Classical Mechanics
Phy 235, Lecture 04.

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Nothing to do with airplanes, nothing to with PHY 235, just a nice picture of part of my group.

Extra Credit Homework Assignment. Study projectile motion.

- There are three extra credit homework assignments.
- Each assignment will require numerical simulations.
- Each assignment will have the same weight as a regular assignment.
- You can thus earn 130% credit for the Phy 235 homework component of your final grade.

PHY 235, Extra Homework Set 1 — Due: September 26, 2025 at 11.59 am

Write the following text on the front cover of your homework assignment and sign it. If the text is missing, 20 points will be subtracted from your homework grade.

Honor Pledge for Graded Assignments
"I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

Signature ____________________

Consider the simulation of projectile motion, demonstrated in lecture 2. The script of this simulation, Phy235-ProjectileMotion, can be found in the following glowscript folder:
https://www.glowscript.org/#/user/wolfs/folder/Public/
Use this script as your starting point to complete the following tasks:

a. First consider pure projectile motion in vacuum (turn the dragg force off and set the angle to 60°). Compare the difference between the analytical solution and the numerical solution as function of stepsize dt. Make a plot of this difference as function of dt. Based on this plot, determine an optimum value of dt to run the simulation. Note: you need to make sure you pick the proper range of dt values.

b. Repeat the study carried out in part a) for two different launch angles (45° and 30°) and determine if your optimum choice of dt is angle dependent.

c. Now turn on the drag force ($k = 0.005$) and set the launch angle to 60°. When we include the drag force, we can no longer compare obtain an analytical solution and we have to determine the optimum dt in a different way. One possible approach is to look at the point of impact and determine how the point of impact depends on dt. Make a graph of the impact point as function of dt. Based on this plot, determine an optimum value of dt to run the simulation. Note: you need to make sure you pick the proper range of dt values.

d. Repeat the study carried out in part c) for two different values of the drag constant ($k = 0.01$ and $k = 0.05$) and determine if your optimum choice of dt is different for different drag constants.

e. Set the drag constant to $k = 0.005$ and set the launch angle to 60°. Add a constant thrust force F to the projectile, directed in the direction of motion and acting between time $t = 0$ and time $t = T$, to the counter the effect of the drag force. What combination(s) of thrust force F and thrust time T brings the projectile to the impact point it would reach when the only force acting on it would be gravitational force?

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Outline

- Damped and driven harmonic motion:
  - Damped harmonic motion occurs when friction or drag forces are acting on the system. Energy is dissipated and the system will gradually come to rest.
  - Driven harmonic motion adds a driving force in order to compensate for damping losses.

Solving Second-Order Differential Equations. Damped and driven.

- General form:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$$

- If you find two linearly independent solutions, every other solution will be a linear combination of these two solutions.
- The general solution has two constants, defined by the initial conditions.

- **Homogeneous equation:**
  - $f(x)$ is equal to 0.
- **Inhomogeneous equation:**
  - $f(x)$ is not equal to 0.

Homogeneous Equation: $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$

- Three different scenarios:
  - $a^2 > 4b$
  - $a^2 = 4b$
  - $a^2 < 4b$

Inhomogeneous Equation: $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$

- Suppose:
  - v is a solution of the inhomogeneous equation.
  - u is the general solution of the homogeneous equation.
- Then:
  - $u + v$ is the general solution of the inhomogeneous equation.

Homogeneous Equation

- Consider a damping force *–bv* and a restoring force *–kx*. The equation of motion for such system is: *ma* = *–bv* – *kx*.
- This provides us with the homogeneous equation:

