Problem 1 (2.5 points) **Answer on Scantron form**

What is the proper translation of the name Schiphol?

Figure 1: Schiphol.

1. Amsterdam Airport.
2. **Cemetery of ships.**
3. Hole in the dike.
4. Where a lake used to be.
5. Below sea level.
6. The best airport in the world.
7. Home of the KLM.
8. Prof. Wolfs delivered newspapers there.

Option 2 is the correct answer.

Problem 2 (2.5 points) **Answer on Scantron form**

A device consists of eight balls, each of mass M, attached to the ends of low-mass spokes of length L. The device is mounted in the vertical plane, as shown in Fig. 2. The axle is held up by supports that are not shown and the wheel is free to rotate on the nearly frictionless axle. A lump of clay with mass m falls and sticks to one of the balls at the location shown. Just before the collision, the device was rotating counter-clockwise with a constant angular velocity.

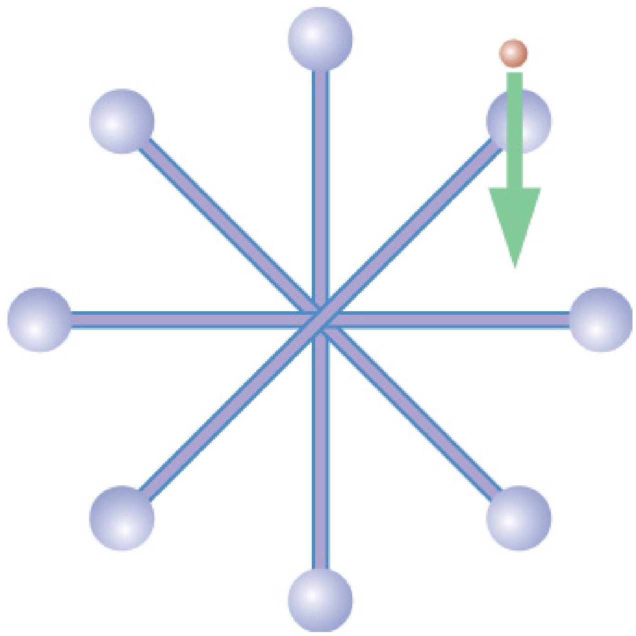

Figure 2: Rotating device colliding with a lump of clay.

Consider the following statements if the angular momentum is calculated relative to the axle of the device.

(a) The angular momentum of the device + clay just after the collision is equal to the angular momentum of the device + clay just before the collision.

(b) The angular momentum of the clay is zero because the clay is moving in a straight line.

(c) Just before the collision, the angular momentum of the device is 0.

(d) The angular momentum of the device is the sum of the angular momenta of the eight balls.

(e) The angular momentum of the device is the same before and after the collision.

Which of the above statements are true?

1. Statements (a) and (b) are correct.

2. Statements (a) and (c) are correct.
3. **Statements (a) and (d) are correct.**
4. Statements (a) and (e) are correct.
5. Statements (b) and (c) are correct.
6. Statements (b) and (d) are correct.
7. Statements (b) and (e) are correct.
8. Statements (c) and (d) are correct.
9. Statements (c) and (e) are correct.
10. Statements (d) and (e) are correct.

Option 3 is the correct answer.

Problem 3 (2.5 points) **Answer on Scantron form**

A ball falls straight down in the xz plane, as shown in Fig. 3. The linear momentum of the ball is shown by the arrow.

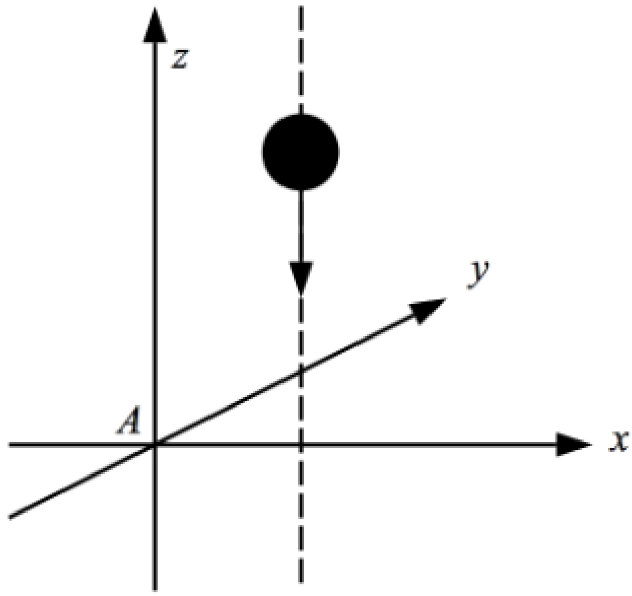

Figure 3: A ball falling in the xz plane.

