Classical Mechanics
Phy 235, Lecture 10.

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A great way to start today.

Hamilton's Principle

" *Of all the possible paths along which a dynamical system may more from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.* "

$$\delta \int_{t_1}^{t_2} (T - U) dt = 0$$

The quantity *T* - *U* is called the **Lagrangian *L***.

Lagrange Equation(s) of Motion.

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial x_i'}\right) = 0$$

Generalized coordinates $q_1, q_2, \ldots$

- Generalized coordinates are coordinates that completely specify the state of the system.
- Generalized coordinates do **<u>not</u>** need to be coordinates of a coordinate system.
- Hamilton's principle: "*Of all the possible paths along which a dynamical system may move from one point to another in configuration space within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the Lagrangian function for the system.*"

$$\delta \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t) f dt = 0$$

Problem 7.4

A particle moves in a plane under the influence of a force $f = -Ar^{\alpha-1}$ directed toward the origin; A and α are constants.

Choose appropriate generalized coordinates, and let the potential energy be zero at the origin.

- Find the Lagrangian equations of motion.
- Is the angular momentum about the origin conserved?
- Is the total energy conserved?

2 Minute 37 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 37 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Problem 7.8

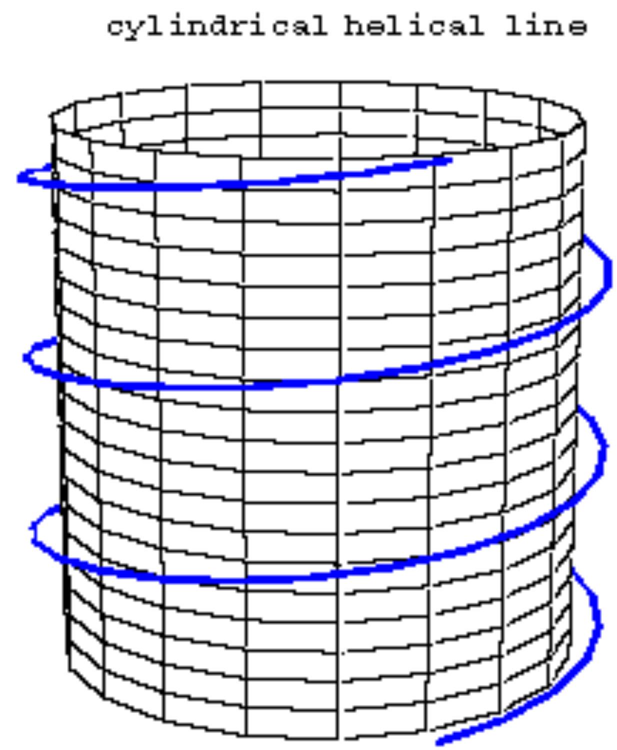
Consider a region of space divided by a plane. The potential energy of a particle in region 1 is U_1 and in region 2 it is U_2. If a particle of mass m and with speed v_1 in region 1 passes from region 1 to region 2 such that its path in region 1 makes and angle θ_1 with the normal to the plane of separation and an angle θ_2 with the normal when in region 2, show that

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \sqrt{1 + \frac{U_1 - U_2}{T_1}}$$

where $T_1 = (1/2)mv_1^2$.

Equations of Constraints

- One can remove the equations of constraint by a suitable choice of coordinates.
- For example, in the cylinder problem:
  - We can use three coordinates and one equation of constraint.
  - We can use two coordinates (azimuthal angle and vertical position) and no equations of constraint.



https://www.encyclopediaofmath.org/index.php/Helical_line

Lagrange's Equations with Undetermined Multipliers.

