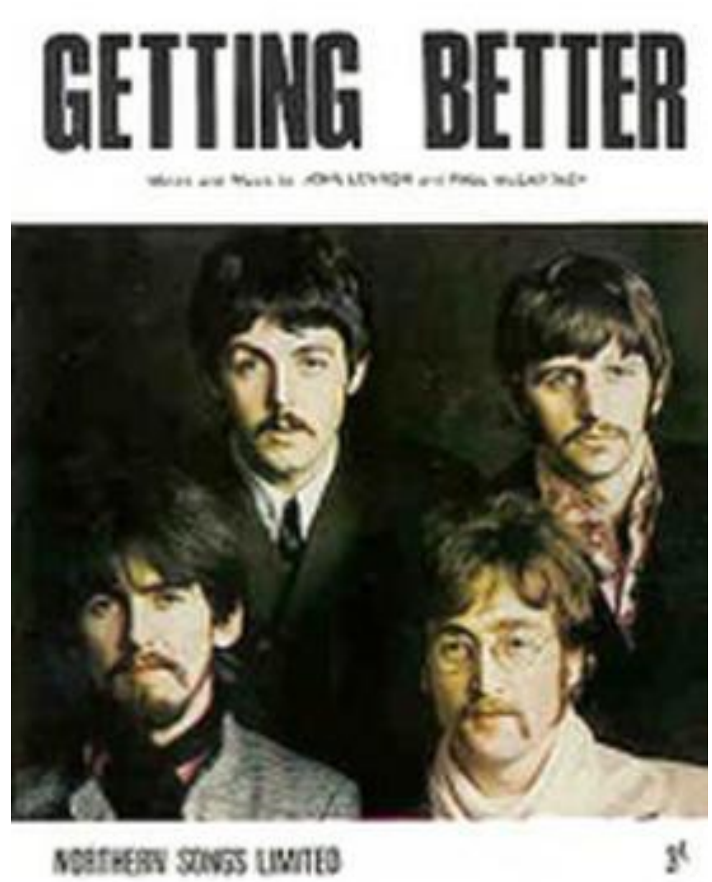
- Change in polar angle during one period:

$$\Delta\theta = 2\int_{r_{min}}^{r_{max}} \frac{\ell}{r^2\sqrt{\frac{2}{\mu}(E-U) - \frac{\ell^2}{\mu^2 r^2}}}dr$$



2 Minute 47 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 47 second intermission.



Problem 8.8

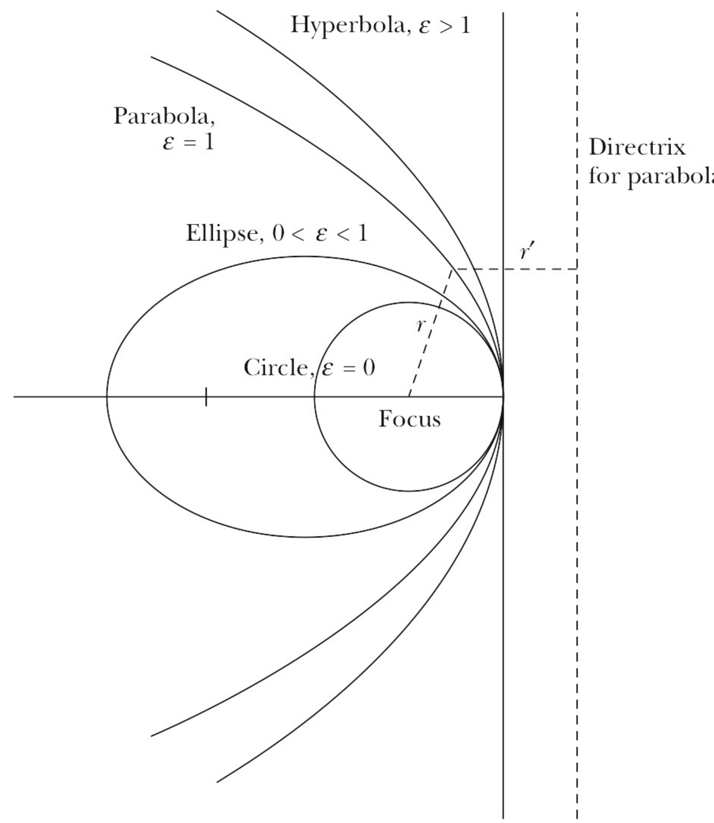
- Investigate the motion of a particle repelled by a force center according to the law $F(r) = kr$. Show that the orbit can only be hyperbolic.