What is the direction of the angular momentum of the ball about the origin of the coordinate system (point A)?

1. $\hat{\boldsymbol{x}}$
2. -$\hat{\boldsymbol{x}}$
3. $\hat{\boldsymbol{y}}$
4. -$\hat{\boldsymbol{y}}$
5. $\hat{\boldsymbol{z}}$
6. -$\hat{\boldsymbol{z}}$
7. 0

Option 3 is the correct answer.

Problem 4 (2.5 points) **Answer on Scantron form**

A diatomic molecule, such as molecular nitrogen, consists of two atoms, each of mass M, whose nuclei are a distance d apart, as shown in Fig. 4.

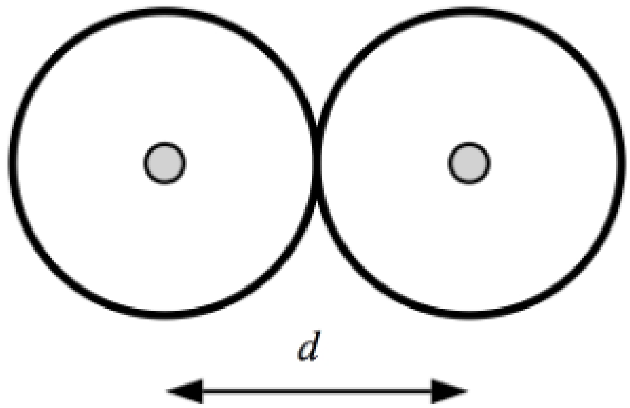

Figure 4: A diatomic molecule.

What is the moment of inertia of the molecule about its center of mass?

1. Md^2.
2. $2Md^2$.
3. $4Md^2$.
4. $\frac{1}{2}Md^2$
5. $\frac{1}{4}Md^2$

Option 4 is the correct answer.

Problem 5 (2.5 points) **Answer on Scantron form**

A bicycle wheel with a heavy rim is mounted on a lightweight axle, and one end of the axle rests on top of a post, as shown in Fig. 5. The wheel is observed to precess in the horizontal plane.

