

**Classical Mechanics**  
**Phy 235, Lecture 05.**

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October 28, 2016.  
 The last KLM 747 landing on St. Maarten.



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**Non-linear oscillations.**

- **Linear differential equations:**
  - Terms are proportional to acceleration, velocity, and position:
$$\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = f(x)$$
- **Non-linear differential equations:**
  - Include terms that non-linear in term of acceleration, velocity, and position.
- **Non-linear terms are divided in two groups:**
  - Symmetric around the equilibrium position. This requires terms proportional to  $\epsilon^3$ . If  $\epsilon > 0$ : soft system. If  $\epsilon < 0$ : hard system.
  - Asymmetric around the equilibrium position. This requires terms proportional to  $r^2$ .

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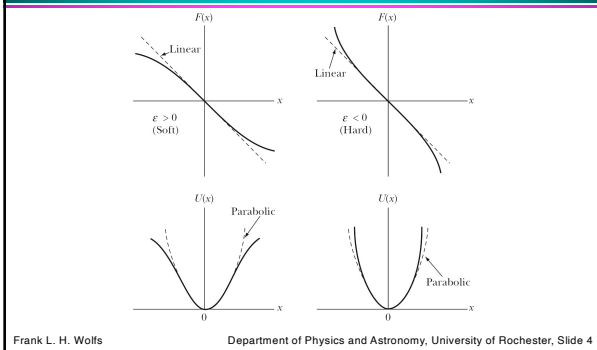
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## Non-linear Forces: $\epsilon x^3$ . Symmetric potentials.



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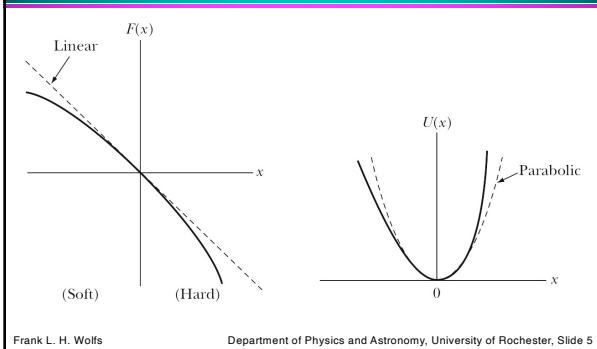
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## Non-linear Forces: $\lambda x^2$ . Asymmetric potentials.



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## Numerical studies.

- Using tools such as VPython, it is easy to explore what happens when we add these additional components to the restoring force.

• Let us have a look:

<https://www.glowscript.org/#/user/wolfs/folder/Public/program/Phy235-HardandSoftMotionWithRadialPhasePlot>

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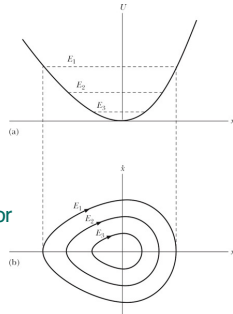
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## Phase Diagrams. Asymmetric for asymmetric potentials.



Closed contours for motion around stable equilibrium positions.

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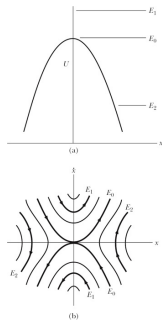
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## Phase diagrams.



Open contours for motion around unstable equilibrium positions.

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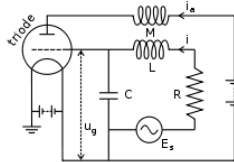
## Van der Pol Equation.

- Used to describe non-linear oscillations in circuits containing vacuum tubes.

- Important facts:**
  - Van der Pol was a Dutch physicist.
  - He studied physics in Utrecht and received his PhD in 1920.

- Other facts: de Van der Pol equation:**

$$\ddot{x} + \mu(x^2 - a^2)\dot{x} + \omega^2 x = 0$$



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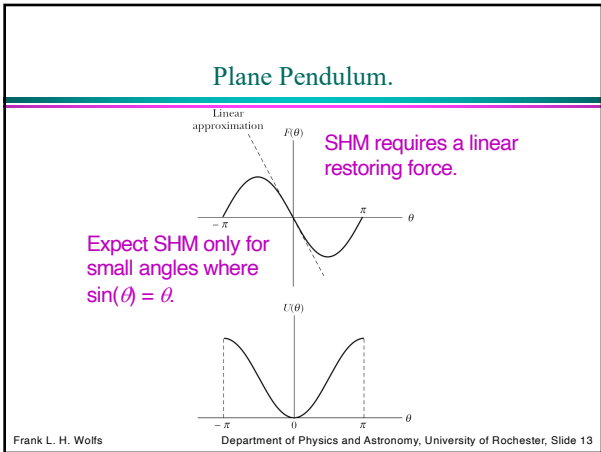
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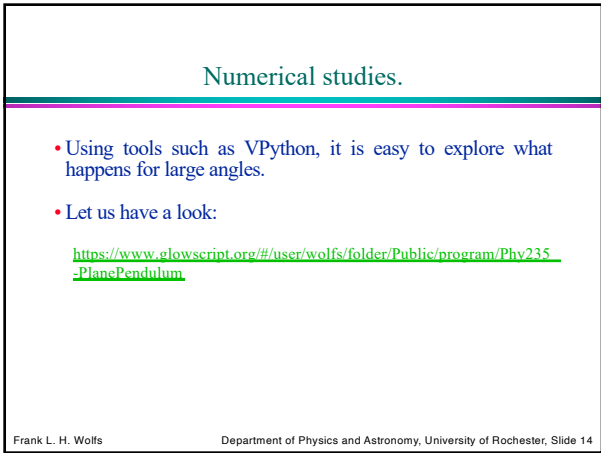
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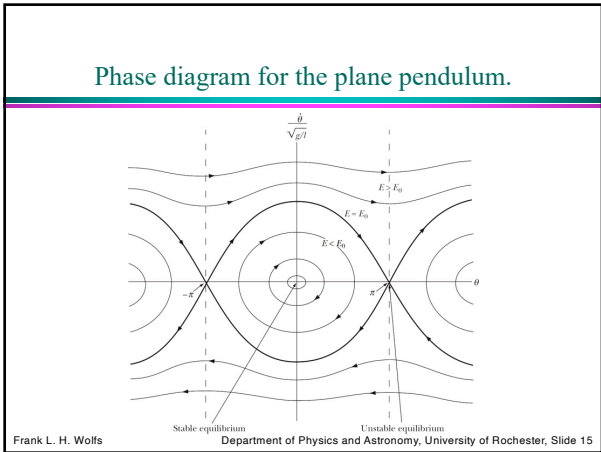
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### Problem 4.6.

- Derive the expression for the phase paths of the plane pendulum if the total energy  $E > 2mgl$ . Note that this is just the case of a particle moving in a periodic potential  $U(\theta) = mgl(1 - \cos(\theta))$ .

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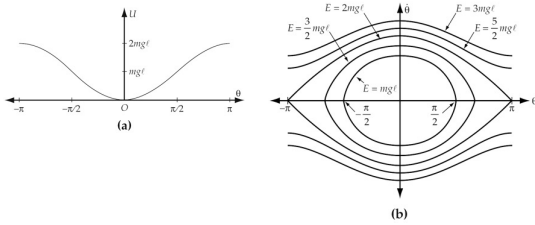
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### Problem 4.6.



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**ENOUGH FOR TODAY?**

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