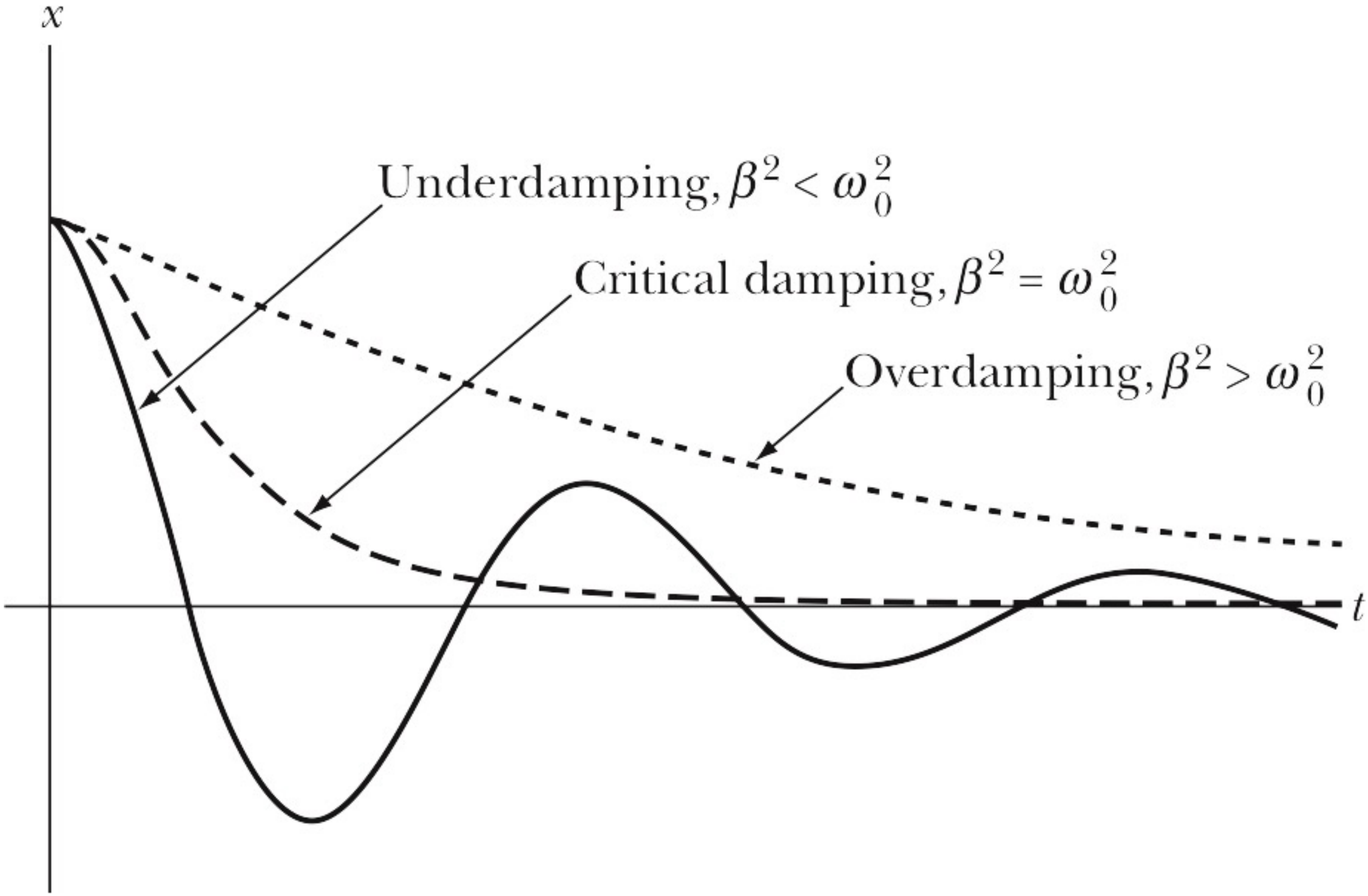
$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$$

- Try the following solution: $x = e^{rt}$.
- This is a valid solution if $r^2 + 2\beta r + \omega_0^2 = 0$. This requires:

$$r = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

- Three different scenarios:
  - $\beta^2 > \omega_0^2$: over damping. Two values of *r*.
  - $\beta^2 = \omega_0^2$: critical damping. One value of *r*. Second solution is $te^{-\beta t}$.
  - $\beta^2 < \omega_0^2$: under damping. Two values of *r*; r is a complex number

Damped motion.



Numerical studies.

- Using tools such as VPython, it is easy to explore how damped harmonic motion changes as the damping conditions are changed.

- Let us have a look:

  https://www.glowscript.org/#/user/wolfs/folder/Public/program/Phy235-3D-DampedHarmonicMotion

Problem 3.22

- Let the initial position and speed of an overdamped, non-driven oscillator be x_0 and v_0, respectively.
  - Determine the values of the amplitudes A_1 and A_2 in equation 3.44.

3 Minute 12 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 12 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Inhomogeneous Equation

- Consider a damping force $-bv$, a restoring force $-kx$, and a driving force $f(t)$. The equation of motion for such system is: $ma = -bv - kx + f(t)$.
- The equation of motion can be rewritten as:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = f(t)$$

- Suppose:
  - v is a solution of the inhomogeneous equation (this is called the **particular solution**).
  - u is the general solution of the homogeneous equation (this is called the **complementary solution**).

then $u + v$ is the general solution of the inhomogeneous equation.

