Red Sox - Yankees. Baseball can not get more exciting than these games.

Physics 121, April 17, 2008.
Kinetic theory of gases.

http://eml.ou.edu/Physics/module/thermal/ketcher/Idg4.avi

Physics 121.
April 17, 2008.

• Course Information
• Topics to be discussed today:
  • The ideal gas law (review).
  • Molecular interpretation of temperature.
  • The real equation of state.
• Quiz
• Homework set # 9 is due on Saturday morning, April 19, at 8.30 am.
• Midterm Exam # 3 will take place on Tuesday April 22 between 8.00 am and 9.30 am. The material to be covered is the material contained in Chapters 10, 11, 12, and 14.
• I will distribute the formula sheet that will be attached to the exam later this week via email.
• I will review the material covered on the exam on Sunday evening. Further details will be announced via email.
• Extra office hours will be offered by the TAs on Sunday and Monday. The schedule will be announced via email.

• There will be no class after the exam on Tuesday.

• Workshops schedule next week:
  • There will be workshops on Monday to answer questions related to Exam # 3.
  • There will be workshops on Thursday and Friday to discuss the solutions of Exam # 3 and return your blue booklets.
  • During the week of April 28 there will only be workshops on Monday, Tuesday, and Wednesday.
  • To increase the efficiency with which we can return the blue booklets of Exam # 3, you will need to write your workshop day/time on the front of each booklet.
  • The results of Exam # 3 will be emailed to you on Monday April 28

• Grades: Worst case scenario:
  • Let’s assume you got 0% on exam 1 and 0% on exams 2.
  • Clearly you would benefit if we drop one of these grades!
  • If we drop one of these grades, than the one we count will count for 20% of your final grade.
  • If you get 100% on exam # 3 and 100% on the final (and assuming you have 100% on the homework sets, quizzes, and labs) you end up with a final grade for 80% or an A!!!
  • No one matches this worst case scenario, but realize that even if you did poorly on the first two exams, you can still get an A in this course.
The equation of state of a gas.
A quick review.

- The equation of state of a gas was initially obtained on the basis of observations.
  - Boyle’s Law (1627 - 1691): \( pV = \text{constant} \) for gases maintained at constant temperature.
  - Charles’s Law (1746 - 1823): \( V/T = \text{constant} \) for gases maintained at constant pressure.
  - Gay-Lussac’s Law (1778 - 1850): \( pT = \text{constant} \) for gases maintained at constant volume.

The equation of state of a gas can be written as

\[ pV = nRT \]

where

- \( p \) = pressure (in Pa).
- \( V \) = volume (in m\(^3\)).
- \( n \) = number of moles of gas (1 mole = \( 6.02 \times 10^{23} \) molecules or atoms). Note the number of molecules in a mole is also known as Avogadro’s number \( N_A \).
- \( R \) = the universal gas constant (\( R = 8.315 \text{ J/(mol K)} \)).
- \( T \) = temperature (in K).

Note: the equation of state is the equation of state of an ideal gas. Gases at very high pressure and/or close to the freezing point show deviations from the ideal gas law.

The equation of state of a gas can also be written as

\[ pV = nRT = (N/N_A)RT = NK \]

where

- \( N \) = total number of molecules
- \( N_A \) = Avogadro’s number
- \( k = R/N_A \) is the Boltzmann constant = \( 1.38 \times 10^{-23} \text{ J/K} \)

This is the most frequently used form of the equation of state of the gas.
The molecular point of view of a gas.

- Consider a gas contained in a container.
- The molecules in the gas will continuously collide with the walls of the vessel.
- Each time a molecule collides with the wall, it will carry out an elastic collision.
- Since the linear momentum of the molecule is changed, the linear momentum of the wall will change too.
- Since force is equal to the change in linear momentum per unit time, the gas will exert a force on the walls.

The molecular point of view of a gas.

- Consider the collision of a single molecule with the left wall.
- In this collision, the linear momentum of the molecule changes by \( mv_x - (-mv_x) = 2mv_x \).
- The same molecule will collide with this wall again after a time \( \Delta t = \frac{2l}{vx} \).
- The force that this single molecule exerts on the left wall is thus equal to \( \Delta p/\Delta t = \frac{(2mv_x)}{(2l/vx)} = m v_x^2/l \).

The molecular point of view of a gas.