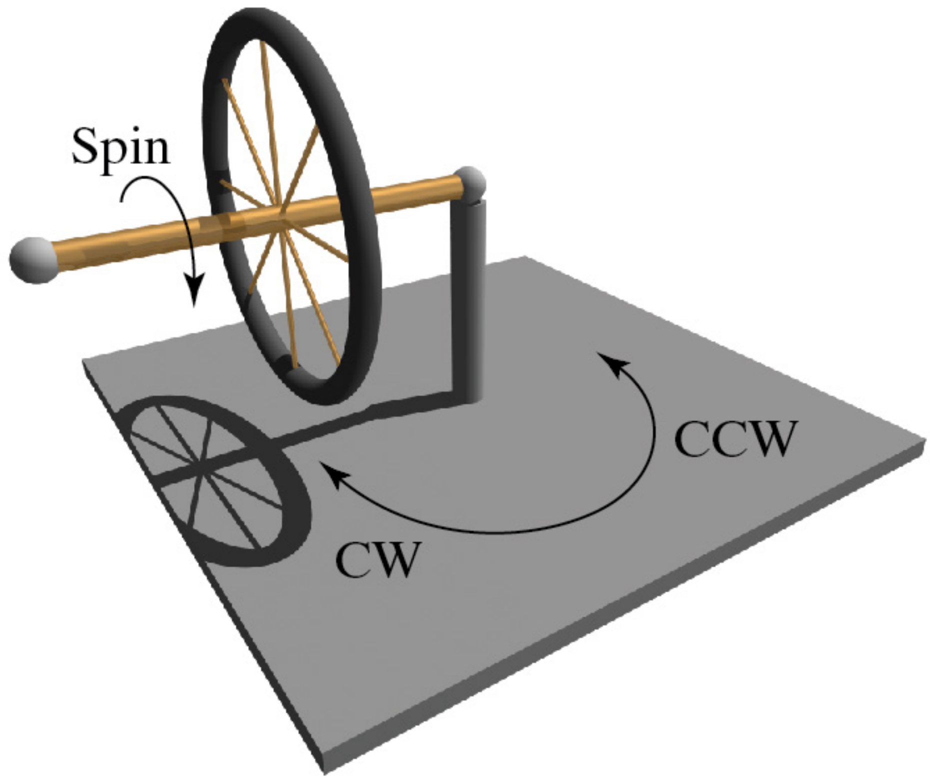

Figure 5: The path of a comet orbiting a star.

With the spin direction shown in the Fig. 5, in what direction will the wheel precess?

1. Counter clockwise.
2. **Clockwise.**

Option 2 is the correct answer.

Problem 6 (2.5 points) **Answer on Scantron form**

Two wheels with fixed hubs, each having a mass of 1 kg, start from rest, and two forces are applied as shown in Fig. 6. Assume the hubs and spokes are massless, so that the rotational inertia of each wheel is $I = mR^2$.

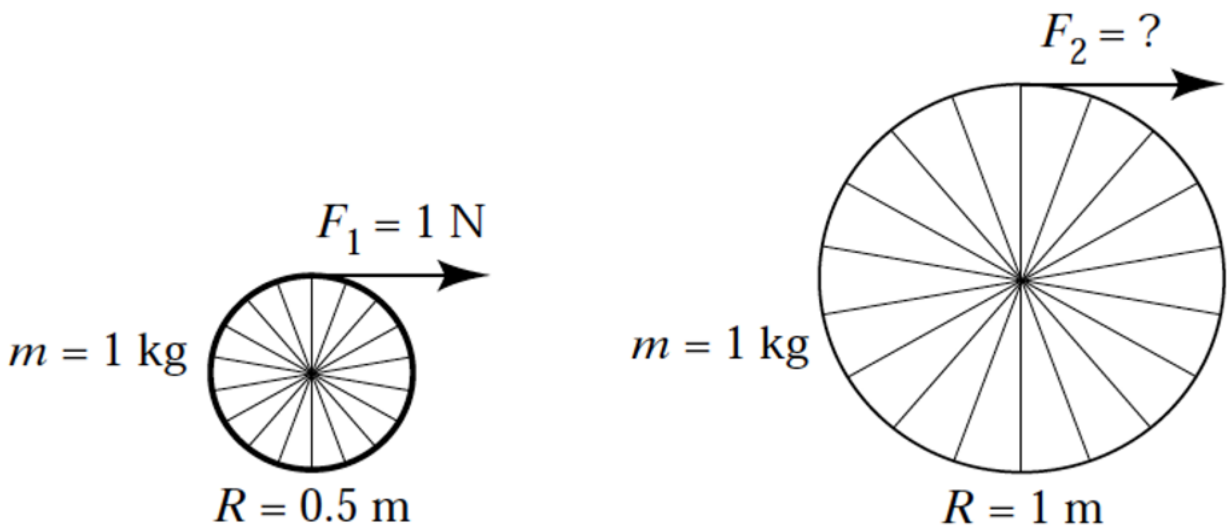

Figure 6: Two rotating wheels.

In order to impart identical angular accelerations, how large must F_2 be?

1. 0.25 N
2. 0.5 N
3. 1 N
4. **2 N**
5. 4 N

Option 4 is the correct answer.

Problem 7 (2.5 points) **Answer on Scantron form**

A chain of metal links is coiled up in a tight ball on a low-friction table as shown in Fig. 7. You pull on a link at one end of the chain with a constant force. Eventually the chain straightens out to its full length and you keep pulling until you have pulled your end of the chain a total distance of 4.5 m.

