Classical Mechanics
Phy 235, Lecture 02

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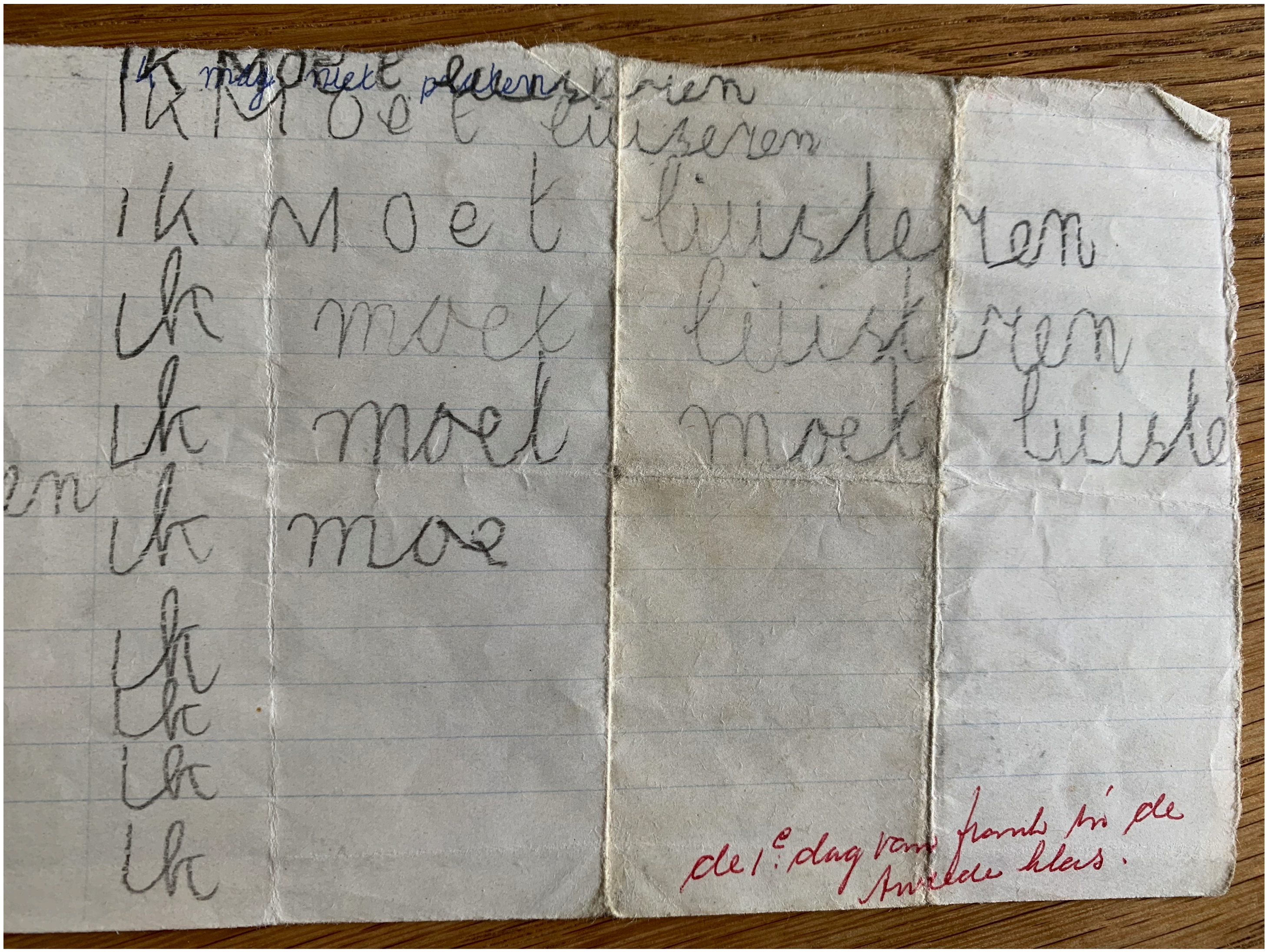