Simplest case of driven harmonic motion.

- The simplest case of driven harmonic motion is the case when the driving force varies harmonically with time:

$$f(t) = F_0 \cos(\omega t)$$

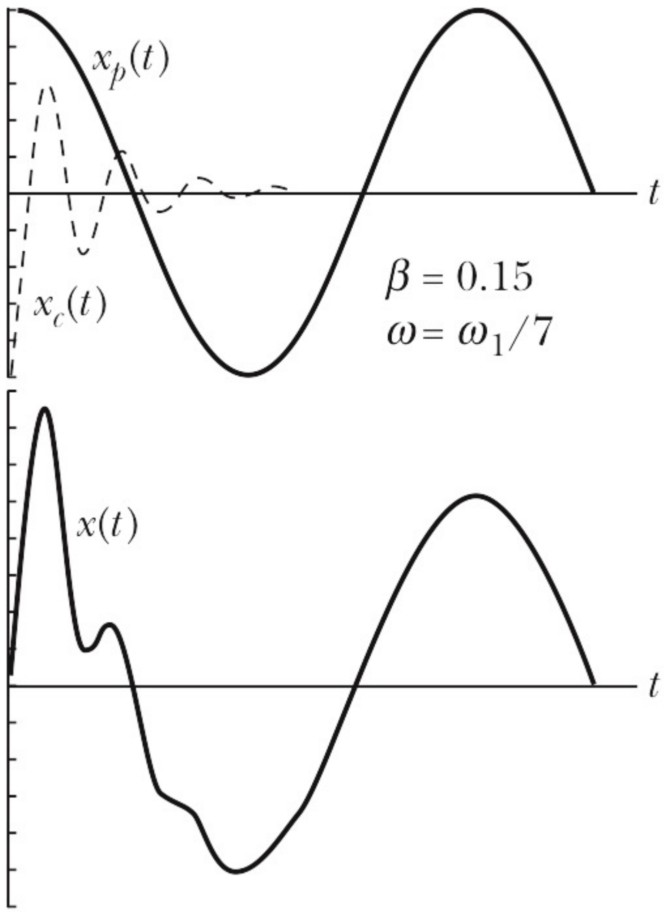
- Important properties of the solution:
  - The complementary solution approaches 0 for large t.
  - The particular solution will have a frequency equal to the driving frequency.
  - The amplitude of the particular solution has a maximum when the driving frequency is equal to the resonance frequency:

$$\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$$

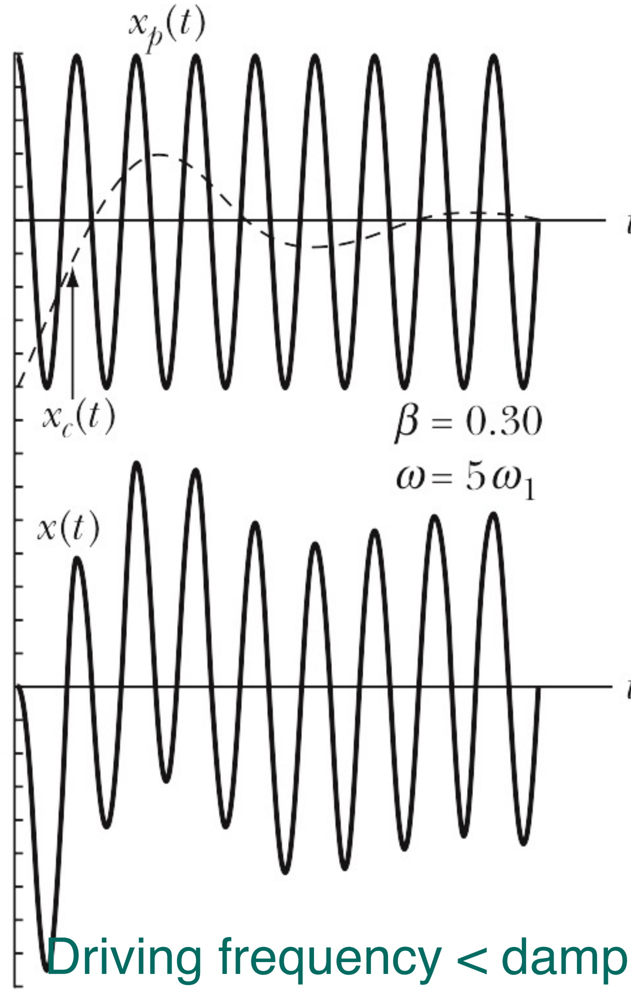
  - The resonance frequency is less than the natural frequency when damping is present.