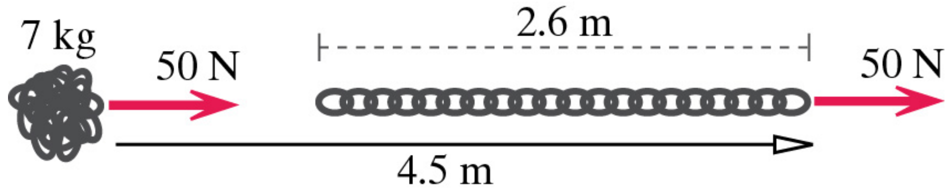

Figure 7: Pulling a chain coiled up in a tight ball.

By what distance did the center of mass of the chain move?

1. 4.5 m.
2. 7.1 m.
3. 1.9 m.
4. **3.2 m**.
5. 5.8 m.

Option 4 is the correct answer.

Problem 8 (2.5 points) **Answer on Scantron form**

Which energy levels shown in Fig. 8 are appropriate for the following situations?

(a) Nuclear states.

(b) Electronic states of a single atom.

(c) Hadronic states.

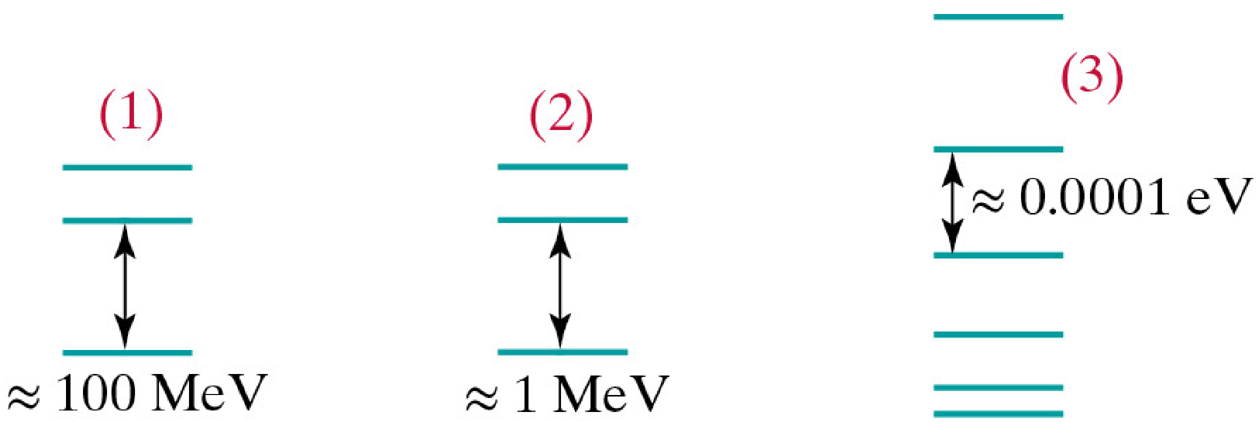

Figure 8: Energy levels.

1. (1=a), (2=b), (3=c).
2. (1=a), (2=c), (3=b).
3. (1=b), (2=a), (3=c).
4. (1=b), (2=c), (3=a).
5. **(1=c), (2=a), (3=b)**.
6. (1=c), (2=b), (3=a).

Option 5 is the correct answer.

Problem 9 (2.5 points) **Answer on Scantron form**

Consider the energy diagram shown in Fig. 9.

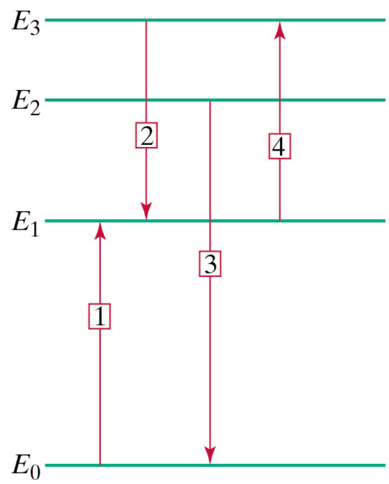

Figure 9: Atomic Transitions.

Match the description of the processes in the following list

(a) Absorption of a photon whose energy is $E_1 - E_0$.
(b) Absorption from an excited state (a rare event at low temperatures).
(c) Emission of a photon whose energy is $E_3 - E_1$.
(d) Emission of a photon whose energy is $E_2 - E_0$.

with the corresponding arrows in Fig. 9.