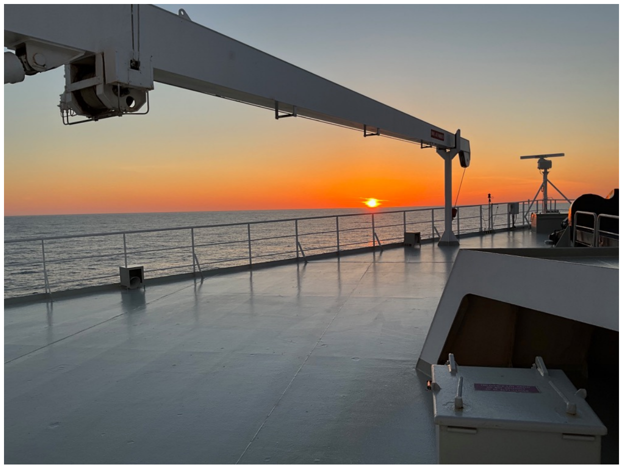
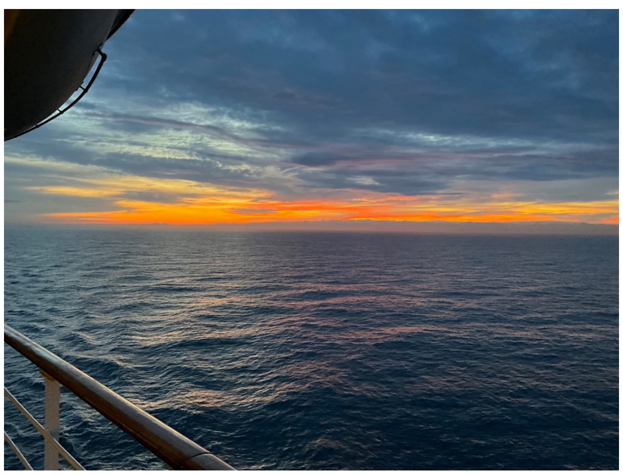
Recently found by my sister: my first day in second grade.



