
Classical Mechanics

Phy 235, Lecture 12.

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A Boeing 747 of the KLM, landing at Schiphol.



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But today we did not need to use the KLM as a backup.



Course Comments

- The next exam will take place on Thursday October 23.
 - Material covered: Chapters 5 – 7.
 - Material will be reviewed on Monday October 20.
- You must submit a written proposal of the topic to be covered in your term paper by Friday October 24, at noon. Details about the term paper can be found on the web at:

<http://teacher.pas.rochester.edu/PHY235/CourseInformation/TermPaper.htm>

Overview.

- Central force problems are important problems in physics.
- In Chapter 8 we focus on the motion associated with central forces.
- Today we will solve the problem of orbital motion.
- What is different today compared to your introductory course?
 - We do not assume that one object is more massive than the other object.
 - We do not assume that the more massive object is at rest.
 - We determine the general properties of the orbit and not restrict ourselves to circular motion.

Chapter 8.

Central-Force Motion.

- Many important problems in physics involve the motion of two bodies with a central force acting between them.
- Assume the potential depends on the position between the two objects.
- The Lagrangian can be written in terms of the coordinates of the two masses:

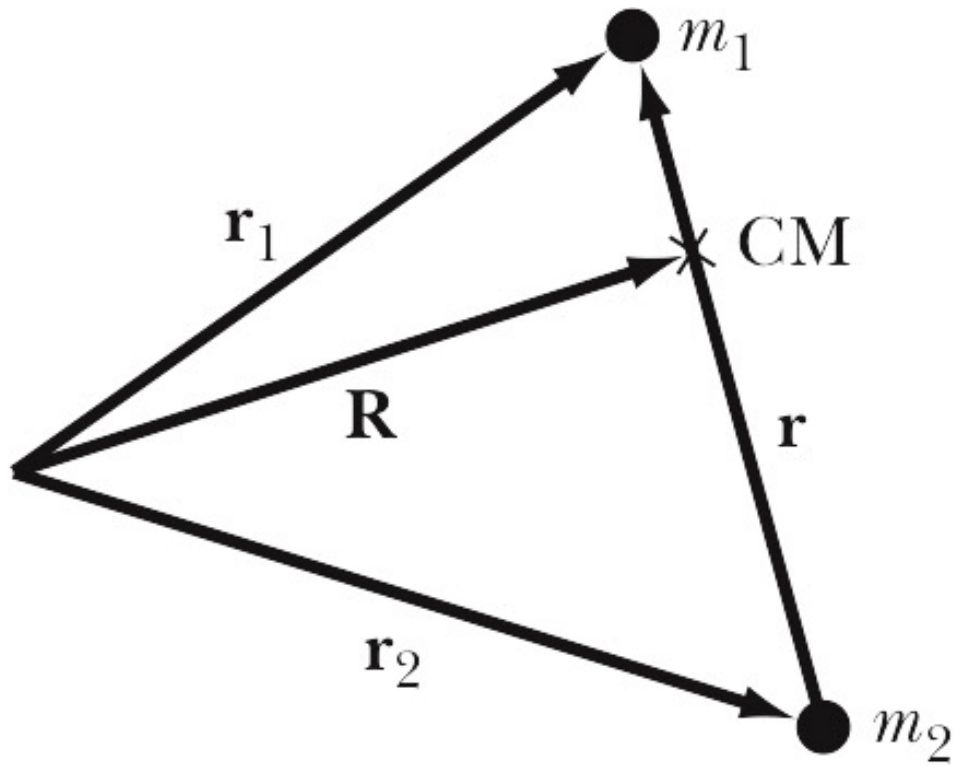
$$L = \frac{1}{2} m_1 |\dot{\vec{r}}_1|^2 + \frac{1}{2} m_2 |\dot{\vec{r}}_2|^2 - U(\vec{r}_1 - \vec{r}_2)$$

- In terms of their relative position:

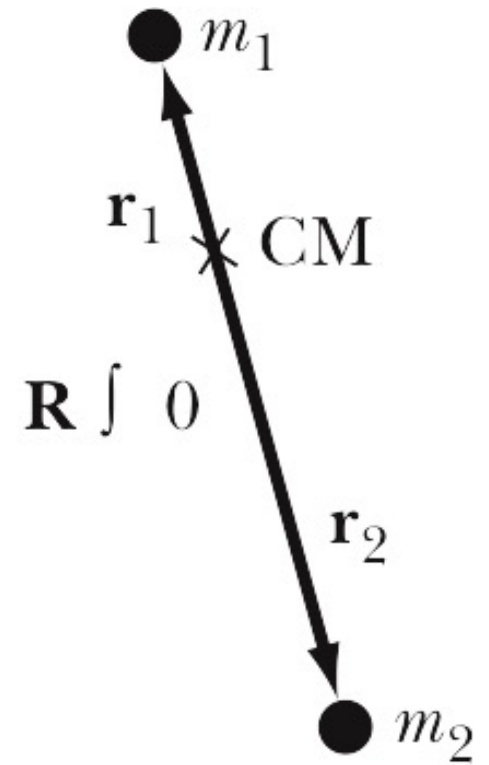
$$L = \frac{1}{2} \mu |\dot{\vec{r}}|^2 - U(\vec{r})$$

- Note: the two-body problem has been reduced to a one-body problem.

Changing a 2-body problem into a 1-body problem.



(a)



(b)

Conservation of angular momentum.

Spherical symmetry: U only depends on r .

Starting from the Lagrangian:

$$L = \frac{1}{2}\mu|\dot{\vec{r}}|^2 - U(\vec{r}) = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) - U(\vec{r})$$

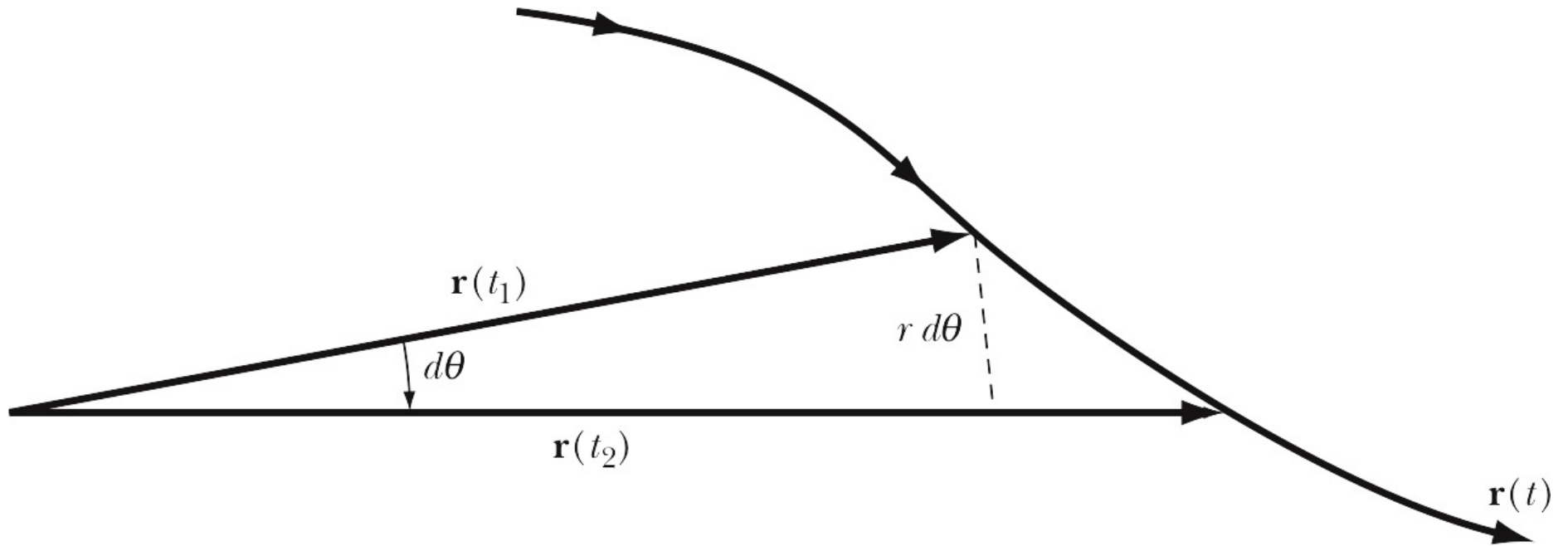
we define the generalized momenta:

$$p_r = \frac{\partial L}{\partial \dot{r}} = \mu\dot{r}$$
$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \mu r^2 \dot{\theta}$$

The time derivatives of the generalized momenta are:

$$\dot{p}_r = \frac{d}{dt} \frac{\partial L}{\partial \dot{r}} = \frac{\partial L}{\partial r} = \mu r \dot{\theta}^2 - \frac{\partial U}{\partial r}$$
$$\dot{p}_\theta = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} = 0$$

Areal Velocity.



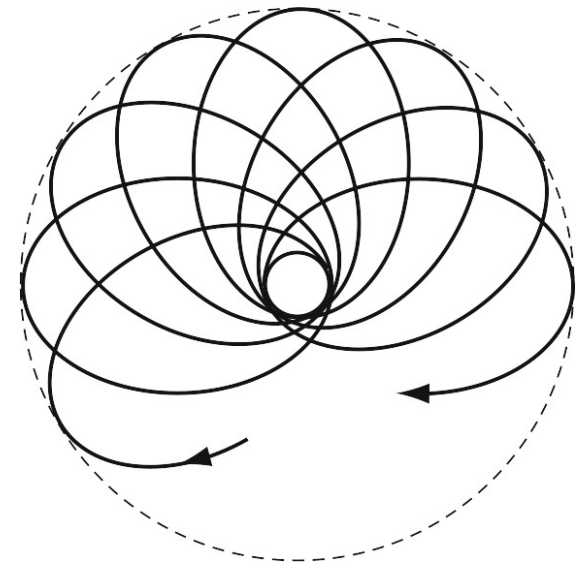
Orbital Motion.

- Equation of motion:

$$\frac{dr}{dt} = \pm \sqrt{\frac{2}{\mu}(E - U) - \frac{\ell^2}{\mu^2 r^2}}$$

- Change in polar angle during one period:

$$\Delta\theta = 2 \int_{r_{\min}}^{r_{\max}} \frac{\ell}{r^2 \sqrt{\frac{2}{\mu}(E - U) - \frac{\ell^2}{\mu^2 r^2}}} dr$$





