

Write the following text on the front cover of your homework assignment and sign it. If the text is missing, 20 points will be subtracted from your homework grade.

Honor Pledge for Graded Assignments

"I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

Signature _____

Problem 1 (20 points)

Show that the areal velocity is constant for a particle moving under the influence of an attractive force, given by $F(r) = -kr$. Calculate the time average of the kinetic and the potential energy.

Problem 2 (20 points)

A communication satellite is in a circular orbit of radius R around the earth. Its velocity is v . Its engine accidentally fires, giving the satellite an outward radial velocity v in addition to its original velocity.

1. Calculate the ratio of the new energy and angular momentum to the old.
2. Describe the subsequent motion of the satellite and plot $T(r)$, $U(r)$, and $E(r)$ after the engine fires.

Note: for a circular orbit, $T = -U/2$.

Problem 3 (20 points)

A particle of unit mass moves from infinity along a straight line that, if continued, would allow it to pass a distance $\sqrt{2}b$ from a point P . If the particle is attracted toward P with a force varying as k/r^5 , and if the angular momentum about P is $(\sqrt{k})/b$, show that the trajectory is given by

$$r = b \coth\left(\frac{\theta}{\sqrt{2}}\right) \quad (1)$$

Problem 4 (20 points)

A particle moves in an elliptical orbit in an inverse-square-law central-force field. If the ratio of the maximum angular velocity to the minimum angular velocity of the particle in its orbit is n , then show that the eccentricity of the orbit

$$\varepsilon = \frac{\sqrt{n} - 1}{\sqrt{n} + 1} \quad (2)$$

Problem 5 (20 points)

Consider the family of orbits in a central potential for which the total energy is a constant. Show that if a stable circular orbit exists, the angular momentum associated with this orbit is larger than that for any other orbit in the family.