Images from crossing the Atlantic Ocean.





Course Information

- Recitations will start on September 2, 2025.
  - Tuesdays, 4.50 pm - 6.05 pm, Gylan 201.
  - Tuesdays, 6.15 pm - 7.30 pm, Dewey 2110E.
  - Thursdays, 4.50 pm - 6.05 pm, B&L 480
- Note: I will ask the TAs to take attendance during the recitations.

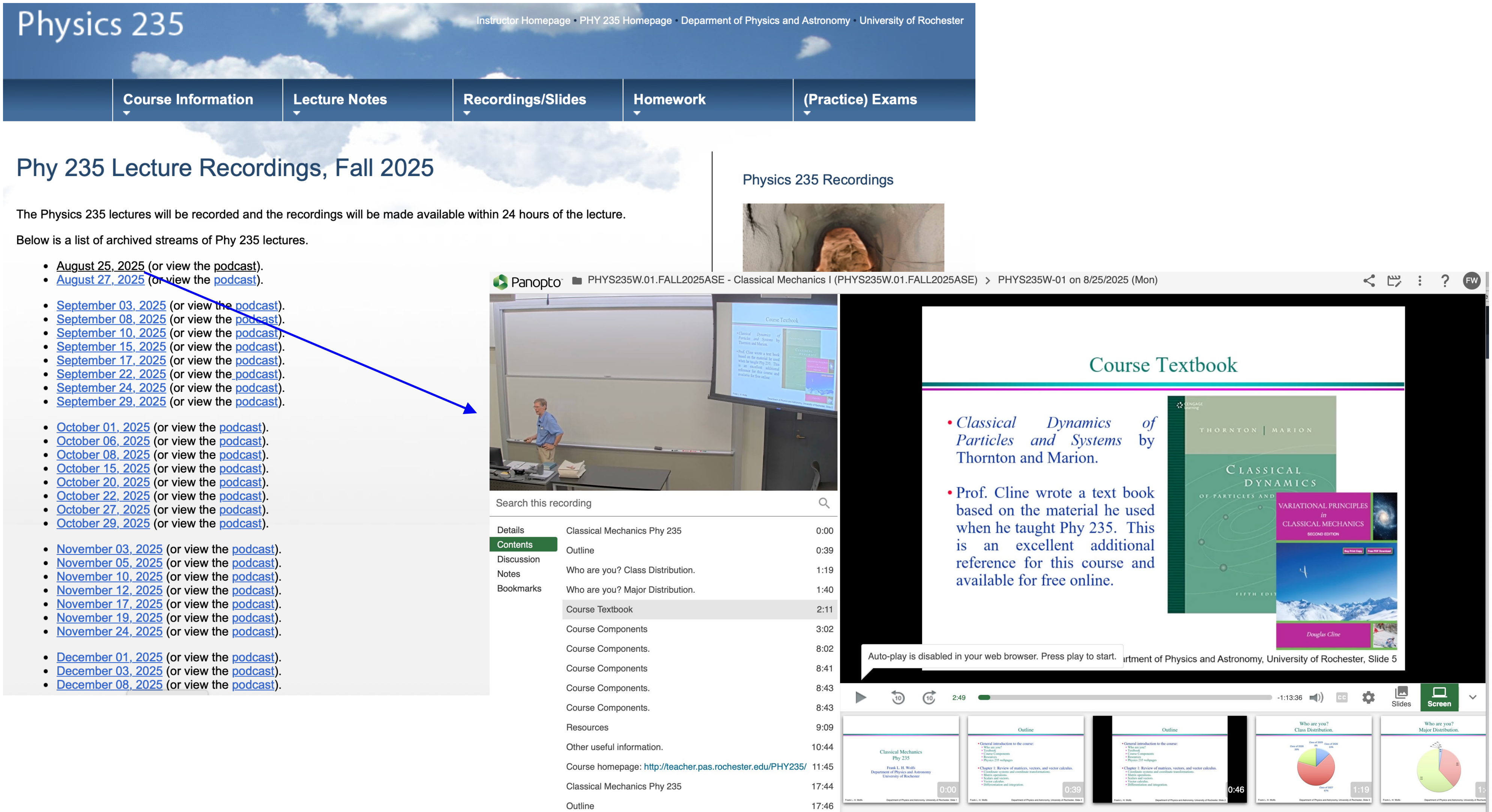
- Office hours Tas will start on September 3, 2025:
  - TBA

- First homework is due on Friday 9/5 at noon.
  - Note: there are optional homework assignments in exam weeks. These assignments are computational based assignments. E.g.:
    http://teacher.pas.rochester.edu/PHY235/HomeWork/Sets/ExtraCreditSet01/ExtraCreditHWSet01.pdf

Computational Techniques,

- Computational techniques are important in order to understand realistic systems.
- You should be familiar with the limitations of computational techniques.
- Many different tools can be used to carry out simulations of mechanical systems:
  - C++
  - Python
  - VPython
  - ……
  - Excel
- In some situations, you want to visualize the solution of a set of (differential) equations. Mathematica is a good tool to do this, although Python with SciPy can provide you with the same functionality.

Lecture recordings will be available after lecture.



