Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use **only** a writing instrument while taking this test. You may **not** consult any calculators, computers, books, nor each other.

Problems 1 and 2 must be answered in exam booklet 1. Problems 3 and 4 must be answered in exam booklet 2. Your answers to Problems 1, 2, and 3 must be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only if we can read your solution. Answers that are not motivated will not receive any credit, even if correct. No justification is required for your answers to Problem 4, although you are of course free to provide it.

At the end of the exam, you must hand in your exam with the honor pledge completed, the two blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Signature: ________________________________

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Good Luck !

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Problem 1 (25 points) **Answer in booklet 1**

Consider a particle of mass m suspended between two identical springs, as shown in Fig. 1. When the particle is in its equilibrium position, both springs are in their unextended condition (that is, the rest length of each spring is l). In this problem, we will ignore the gravitational force acting on the particle.

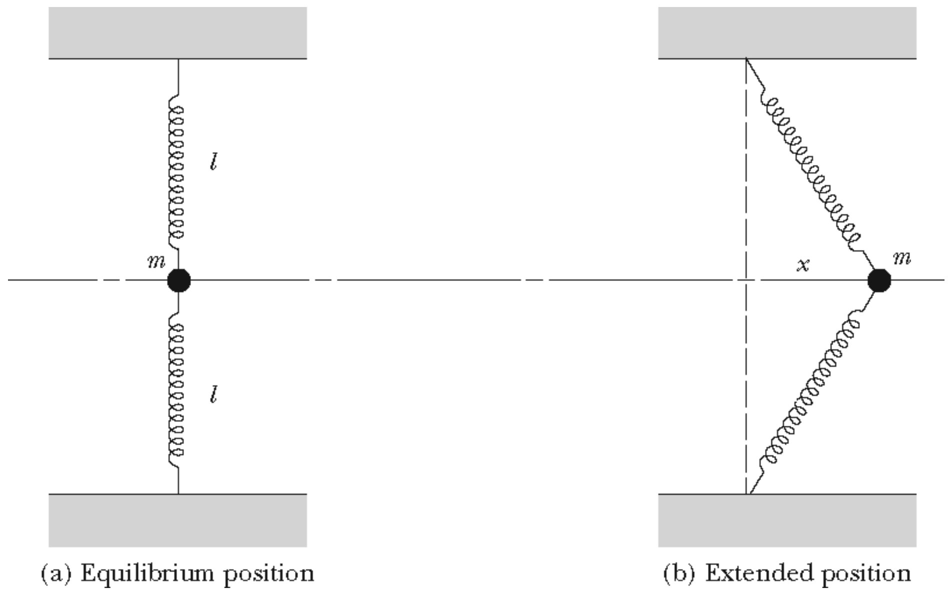

Figure 1: A double-spring system in (a) its equilibrium position and (b) its extended position.

a) **(10 points)** What is the net force acting on the particle when it is in its extended position shown in Fig. 1b? **Express your answer in terms of** k**,** m**,** x**, and** l**.** Note: you need to specify both its magnitude and its direction.

b) **(5 points)** Consider x/l to be small ($x/l \ll 1$). Use a Taylor expansion to rewrite the force on the particle obtained in part a) in terms of powers of x/l.

c) **(5 points)** Using the result obtained in part b), write down the equation of motion of the particle.

d) **(5 points)** Is the equation of motion linear or non-linear? If it is non-linear, is the system a hard or soft system?

Your answer needs to be well motivated and expressed in terms of the variables provided.

Problem 2 (25 points) **Answer in booklet 1**

Two masses m_1 and m_2 slide freely on a horizontal frictionless track and are connected by a spring with a force constant k and rest length L. The system is shown schematically in Fig. 2.