Examples.

Driving frequency > damping frequency



(a)



Driving frequency < damping frequency

(b)

Numerical studies.

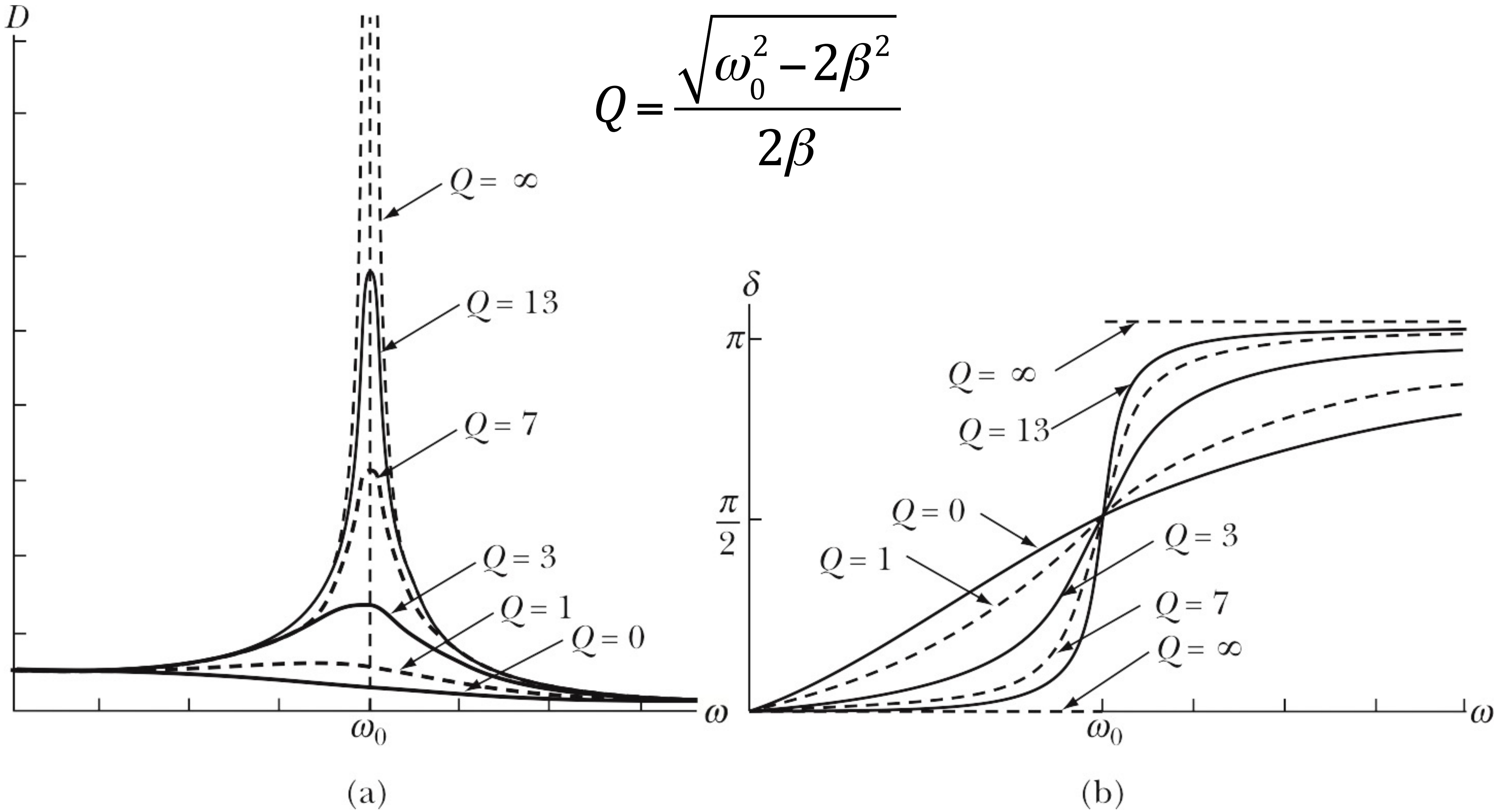
- Using tools such as VPython, it is easy to explore how driven harmonic motion changes as the driving conditions are changed.