Outline

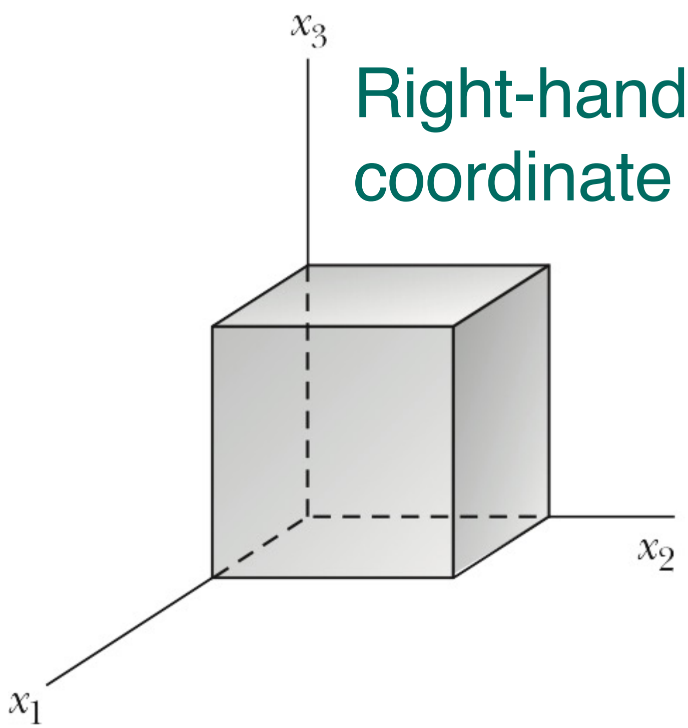
- Chapter 1:
  - A few more comment on rotations (proper and improper).
- Chapter 2: Newtonian Mechanics.
  - Newton's laws.
  - Inertial reference frames.
  - Conservation laws.
  - Motion of a single particle.

Chapter 1: A few additional comments on rotation matrices.

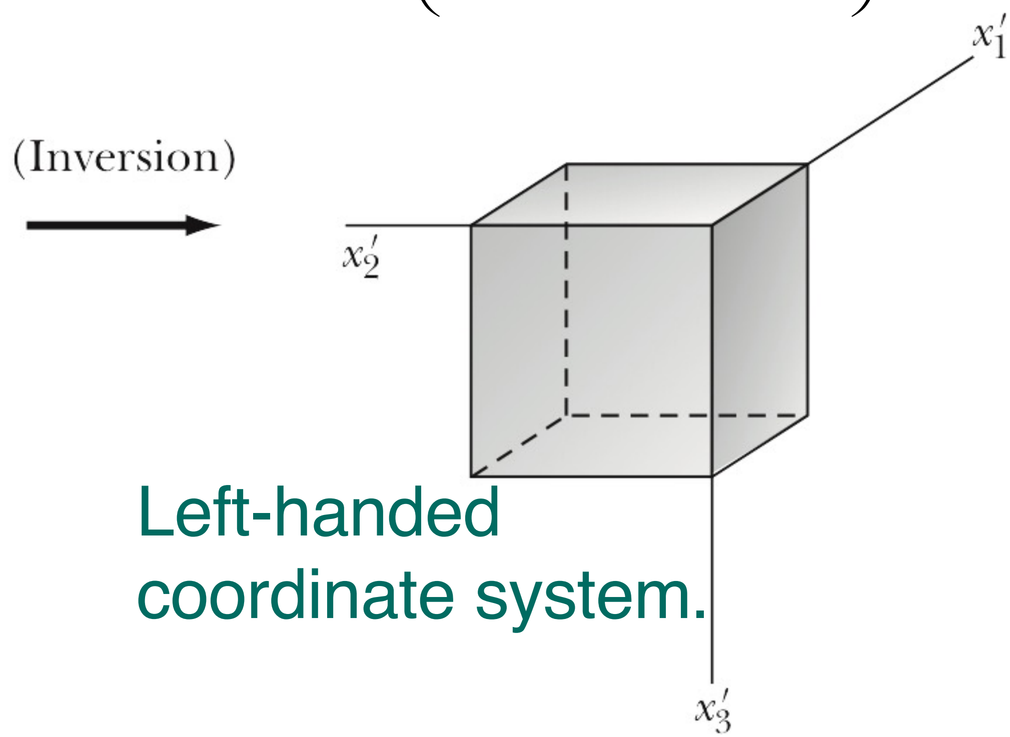
- The determinant of an orthogonal rotation matrix is either +1 or -1.
- Proper rotations:
  - Determinant is +1.
  - Changes a right-handed coordinate system into another right-handed coordinate system.
  - The "laws" of physics are unchanged.
- Improper rotations:
  - Determinant is -1.
  - Cannot be written as a combination of rotations.
  - Always involves a reflection.
  - Changes a right-handed coordinate system into a left-handed coordinate system.
  - The "laws" of physics may be changed.