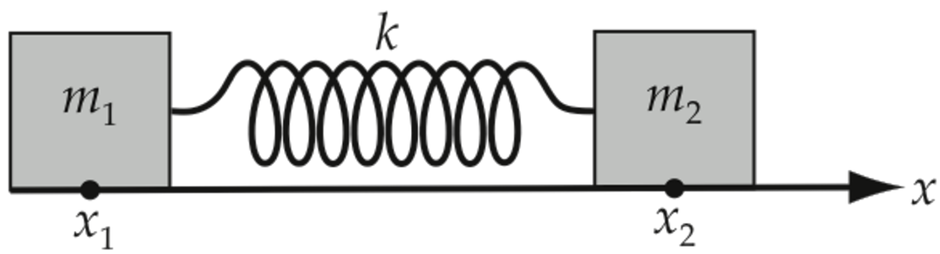

Figure 2: Motion of a spring-mass system.

a) **(5 points)** What is the equation of motion of mass m_1?

b) **(5 points)** What is the equation of motion of mass m_2?

c) **(5 points)** Uncouple the two equations of motion obtained in parts a) and b) to determine an equation of motion of mass m_2, in terms of the position of mass m_2.

d) **(5 points)** Show that simple harmonic motion is one possible solution to the uncoupled equation obtained in part c). What is the frequency of the harmonic motion?

e) **(5 points)** Show that linear motion of mass m_2 with constant velocity is also a solution to the uncoupled equation obtained in part c).

Your answer needs to be well motivated and expressed in terms of the variables provided.

Problem 3 (25 points) **Answer in booklet 2**

a) **(10 points)** Consider the following force.

$$F_x = ayz + bx + c, \quad F_y = axz + bz, \quad F_z = axy + by \tag{1}$$

In this equation, a, b, and c are constants.

Is this a conservative force? If your answer is yes, find the corresponding potential energy.

b) **(15 points)** Consider the following force.

$$\vec{F} = \left(\frac{a}{r}\right)\hat{r} \tag{2}$$

In this equation, a is a constant.

Is this a conservative force? If your answer is yes, find the corresponding potential energy.

Your answer needs to be well motivated and expressed in terms of the variables provided.

Problem 4 (25 points) **Answer in booklet 2**

a) **(5 points)** Consider a damped and driven pendulum. The Poincare sections for seven different driving force amplitudes are shown in Fig. 3. The corresponding driving forces are $F = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, and 1.0 N (from top to bottom). For each Poincare section, indicate if the motion is

1. Simple Harmonic.
2. Periodic.
3. Chaotic.

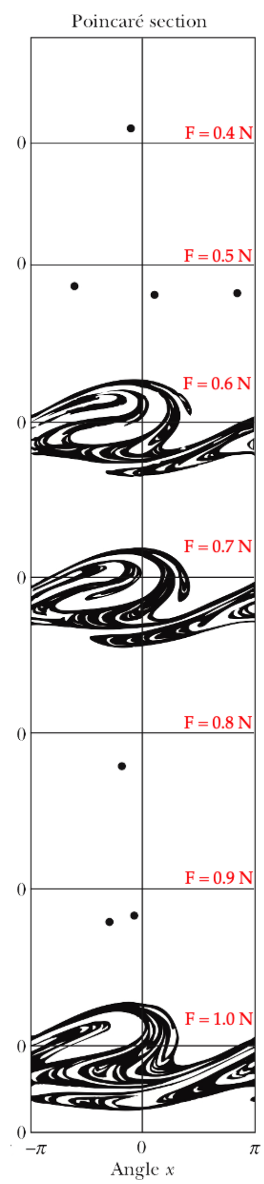

Figure 3: Poincare plots for a damped and driven pendulum.

b) **(5 points)** Consider the phase diagrams of the damped harmonic motion of a ball shown in Fig. 4. For each of the three diagrams, indicate if the motion is overdamped, critically damped, or underdamped.