- Assume constraints can be expressed in differential form:

$$\sum_{j=1}^{s} \frac{\partial f_k}{\partial q_j} dq_j = 0$$

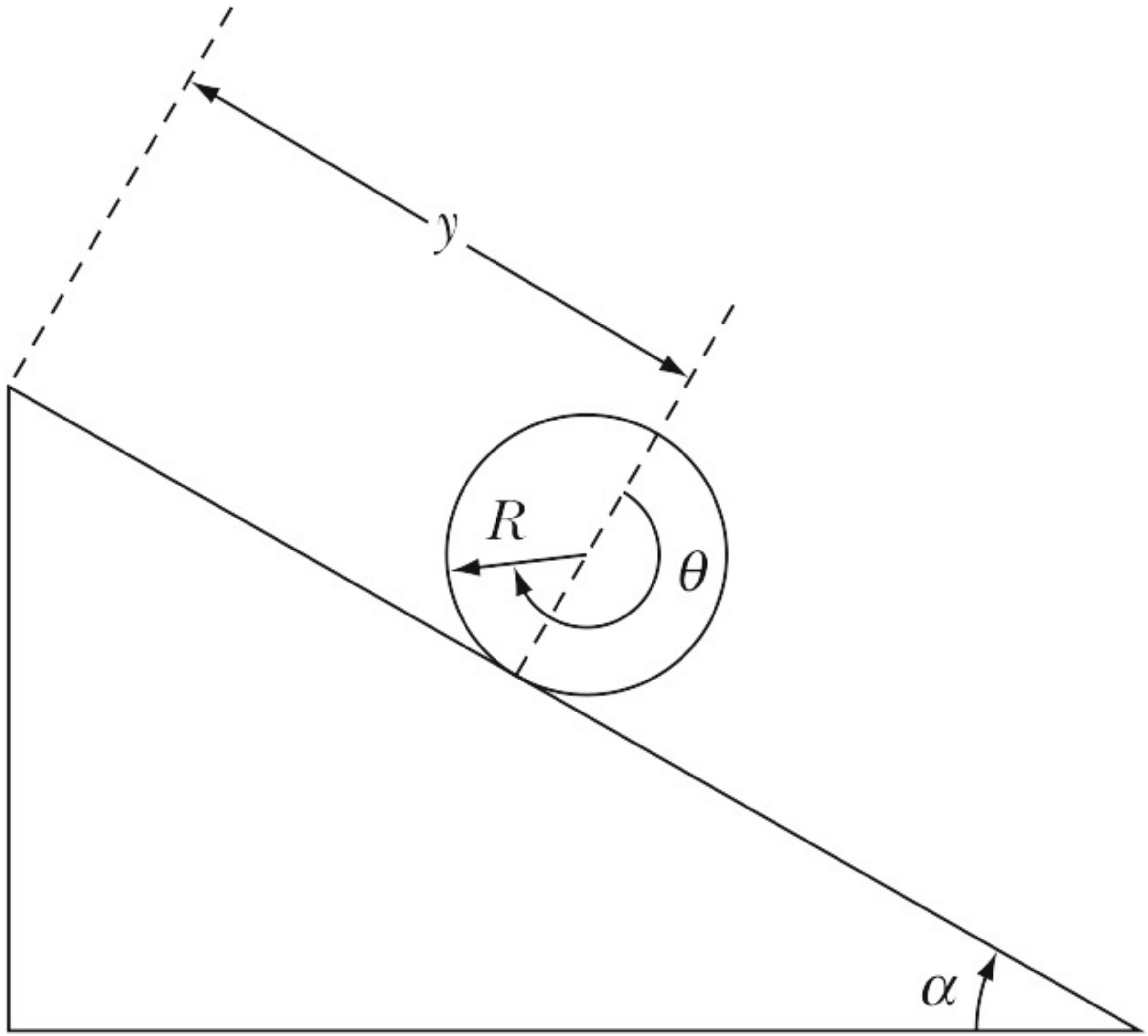
- Constraints can be incorporated into the Lagrange equations:

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \sum_{k=1}^{m} \lambda_k(t) \frac{\partial f_k}{\partial q_j} = 0$$