Inversion: **improper rotation.**
A few more details.

Right-handed coordinate system.

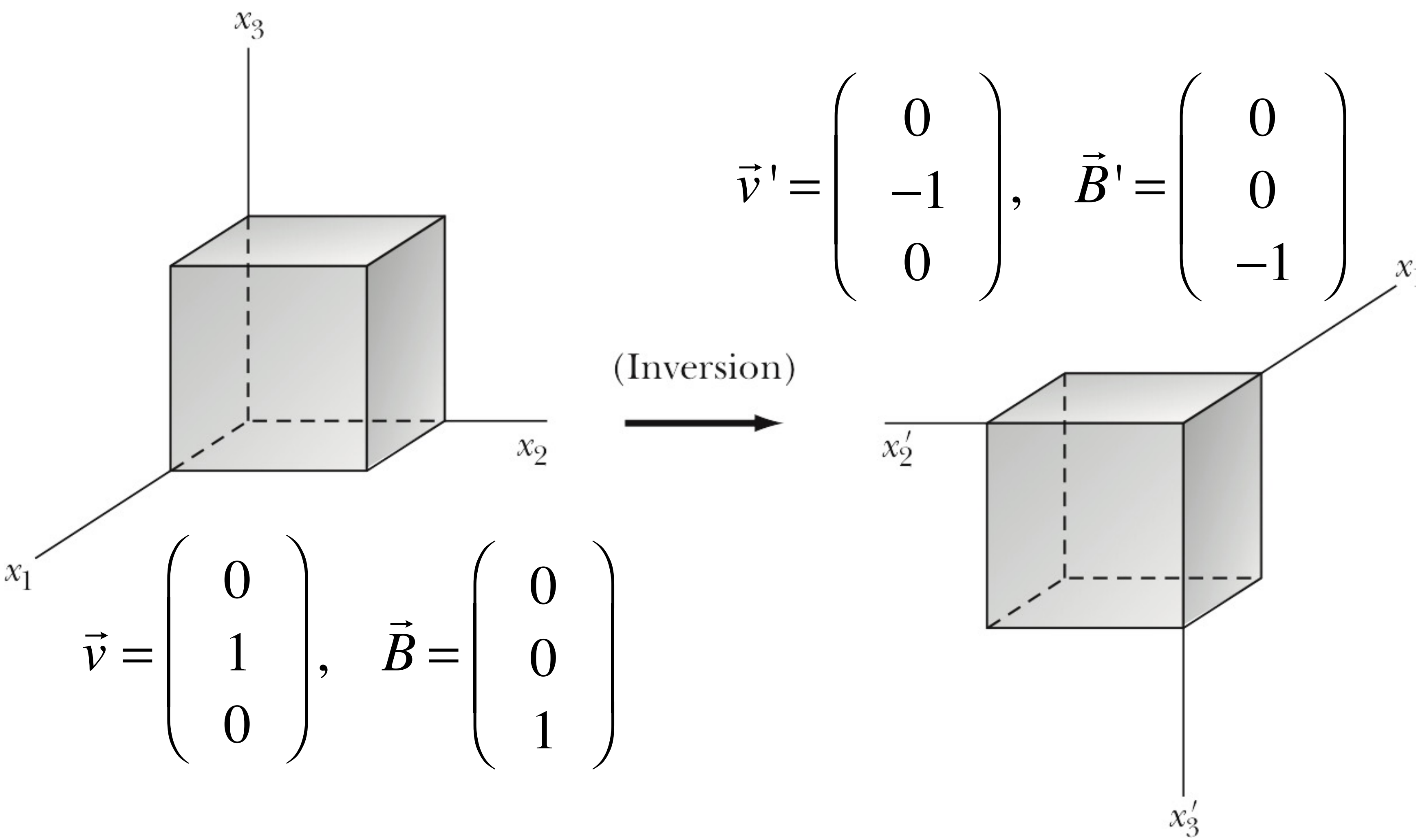


$$\vec{a}\,' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \vec{a}$$



Left-handed coordinate system.