- Let us have a look:

  https://www.glowscript.org/#/user/wolfs/folder/Public/program/Phy235-DrivenHarmonicMotion

Amplitude and phase angle for driven harmonic motion.

$$Q = \frac{\sqrt{\omega_0^2 - 2\beta^2}}{2\beta}$$



Non-harmonic driving forces.

- Most driving forces are not pure harmonic driving forces.
- But every driving force and be written as a sum of pure harmonic functions (Fourier analysis).
- Consider that x_1 is the solution of the following equation:

$$\frac{d^2x_1}{dt^2} + 2\beta\frac{dx_1}{dt} + \omega_0^2 x_1 = f_1(t)$$

- Consider that x_2 is the solution of the following equation:

$$\frac{d^2x_2}{dt^2} + 2\beta\frac{dx_2}{dt} + \omega_0^2 x_2 = f_2(t)$$

- Then $x_1 + x_2$ is the solution of the following equation:

$$\frac{d^2(x_1 + x_2)}{dt^2} + 2\beta\frac{d(x_1 + x_2)}{dt} + \omega_0^2(x_1 + x_2) = f_1(t) + f_2(t)$$

Non-harmonic driving forces.

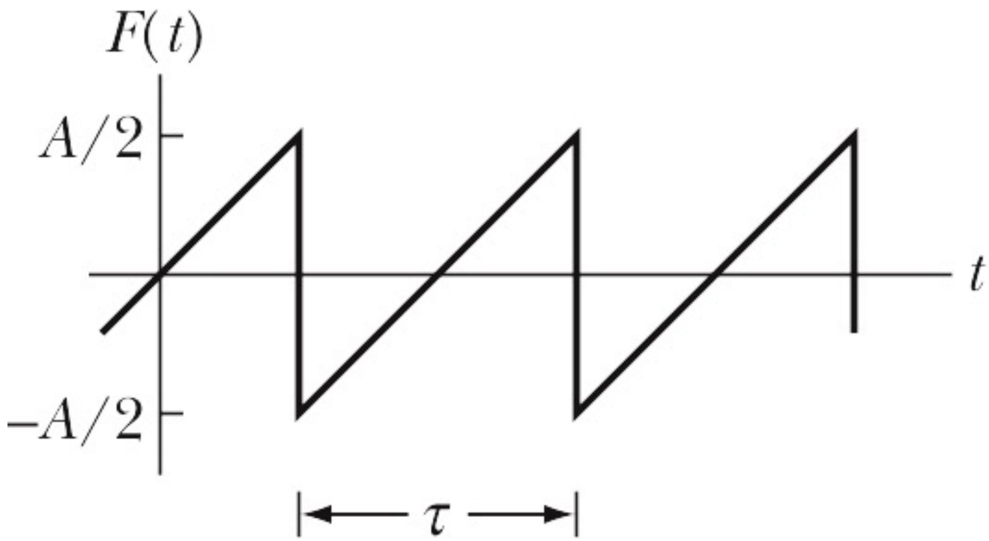
- If the function F has a period τ, then we can write F as:

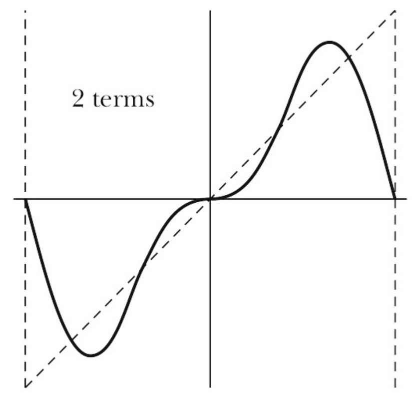
$$F(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$

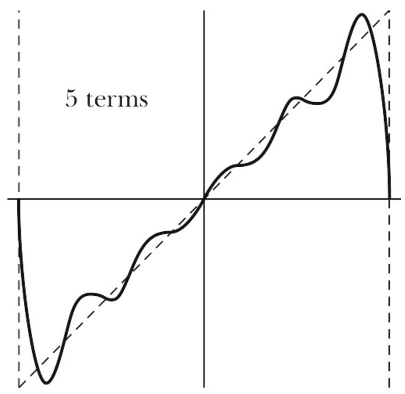
$$\omega = \frac{2\pi}{\tau}$$

- For each term in this series, we can find the solution to the corresponding inhomogeneous equation.
- The total solution is then the linear sum of all of these individual solutions.

