Write the following text on the front cover of your homework assignment and sign it. If the text is missing, 20 points will be subtracted from your homework grade.

Honor Pledge for Graded Assignments
"I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

Signature ______________________________

Problem 1 (20 points)
Construct a phase diagram for the potential $U(x)$ = -(l/3)x^3.

Problem 2 (20 points)
Use numerical calculations to find a solution of the van der Pol oscillator:

$$\ddot{x} + \mu\left(x^2 - a^2\right)\dot{x} + {\omega_0}^2 x = 0 \tag{1}$$

Let a and ω_0 be equal to 1. Plot the phase diagram of the solution for the following conditions:

1. $\mu = 0.007$, $x_0 = 1.0$, $dx_0/dt = 0$.
2. $\mu = 0.007$, $x_0 = 3.0$, $dx_0/dt = 0$.
3. $\mu = 0.05$, $x_0 = 1.0$, $dx_0/dt = 0$.
4. $\mu = 0.05$, $x_0 = 3.0$, $dx_0/dt = 0$.

In each case, describe whether the motion appears to approach a limit cycle.

Problem 3 (20 points)
Calculate the potential due to a thin circular ring of radius a and mass M for points lying in the plane of the ring and exterior to it. The result can be expressed as an elliptical integral (see Appendix B for a list of elliptical integrals).
For the region where the distance from the center of the ring is large compared to the radius of the ring, expand the expression for the potential and find the first correction term.

Problem 4 (20 points)
The orbital revolution of the Moon about the Earth takes 27.3 days and is in the same direction as the Earth's rotation (24 hours). Use this information to show that high tides occur everywhere on Earth every 12 hours and 26 minutes.

Problem 5 (20 points)
The so-called *tent* map is represented by the following iterations:

$$\begin{aligned} x_{n+1} &= 2\alpha x_n && \text{for } 0 < x < \tfrac{1}{2} \\ x_{n+1} &= 2\alpha(1 - x_n) && \text{for } \tfrac{1}{2} < x < 1 \end{aligned} \tag{2}$$

where $0 < \alpha < 1$.

1. Make a map up to 20 iterations for $\alpha = 0.4$ and $\alpha = 0.7$ with $x_1 = 0.2$.
2. Plot a bifurcation diagram for the *tent* map.
3. Show analytically that the Lyapunov exponent for the *tent* map is $\lambda = \ln(2a)$. This indicates that chaotic behavior occurs for $a > 1/2$.