Inversion: **improper rotation.**
Consider the magnetic force.

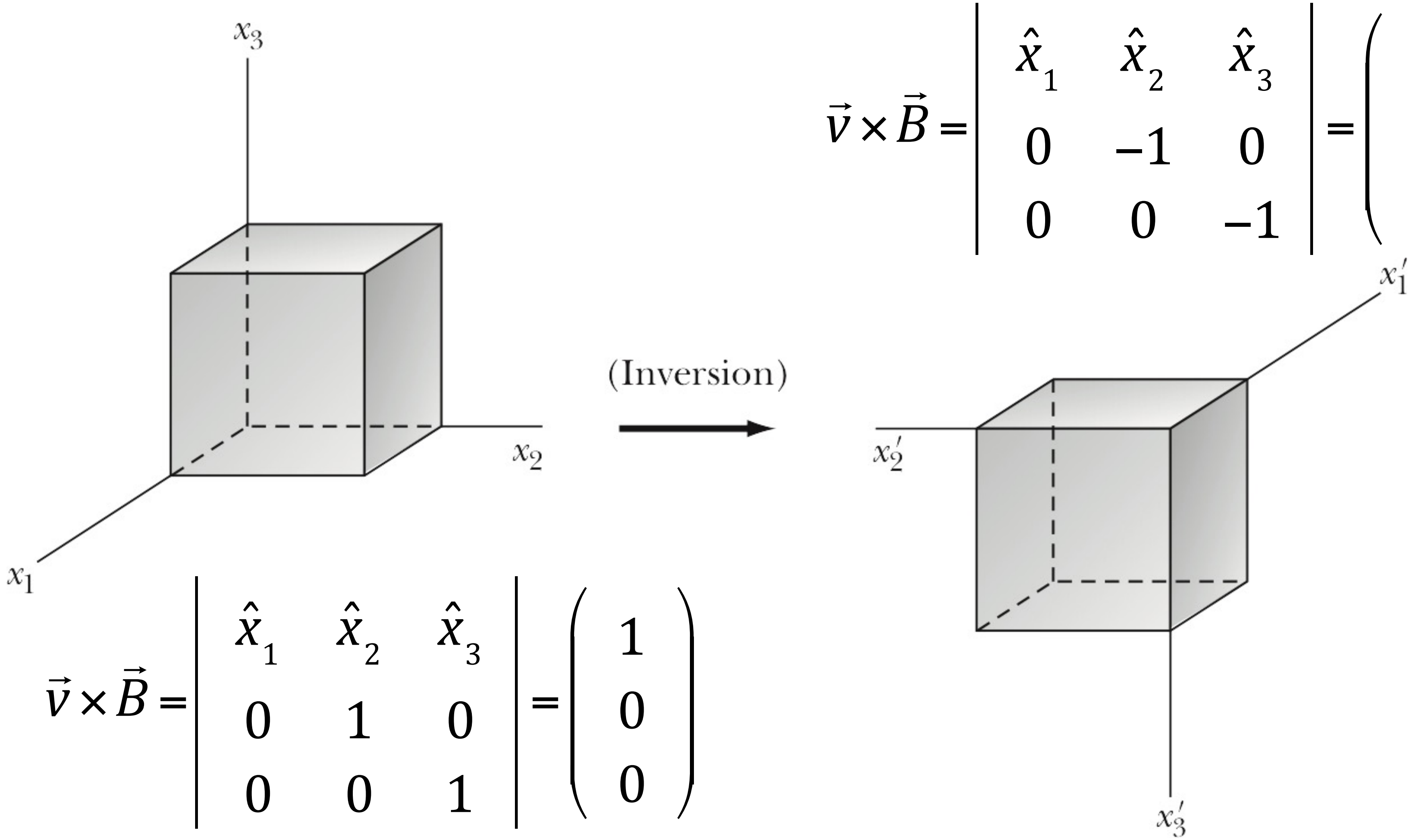


$$\vec{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{v}\,' = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \quad \vec{B}\,' = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

Inversion: **improper rotation.** Consider the magnetic force.