1. (1=a), (2=b), (3=c), (4=d).
2. (1=a), (2=c), (3=b), (4=d).
3. (1=a), (2=d), (3=c), (4=b).
4. **(1=a), (2=c), (3=d), (4=b)**.
5. (1=b), (2=a), (3=c), (4=d).
6. (1=c), (2=a), (3=b), (4=d).
7. (1=c), (2=d), (3=a), (4=b).
8. (1=d), (2=c), (3=a), (4=b).
9. (1=d), (2=b), (3=c), (4=a).
10. (1=d), (2=a), (3=b), (4=c).

Option 4 is the correct answer.

Problem 10 (2.5 points) **Answer on Scantron form**

In a nuclear fission reactor, each fission of a uranium nucleus is accompanied by the emission of one or more high-speed neutrons which travel through the surrounding material. If one of these neutrons is captured by another uranium nucleus, it can trigger fission, which produces more fast neutrons, as shown schematically in Fig. 10.

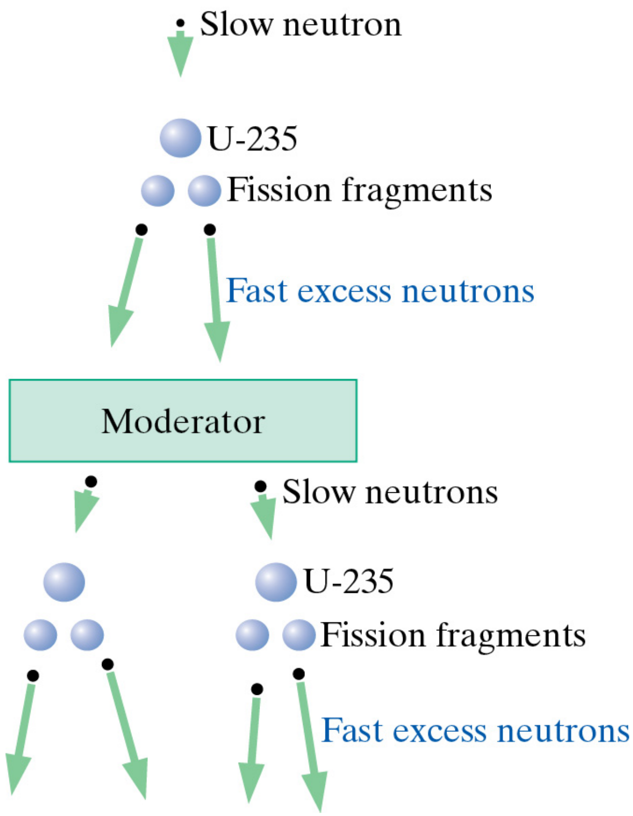

Figure 10: A chain of nuclear fission reactions.

However, fast neutrons have a low probability of capture and usually scatter of uranium nuclei without triggering fission. In order to sustain a chain reaction, the neutrons must be slowed down in some material, called a ”moderator”. Slow neutrons have a high probability of being captured by the uranium nuclei.
Identify the most efficient material to use as a moderator.

1. ^{238}U
2. ^{208}Pb
3. ^{56}Fe
4. ^{12}C
5. H_2O

Option 5 is the correct answer.

Problem 11 (25 points) **Answer in booklet 1**

Consider an object consisting of two masses M connected by a low-mass spring with spring constant k. When you exert an upward force of $2Mg$, the object remains at rest, as shown in Fig. 11. In this situation, the spring is stretched by a distance s_i from its rest length. At one point, a larger constant force is applied ($F > 2Mg$) and the object starts moving up. At some later time, the stretch of the spring has increased to s_f, and the object is located at the position shown in Fig. 11.