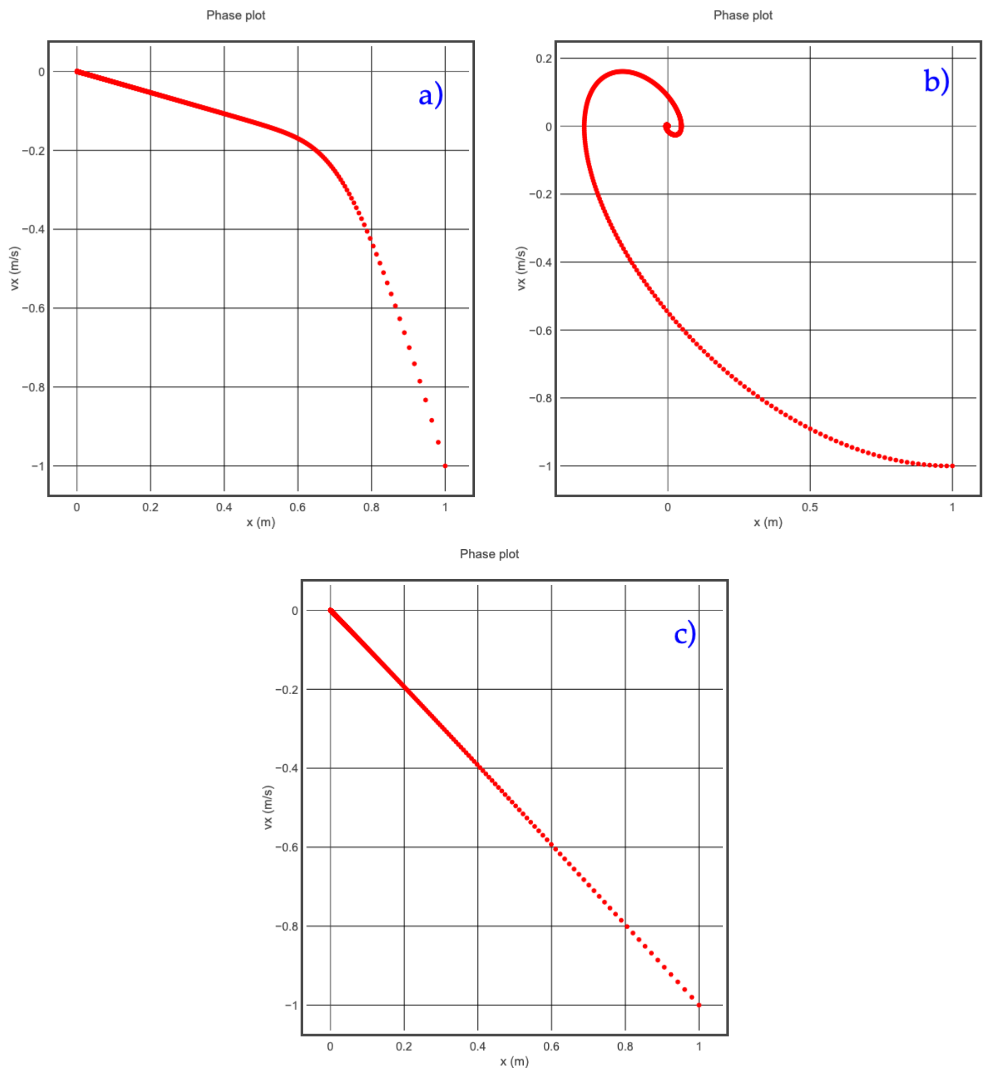

Figure 4: Phase diagrams of the damped harmonic motion of a ball with different damping constants. The harmonic motion displayed in each graph has the same angular frequency ω_0 but different damping constants β. In these simulations, the initial position of the ball is (1 m, 0 m, 0 m) and the initial velocity is (-1 m/s, 0 m/s, 0 m/s).

c) **(5 points)** Figure 5 shows phase paths for three different paths (I, II, and III), each having different initial conditions. Figure 6 shows the position and velocity as function of time for the three phase paths shown in Fig, 5. Match the graphs shown in Fig, 6 to the paths shown in Fig, 5.

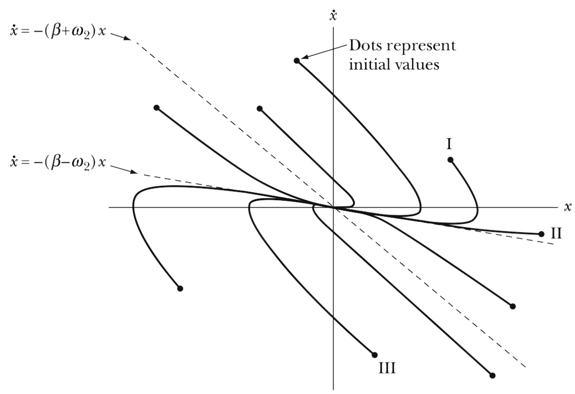

Figure 5: Phase paths of an overdamped harmonic oscillator are shown for several initial values of $(x, \dot{x})$.