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Chapter 2
Newton's Laws

- First law:
  - A body remains at rest or in uniform motion unless acted upon by a force.
  - Note: uniform motion requires constant speed and constant direction.
- Second law:
  - A body acted upon by a force moves in such a manner that the time rate of change of its linear momentum equals the force.
- Third law:
  - If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.

Reference Systems

- Inertial reference frame:
  - A reference frame in which Newton's laws are valid.
- Specific requirements:
  - The equation of motion of a single particle should be independent of the origin of the coordinate system.
  - The equation of motion of a single particle should be independent of the orientation of the coordinate system.
  - Time must be homogeneous.
- Accelerating reference frames are not good inertial reference frames (e.g. accelerating airplane).
  - The earth is a non-inertial reference frame since it rotates around its axis, since it rotates around the sun, and since the sun rotates around the center of the Milky-Way.

Single-Particle Motion.

- If the force is constant (independent of position and time), we usually can obtain an analytical expression to describe the motion.
- If the force is variable (dependent on for example, position, time, velocity, etc.) we may need to rely on **numerical techniques** to predict the motion.
- Numerical techniques rely on the assumption that during short time intervals dt, the force is constant **and** the direction/ magnitude of the velocity is constant.
- If we make dt small enough, this becomes a reasonable assumption, but the calculation time increases and "rounding errors" may become significant.

Single-Particle Motion. Numerical methods.

- Determining the best time step dt is one of the most important steps in numerical simulations.
- Simple numerical simulations can be carried out with various tools. VPython is a powerful tools for simulations of motion, but even programs such as Excel can be used.
- Getting your code to run is the easy part.
- You will spend 80% of your time convincing yourself that the results you get make sense.
- Comparing results of simulations with the results of analytical calculation is a critical step in verifying the proper operation of your program.

Simulations

- Let us examine how we can use numerical simulations to solve Example problem 2.7.
  - Start with looking at a comparison between the numerical results and analytical calculation to determine proper values of dt.
  - Add a drag force that is proportional to the velocity of the projectile.
  - Code used for the simulations is written for VPython. It is available on the Phy 235 website
- Let us have a look! See:
  https://www.glowscript.org/#/user/wolfs/folder/Public/program/Phy235-ProjectileMotion

2 Minute 59 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let’s take a 2 minute 59 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

