Classical Mechanics
Phy 235, Lecture 11.

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I cannot mention the Yankees today (1-10, 7-13). But KLM is always a great backup.

Course Information

- Due to fall break, there will be **no recitations next week**.
- But …. **there will be office hours next week** to help you with homework set # 6 (due next week on Friday).
- Your fall break may be a good time to start thinking about the final paper in this course. **Note**: this paper is due in 49 days from today.

Hamilton's Principle

" *Of all the possible paths along which a dynamical system may more from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.* "

$$\delta \int_{t_1}^{t_2} (T - U)dt = 0$$

The quantity *T* - *U* is called the **Lagrangian *L***.
The Lagrange equation(s) of motion are:

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial x_i'}\right) = 0$$

Generalized coordinates.

- So far, we have expressed the Lagrangian in terms of (generalized) position and (generalized) velocities:

$$L = T - U = \frac{1}{2} m \sum_{i=1}^{3} \dot{x}_i^2 - U(x_i)$$

- An alternative is to express the Lagrangian in terms of (generalized) position and (generalized) momenta. For example:

$$p_r = \frac{\partial L}{\partial \dot{r}} = \frac{\partial T}{\partial \dot{r}} = m\dot{r}$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{\partial T}{\partial \dot{\theta}} = mr^2\dot{\theta}$$

Conservation Laws – Part I.

- Conservation of energy:
  - Lagrangian does not depend on time explicitly.
  - If L does not depend explicitly on time, it can be shown that

$$L - \sum_j \left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) = \text{constant} = -H$$

  - The constant H is called the Hamiltonian of the system:

$$H = \sum_j \left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) - L$$

Problem 7.22.

A particle of mass m moves in one dimension under the influence of a force F:

$$F(x,t) = \frac{k}{x^2} e^{-t/\tau}$$

where k and τ are positive constants. Compute the Lagrangian and Hamiltonian functions. Compare the Hamiltonian and the total energy and discuss the conservation of energy for the system.

2 Minute 2 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let’s take a 2 minute 2 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Conservation Laws – Part II.

- Conservation of linear momentum:
  - Lagrangian should not be effected by a translation of space.
- Conservation of angular momentum:
  - Lagrangian should not be effected by a rotation of space.

Canonical Equations of Motion

Lagrange equations of motion in terms of generalized momentum:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \dot{p}_i = \frac{\partial L}{\partial q_i}$$

The Hamiltonian H can be written in terms of the generalized momenta as

$$H = \sum_j \left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) - L = \sum_j \dot{q}_j p_j - L$$

Hamilton's Equations of Motion

- For each coordinate: **two** equations of motion.
  - For each coordinate there is only one Lagrange equation of motion.
- Equations of motion are **first order differential equations**.
  - The Lagrange equations of motion are second order differential equations.

$$\frac{\partial H}{\partial p_j} - \dot{q}_j = 0$$

$$\frac{\partial H}{\partial q_j} + \dot{p}_j = 0$$

$$\frac{\partial H}{\partial t} + \frac{\partial L}{\partial t} = 0$$

Problem 7.38

The potential for an anharmonic oscillator is

$$U = k\frac{x^2}{2} + b\frac{x^4}{4}$$

where k and b are constants.

Find Hamilton's equations of motion.

Problem 7.28

Consider a force *F* that is provided by the potential *U*:

$$U = -\frac{k}{r}$$

Use plane polar coordinates and find Hamilton's equations of motion.

Problem 7.24

Consider a simple plane pendulum consisting of a mass m attached to a string of length l. After the pendulum is set into motion, the length of the string is shortened at a constant rate:

$$\frac{dl}{dt} = -\alpha = \text{constant}$$

The suspension point remains fixed. Compute the Lagrangian and Hamiltonian functions. Compare the Hamiltonian and the total energy, and discuss the conservation of energy for the system.

End of Chapter 7

- We now have finished the material that will be covered on midterm exam# 2: chapters 5 – 7.
- Midterm exam # 2 will take place on Thursday October 23 between 8 am and 9.20 am in B&L 109.
- **NOTE: WE WILL SKIP SECTIONS 7.12 AND 7.13.**

ENOUGH FOR TODAY?