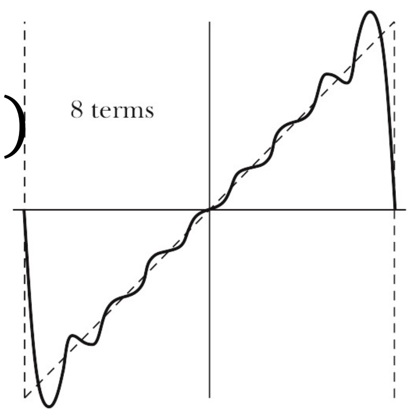
Example of Fourier Analysis.





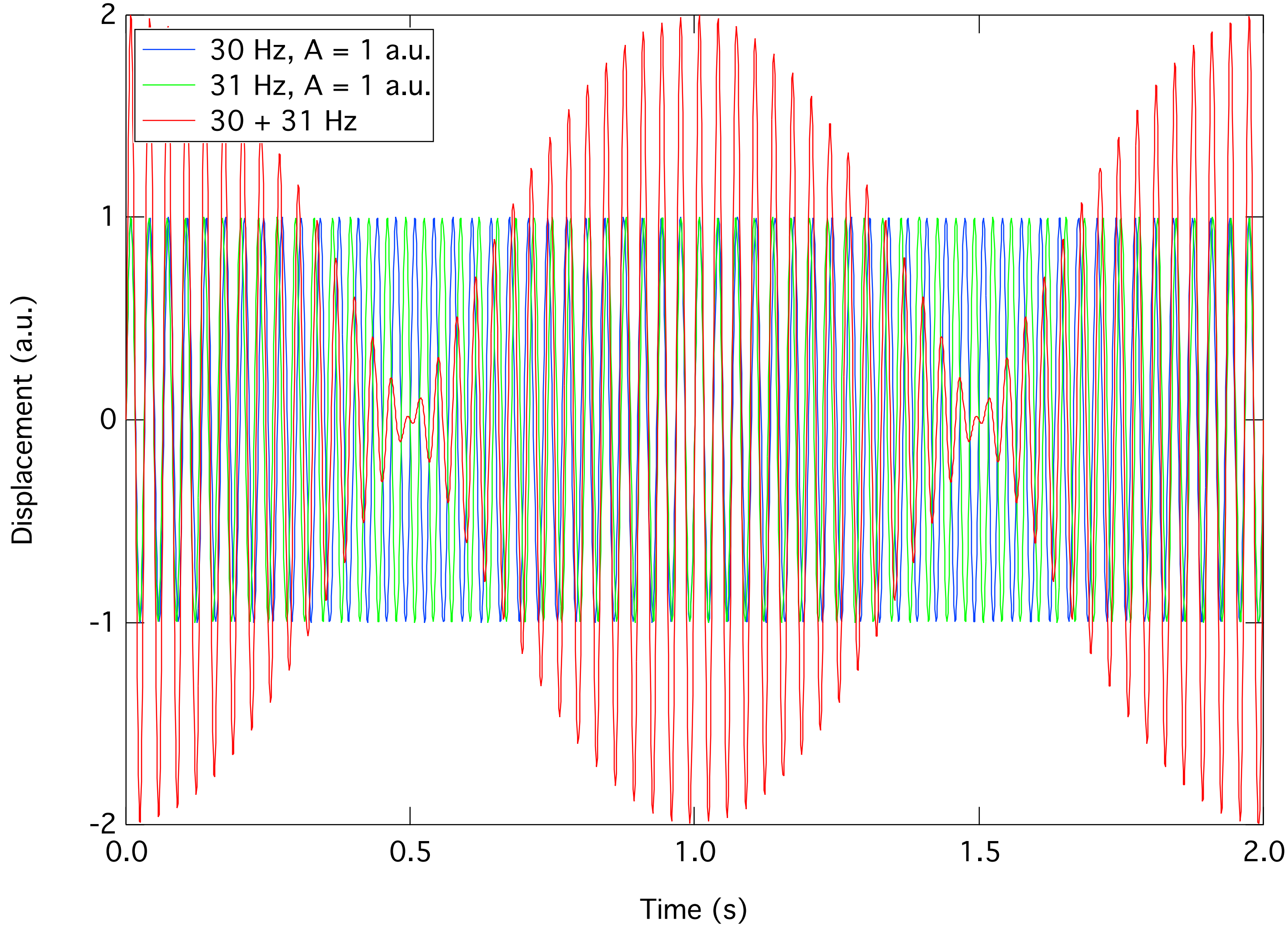


$$F(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$$



$$a_0 = 0$$
$$a_n = 0$$

Adding harmonic functions.



ENOUGH FOR TODAY?