Write the following text on the front cover of your homework assignment and sign it. If the text is missing, 20 points will be subtracted from your homework grade.

Honor Pledge for Graded Assignments
"I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own."

Signature _____

Problem 1 (20 points)
Obtain an expression for the fraction of a complete period that a simple harmonic oscillator spends within a small interval Δx at a position x. Sketch the curves of this function versus x for several different amplitudes. Discuss the physical significance of the results. Comment on the areas under the various curves.

Problem 2 (20 points) A particle of mass m is at rest at the end of a spring (force constant k) hanging from a fixed support. At time $t = 0$, a constant downward force F is applied to the mass and acts for a time t_0. Show that, after the force is removed, the displacement of the mass from its equilibrium position ($x = x_0$, where x is down) is

$$x - x_0 = \frac{F}{k}\left[\cos\left(\omega_0\left(t - t_0\right)\right) - \cos\left(\omega_0 t\right)\right] \tag{1}$$

where

$$\omega_0{}^2 = \frac{k}{m} \tag{2}$$

Problem 3 (20 points)
For a damped driven oscillator, show that the average kinetic energy is the same at a frequency of a given number of octaves (an octave is a frequency interval in which the highest frequency is just twice the lowest frequency) above the kinetic energy resonance as at a frequency of the same number of octaves below the resonance.

Problem 4 (20 points)
Use the general solutions $x(t)$ to the differential equation

$$\ddot{x} + 2\beta\dot{x} + {\omega_0}^2 x = 0 \tag{3}$$

for under damped, critically damped, and over damped motion and choose the constants of integration to satisfy the initial conditions $x = x_0$ and $v = v_0 = 0$ m/s at $t = 0$ s. Use a computer to plot the results for $x(t)/x_0$ as a function of $w_0 t$ for the following three cases:

1. $\beta = (1/2)\omega_0$
2. $\beta = \omega_0$
3. $\beta = 2\omega_0$

Show all three curves on a single plot.

Problem 5 (20 points)
A damped linear oscillator, originally at rest in its equilibrium position, is subjected to a forcing function given by

$$\frac{F(t)}{m} = \begin{cases} 0 & t < 0 \\ a\left(\frac{t}{\tau}\right) & 0 < t < \tau \\ a & \tau < t \end{cases} \tag{4}$$

Find the response function. Allow $\tau \to 0$ s and show that the solution becomes that of a step function.