- The force that this single molecule exerts on the left wall is thus equal to \( F_{left} = m v_x^2/l \).
- If the pressure exerted on the left wall by this molecule is equal to \( p_{left} = \frac{F_{left}A}{A} = m v_x^2/(lA) \) where \( A \) is the area of the left wall.
- The volume of the gas is equal to \( lA \) and we can thus rewrite the pressure on the left wall: \( p_{left} = m v_x^2/V \).
The molecular point of view of a gas.

- The pressure that many molecules exert on the left wall is equal to
  \[ p_{\text{left}} = \frac{m(v_1^2 + v_2^2 + v_3^2 + \ldots)}{V} \]
- This equation can be rewritten in terms of the average of the square of the \( x \) component of the molecular velocity and the number of molecules \( (N) \):
  \[ p_{\text{left}} = \frac{mN(v_x^2)}{3V} \]
- Assuming that there is no preferential direction, the average square of the \( x \), \( y \), and \( z \) components of the molecular velocity will be the same:
  \( (v_x^2)_{\text{average}} = (v_y^2)_{\text{average}} = (v_z^2)_{\text{average}} \)
- The force on the left wall can be rewritten in terms of the average squared velocity
  \[ p_{\text{left}} = \frac{mN(v^2)}{3V} \]
- Assuming there is no preferential direction of motion of the molecules, the pressure on all walls will be the same and we thus conclude:
  \[ pV = \frac{mN(v_x^2)}{3} \]
- Compare this to the ideal gas law:
  \[ pV = NkT \]
- Comparing \( pV = \frac{mN(v_x^2)}{3} \) and the ideal gas law \( pV = NkT \) we find that
  \[ kT = \frac{m(v_x^2)}{3} = \frac{(2/3)K_{\text{average}}}{3} \]
- We thus conclude that:
  - The average kinetic energy of the gas molecules is proportional to the temperature of the gas.
  - When the temperature approaches zero, the average kinetic energy approaches zero.
Distribution of molecular speeds. The Maxwell distribution.

Note: the integral does not change when T changes!

Increasing T increases the fraction of molecules above a certain velocity.

Note: all velocities scale with $\sqrt{T}$

Molecular speed and the mean-free path.

- The RMS velocities of individual gas molecules are large. For example, for hydrogen at room temperature, the RMS velocity is $1920 \text{ m/s}$.
- Despite the large RMS velocity, the average diffusion velocity is much smaller and is largely determined by the mean-free path of the molecules.
- We expect that the mean-free path is inversely proportional to the cross-sectional area of the molecules and inversely proportional to the density.

Typical values of the mean-free path are between $10^{-8}$ and $10^{-7} \text{ m}$.
Molecular speed and the propagation of sound.

- Typical values for $v_{\text{rms}}$:
  - For H at 300 K, $v_{\text{rms}} = 1920$ m/s
  - For N at 300 K, $v_{\text{rms}} = 517$ m/s
- The speed of sound in these two gases is 1350 m/s for H and 350 m/s for N.
- Note: The speed of sound in a gas will always be less than $v_{\text{rms}}$ since the sound propagates through the gas by disturbing the motion of the molecules. The disturbance is passed on from molecule to molecule by means of collisions; a sound wave can therefore never travel faster than the average speed of the molecules. Since $v_{\text{rms}}$ increases with $T$, we expect $v_{\text{sound}}$ to increase with $T$.

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Molecular speed and evaporation.

- Evaporation: transformation from the liquid to the gas phase:
  - A microscopic view of evaporation:
    - Molecules with high velocity moving close to the surface can overcome the strong attractive forces between the molecules and escape from the liquid (evaporation).
    - The average velocity of the molecules left behind in the liquid will be lowered.
    - Since the average velocity is proportional to the temperature, the temperature of the liquid is lowered when evaporation takes place.

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The real equation of state.  
Different points of view.

Note:
1. $T > T_c$: gas
2. $T < T_c$: liquid and/or vapor.

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The real equation of state.
Different points of view.

Note the curvature of the solid-liquid line. Curvature to the left implies expansion on cooling.

Direct change from solid to vapor.

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The real equation of state.
Different points of view.

Note the curvature of the solid-liquid line. Curvature to the left implies expansion on cooling.

Water

CO₂

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Physics 121.
Quiz lecture 24.

• The quiz today will have 3 questions!

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Done for today! Next week:
The first law of thermodynamics.

Massive Star Forming Region DR21 in Infrared.
Credit: A. Marston (ESTRACK) et al., JPL, Caltech, NASA