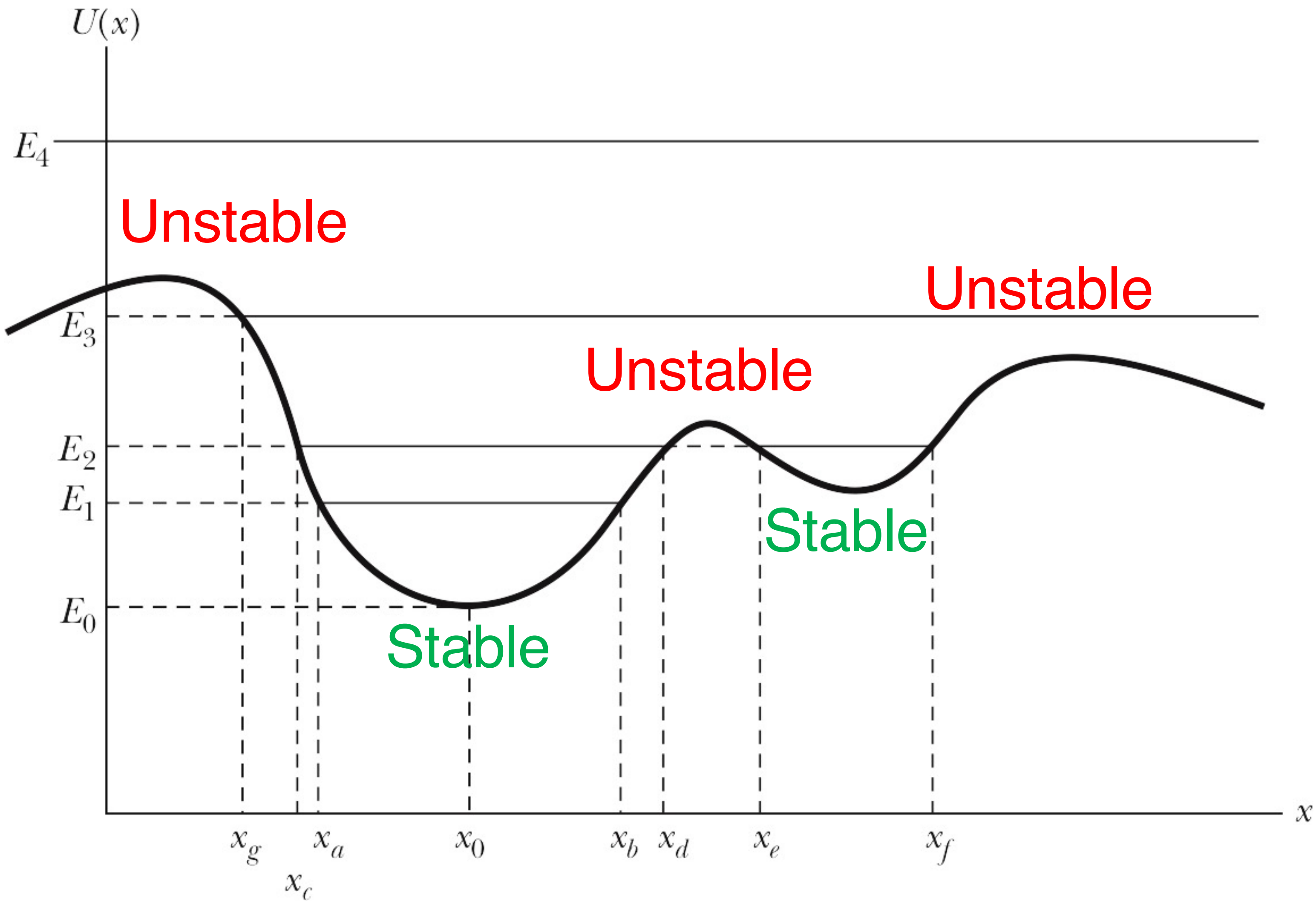
Conservation Laws.

- The following conservation laws are a direct consequence of Newton's laws:
  - Conservation of linear momentum: the total force is 0 N.
  - Conservation of angular momentum: the total torque is 0 Nm.
  - Conservation of energy: energy is constant in a conservative force field that is constant in time. The requirements can be written as:

$$\vec{F} = -\vec{\nabla} U$$

$$\frac{\partial U}{\partial t} = 0$$

Predicting motion based on *U*.



Limitations of Newtonian Mechanics

- Although Newtonian mechanics can be used to describe many macroscopic phenomena, there are three domains where or Newtonian techniques fail:
  - Study of the microscopic world. The description of the microscopic world requires **quantum mechanics**. In this regime, we can not longer measure position and linear momentum with great accuracy and $\Delta x\, \Delta p \geq 10^{-34}$ Js (Heisenberg uncertainty principle).
  - Study of motion with higher velocities (close to c). The description of this regime requires the **theory of relativity.** The speed of light is constant, in any reference frame, clearly inconsistent with the transformation rules of Newtonian mechanics.
  - Study of the properties of systems with large number of particles. The description in this regime requires the theory of **statistical mechanics**. This theory relates the properties of individual interactions between particles to the macroscopic properties of the system.

Problem 2.28.

A superball of mass M and a marble of mass m are dropped from a height h with the marble just on top of the superball. The superball has a coefficient of restitution of nearly 1 (i.e., its collision is essentially elastic). Ignore the size of the superball and marble.

The superball collides with the floor, rebounds, and smacks the marble, which moved back up. How high does the marble go if all the motion is vertical. How high does the superball go?

ENOUGH FOR TODAY?