2 Minute 47 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 47 second intermission.

GETTING BETTER

Words and Music by JOHN LENNON and PAUL McCARTNEY



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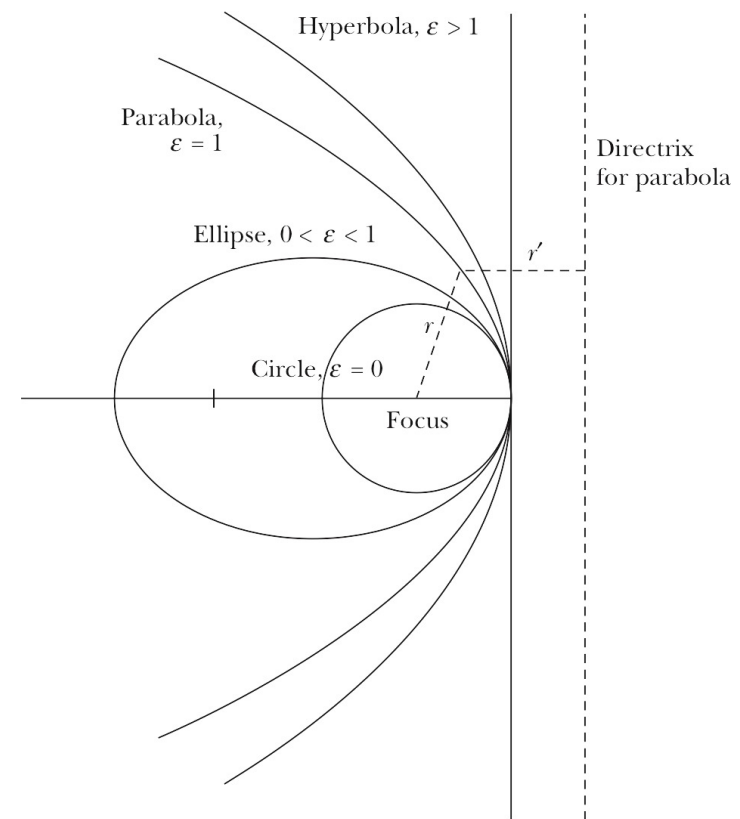
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Problem 8.8

- Investigate the motion of a particle repelled by a force center according to the law $F(r) = kr$. Show that the orbit can only be hyperbolic.

Solving the Orbital Equation.

$$\begin{aligned} \theta(r) &= \pm \int \frac{1}{\sqrt{-u^2 + 2\mu \frac{k}{l} u + 2\mu E}} du = \pm \sin^{-1} \left(\frac{-2u + 2\mu \frac{k}{l}}{\sqrt{\left(2\mu \frac{k}{l}\right)^2 + 8\mu E}} \right) + C = \\ &= \pm \sin^{-1} \left(\frac{\mu \frac{k}{l} - u}{\sqrt{\left(\mu \frac{k}{l}\right)^2 + 2\mu E}} \right) + C = \pm \sin^{-1} \left(\frac{\mu \frac{k}{l} - \frac{l}{r}}{\sqrt{\left(\mu \frac{k}{l}\right)^2 + 2\mu E}} \right) + C = \\ &= \pm \sin^{-1} \left(\frac{\mu k - \frac{l^2}{r}}{\sqrt{(\mu k)^2 + 2\mu l^2 E}} \right) + C \end{aligned}$$



Have a great fall break.

- Have a great fall break!
 - But do not forget about the paper!
- I will go to Germany for fall break.
- See you next week on Wednesday!

The screenshot shows a flight tracking interface for Air Canada 842. At the top left is the Air Canada logo (a red maple leaf). To its right, the text reads: "Air Canada 842", "ACA842 / AC842", "Upgrade account to see tail number", "EXPECTED TO DEPART IN OVER A DAY", and "Where is my plane now?". On the right side, there is a gear icon and an information icon. Below the flight number, the origin is listed as "YYZ TORONTO, CANADA" with details: "departing from GATE E75", "Toronto Pearson Intl. - YYZ", and "THURSDAY 09-OCT-2025 09:10PM EDT (on time)". The destination is listed as "FRA FRANKFURT AM MAIN, GERMANY" with details: "arriving at TERMINAL 1", "Frankfurt Intl. - FRA", and "FRIDAY 10-OCT-2025 (on time) 11:10AM CEST". A green line with dots at each end indicates the flight path, with "8h total travel time" written below it. Below the flight path, there is a link: "NOT YOUR FLIGHT? [ACA842 flight schedule](#)". A light blue box contains the text: "Set Up Unlimited Flight Alerts & More", "Check out premium account features for aviation professionals and enthusiasts.", a blue button "Discover FlightAware Premium", and a link "Show Features". At the bottom is a map showing the flight path from YYZ to FRA. The map includes a scale bar for 1000 km and 500 mi, and copyright information: "© 2025 FlightAware" and "© OpenStreetMap contributors".

ENOUGH FOR TODAY?