- The forces of constrained can be determined from the equations of constraint and the Lagrange multipliers:

$$Q_j = \sum_{k=1}^{m} \lambda_k(t) \frac{\partial f_k}{\partial q_j}$$

Example 7.9



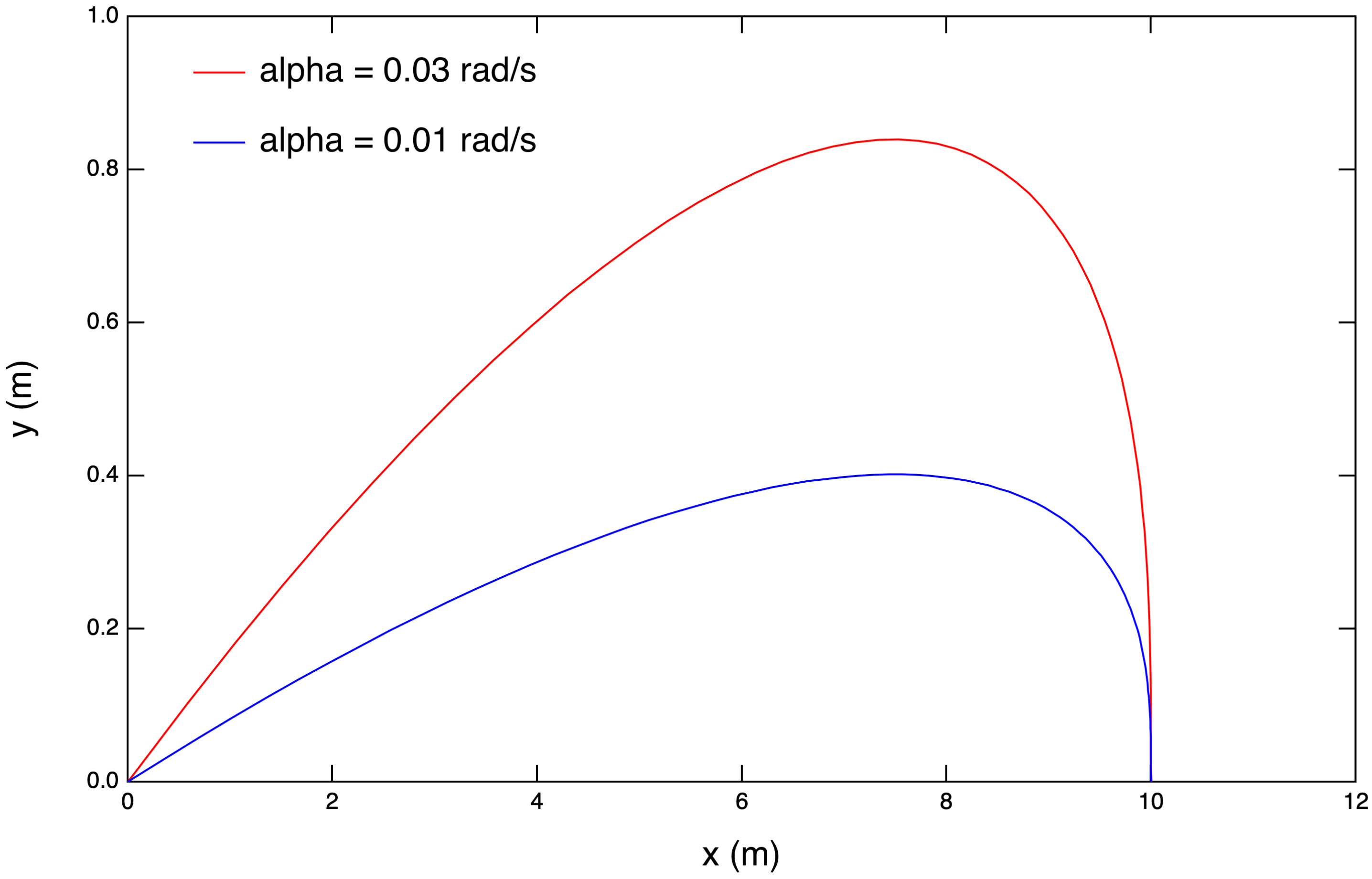
Problem 7.12

A particle of mass m rests on a smooth plane.

The plane is raised to an inclination angle θ at a constant rate α ($\theta = 0^\circ$ at $t = 0$ s), causing the particle to move down the plane.

Determine the motion of the particle.

Problem 7.12



ENOUGH FOR TODAY?