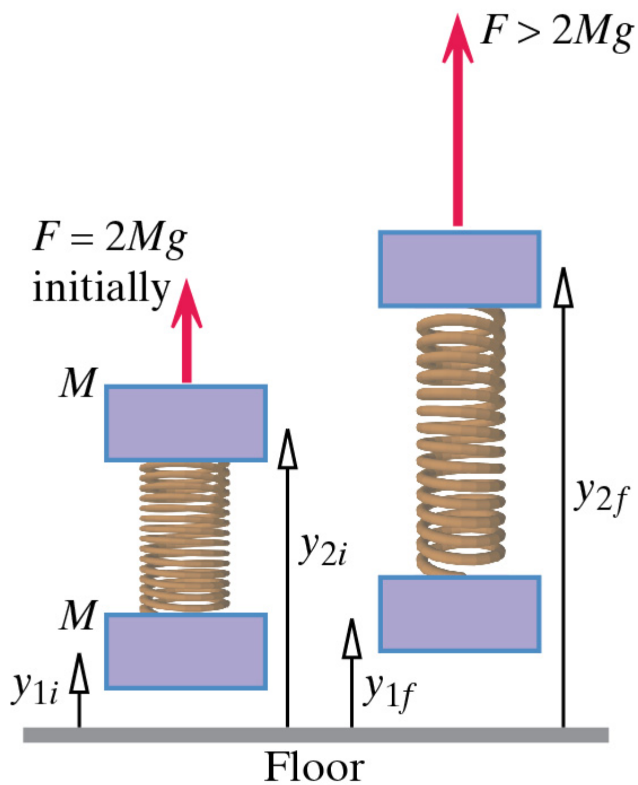

Figure 11: Motion of a mass-spring system.

(a) Calculate the increase in the translational kinetic energy of the two blocks.

The original position of the center-of-mass above the floor is equal to

$$y_{cm,i} = \frac{1}{2}(y_{2,i} + y_{1,i}) \tag{1}$$

The final position of the center-of-mass above the floor is equal to

$$y_{cm,f} = \frac{1}{2}\left(y_{2,f} + y_{1,f}\right) \tag{2}$$

The work done by force F on the center of mass is equal to

$$W_{cm} = F\left(y_{cm,f} - y_{cm,i}\right) \tag{3}$$

Initially the system is at rest and its translational kinetic energy is thus equal to 0 J. The change in the translational kinetic energy will thus be equal to the work done on the center of mass by the applied force F and the change in the gravitational potential energy of the center of mass.

$$\Delta K_{cm} = W - \Delta U = F\left(y_{cm,f} - y_{cm,i}\right) - 2Mg\left(y_{cm,f} - y_{cm,i}\right) \tag{4}$$

Equation 4 can be rewritten in terms of the positions of the individual masses as

$$\Delta K_{cm} = \left(\frac{F}{2} - Mg\right)\left(\left(y_{2,f} + y_{1,f}\right) - \left(y_{2,i} - y_{1,i}\right)\right) = K_{cm} \tag{5}$$

(b) Calculate the vibrational kinetic energy of the two blocks.

The initial energy of the system is the sum of the potential energy of the blocks and the potential energy of the spring.

$$E_i = Mg\left(y_{1,i} + y_{2,i}\right) + \frac{1}{2}ks_i^2 \tag{6}$$

The final energy of the system is the sum of the potential energy of the blocks, the potential energy of the spring, the translational energy K_{cm} of the blocks, and the vibrational energy K_{vib} of the blocks.

$$E_f = Mg\left(y_{1,f} + y_{2,f}\right) + +\frac{1}{2}ks_f^2 + K_{cm} + K_{vib} \tag{7}$$

The final energy of the system is larger than the initial energy of the system due to the work done by force F.

$$E_f = E_i + F\left(y_{2,f} - y_{2,i}\right) \tag{8}$$

This equation can be rewritten using Eqs. 6 and 7 as

$$Mg\left(y_{1,f} + y_{2,f}\right) + +\frac{1}{2}ks_f^2 + K_{cm} + K_{vib} = Mg\left(y_{1,i} + y_{2,i}\right) + \frac{1}{2}ks_i^2 + F\left(y_{2,f} - y_{2,i}\right) \tag{9}$$

The vibrational kinetic energy is thus equal to

$$K_{vib} = F\left(y_{2,f} - y_{2,i}\right) - Mg\left(y_{1,f} - y_{1,i} + y_{2,f} - y_{2,i}\right) - \frac{1}{2}k\left(s_f^2 - s_i^2\right) - K_{cm} \tag{10}$$

Using Eq. 5 we can rewrite Eq. 10 in terms of the variables provided.

Problem 12 (25 points) **Answer in booklet 1**

A yo-yo is constructed of three disks as shown in Fig. 12: two outer disks, each of mass M and radius R, and an inner disk of mass m and radius r. A string is wrapped around the inner disk. The yo-yo is suspended from the ceiling and then released.