Solving the Orbital Equation.

$$\theta(r) = \pm\int \frac{1}{\sqrt{-u^2 + 2\mu\frac{k}{l}u + 2\mu E}}du = \pm\sin^{-1}\left(\frac{-2u + 2\mu\frac{k}{l}}{\sqrt{\left(2\mu\frac{k}{l}\right)^2 + 8\mu E}}\right) + C =$$

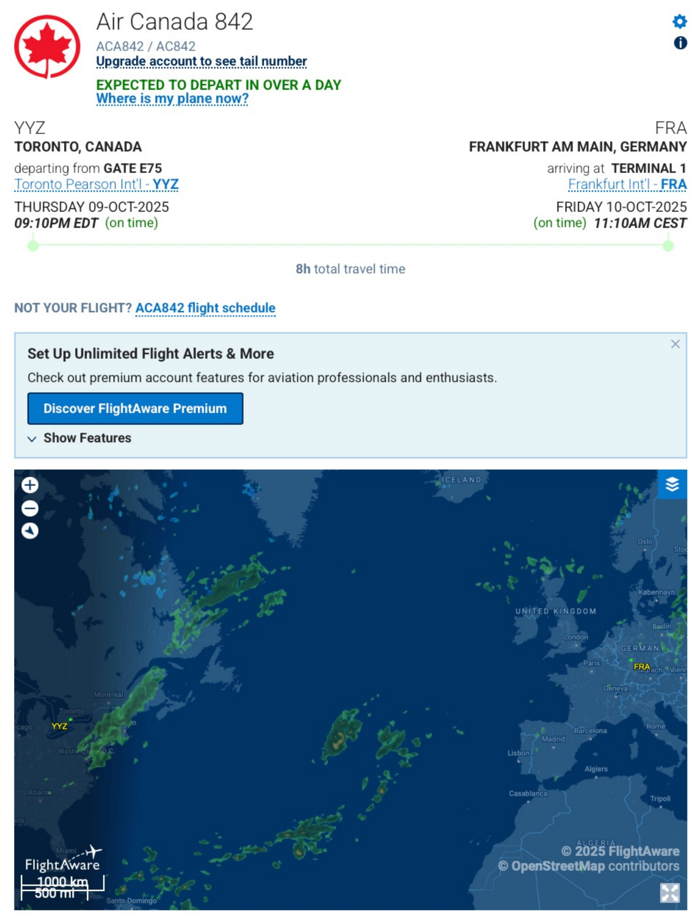
$$= \pm\sin^{-1}\left(\frac{\mu\frac{k}{l} - u}{\sqrt{\left(\mu\frac{k}{l}\right)^2 + 2\mu E}}\right) + C = \pm\sin^{-1}\left(\frac{\mu\frac{k}{l} - \frac{l}{r}}{\sqrt{\left(\mu\frac{k}{l}\right)^2 + 2\mu E}}\right) + C =$$

$$= \pm\sin^{-1}\left(\frac{\mu k - \frac{l^2}{r}}{\sqrt{(\mu k)^2 + 2\mu l^2 E}}\right) + C$$



Have a great fall break.

- Have a great fall break!
  - But .... do not forget about the paper!
- I will go to Germany for fall break.
- See you next week on Wednesday!



ENOUGH FOR TODAY?