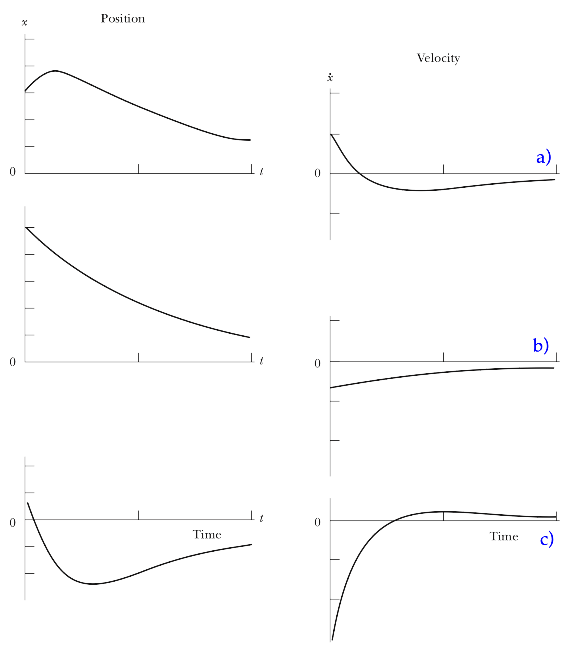

Figure 6: Position and velocity as function of time for the three phase paths shown in Fig. 5.

1) What path in Fig, 5 corresponds to the graphs in Fig. 6a?
2) What path in Fig, 5 corresponds to the graphs in Fig. 6b?
3) What path in Fig, 5 corresponds to the graphs in Fig. 6c?

d) **(5 points)** The solid curve in Figure 7 shows an example of an asymmetric force.

1) Is the system hard or soft in the region where $x < 0$?
2) Is the system hard or soft in the region where $x > 0$?
3) Make a sketch of the potential energy distribution for the asymmetric force shown in Fig. 7. In the same figure, also show the potential energy distribution for the linear system. Use a solid curve for the asymmetric system and a dashed curved for the linear system

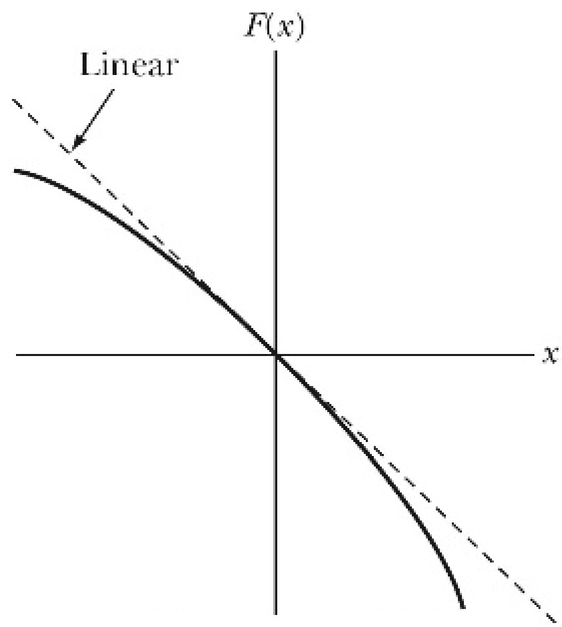

Figure 7: Example of an asymmetric force. The dashed line shows the force versus x graph for a linear system (e.g. the spring force).

e) **(5 points)** What is NAP?

Figure 8: NAP Reference.

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Figure 9: https://www.linkedin.com/posts/doramacovei_10-of-oldest-airlines-still-flying-today-activity-6478763341538955264-dPIT?utm_source=share&utm_medium=member_desktop&rcm=ACoAABfrNLEBMoGJjDhoyYO1UJtS39depjvVM5