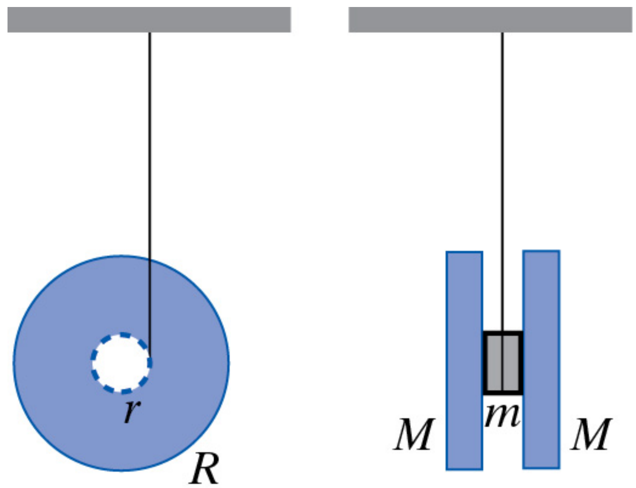

Figure 12: A yo-yo.

The purpose of this problem is to calculate the tension T in the string.

(a) If you knew the tension T, what would be the linear acceleration of the center of mass?

The forces on the yo-yo are shown in Fig. 13.

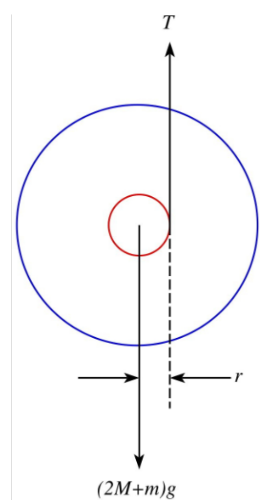

Figure 13: Forces acting on the yo-yo.

The net force acting on the yo-yo is directed in the vertical direction. It has a magnitude equal to

$$F_{net} = (2M + m)g - T \tag{11}$$

This net force is responsible for the motion of the center of mass. The acceleration of the center of mass is thus equal to

$$a_{cm} = \frac{F_{net}}{2M + m} = \frac{(2M + m)g - T}{2M + m} = g - \frac{T}{2M + m} \tag{12}$$

(b) If you knew the tension T, what would be the angular acceleration of the yo-yo with respect to its center of mass?

The total torque with respect to the center of mass is equal to the torque associated with the tension T. The magnitude of this torque is equal to

$$\tau = rT \tag{13}$$

The angular acceleration of the yo-yo with respect to the center of mass can be obtained from the net torque and is equal to

$$\alpha = \frac{\tau}{I} = \frac{rT}{2\left(\frac{1}{2}MR^2\right) + \frac{1}{2}mr^2} = \frac{2rT}{2MR^2 + mr^2} \tag{14}$$

(c) What it the relation between the linear acceleration of the center of mass and the angular acceleration of the yo-yo?

The motion of the yo-yo can be considered to be rolling motion along the string. For this type of motion, the angular velocity and the linear velocity are related in the following way:

$$v_{cm} = \omega r \tag{15}$$

Differentiating the velocities with respect to time, we obtain the following relation between the angular acceleration and the linear acceleration:

$$a_{cm} = \alpha r \tag{16}$$

(d) Use your answers to (a), (b), and (c) to calculate the tension T.

Using the results to parts (a) and (b), we can rewrite eq. 16 as:

$$a_{cm} = g - \frac{T}{2M + m} = \alpha r = \frac{2r^2T}{2MR^2 + mr^2} \tag{17}$$

The relation can be rewritten as

$$g = \frac{T}{2M+m} + \frac{2r^2 T}{2MR^2 + mr^2} = \left[\frac{1}{2M+m} + \frac{2r^2}{2MR^2 + mr^2}\right] T \tag{18}$$

We can solve this equation for the tension T:

$$T = \frac{g}{\frac{1}{2M+m} + \frac{r^2}{2\left(\frac{1}{2}MR^2\right)+\frac{1}{2}mr^2}} \tag{19}$$

Problem 13 (25 points) **Answer in booklet 2**

Consider a two-dimensional elastic collision involving particles of equal mass m in which one of the particles is initially at rest, as shown in Fig. 14.

Figure 14: The collision system before the collision.

After the collision, the angle between the directions of these two particles is A, as shown in Fig. 15.

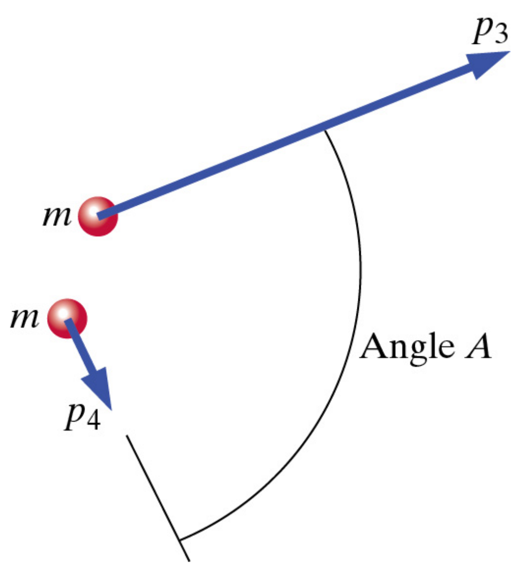

Figure 15: The collision system after the collision.

(a) Use vector notation to write down the relation between the initial linear momentum of the incident particle, $\vec{p}_1$, and the linear momenta of the two particles after the collision, $\vec{p}_3$ and $\vec{p}_4$.

The initial linear momentum of the system is the linear momentum of the incident particle: $\vec{p}_1$. The final linear momentum of the system is the vector sum of $\vec{p}_3$ and $\vec{p}_4$. Conservation of linear momentum thus requires that

$$\vec{p}_1 = \vec{p}_3 + \vec{p}_4 \tag{20}$$

(b) Use the scalar product to obtain an expression for the magnitude of $\vec{p}_1$ in terms of the magnitudes of $\vec{p}_3$ and $\vec{p}_4$ and the angle A. Note: see page 7 for details of the scalar product.

Using the properties of the scalar product we can calculate the magnitude of $\vec{p}_1$:

$$|\vec{p}_1|^2 = \vec{p}_1 \bullet \vec{p}_1 = (\vec{p}_3 + \vec{p}_4) \bullet (\vec{p}_3 + \vec{p}_4) = |\vec{p}_3|^2 + |\vec{p}_4|^2 + 2\vec{p}_3 \bullet \vec{p}_4 \tag{21}$$

The vector product between $\vec{p}_3$ and $\vec{p}_4$ can be expressed in terms of the magnitudes of $\vec{p}_3$ and $\vec{p}_4$ and the angle A:

$$|\vec{p}_1|^2 = |\vec{p}_3|^2 + |\vec{p}_4|^2 + 2|\vec{p}_3||\vec{p}_4|\cos A \tag{22}$$

(c) Use the relation you derived in part (b) to obtain an expression that relates the kinetic energy of the incident particle, K_1, to the kinetic energies of the outgoing particles, K_3 and K_4.

The incident kinetic energy K_1 is equal to

$$K_1 = \frac{|\vec{p}_1|^2}{2m} \tag{23}$$

Using Eq. 22, we can rewrite this equation as

$$K_1 = \frac{\vec{p}_3|^2 + |\vec{p}_4|^2 + 2|\vec{p}_3||\vec{p}_4|\cos A}{2m} = K_3 + K_4 + 2\sqrt{K_3}\sqrt{K_4}\cos A \tag{24}$$

(d) Since the collision is elastic, what can you conclude about the angle A?

Since the collision is elastic, $K_1 = K_3 + K_4$. Using this relation between kinetic energies and Eq. 24 we can conclude that

$$K_1 = K_3 + K_4 + 2\sqrt{K_3}\sqrt{K_4}\cos A = K_1 + 2\sqrt{K_3}\sqrt{K_4}\cos A \tag{25}$$

This requires that

$$2\sqrt{K_3}\sqrt{K_4}\cos A = 0 \tag{26}$$

or

$$\cos A = 0 \tag{27}$$

since $K3$ and $K4$ are not equal to 0. The angle A must thus be equal to 90 degrees.