Problem 1 (2.5 points) **Answer on Scantron form**

What must be the weather condition in order for the Dutch to hold the tegenwindfietsen race? Hint: see Fig. 1.

Figure 1: https://www.youtube.com/watch?v=VMinwf-kRlA

1. Snowy.
2. Rainy.
3. Foggy.
4. **Very windy.**
5. Sunny.
6. No special weather conditions are required.

Option 4 is the correct answer.

Problem 2 (2.5 points) **Answer on Scantron form**

List the four basic forces in order of strength (start with the strongest force and end with the weakest force)

1. Strong > Electromagnetic > Gravitational > Weak.
2. Electromagnetic > Strong > Weak > Gravitational.
3. **Strong** > **Electromagnetic** > **Weak** > **Gravitational.**
4. Strong > Electromagnetic > Gravitational > Weak.

Option 3 is the correct answer.

Problem 3 (2.5 points) **Answer on Scantron form**

Consider the distribution of 11 equal-mass spheres, located on the circumference of a circle, as shown in Fig. 2.

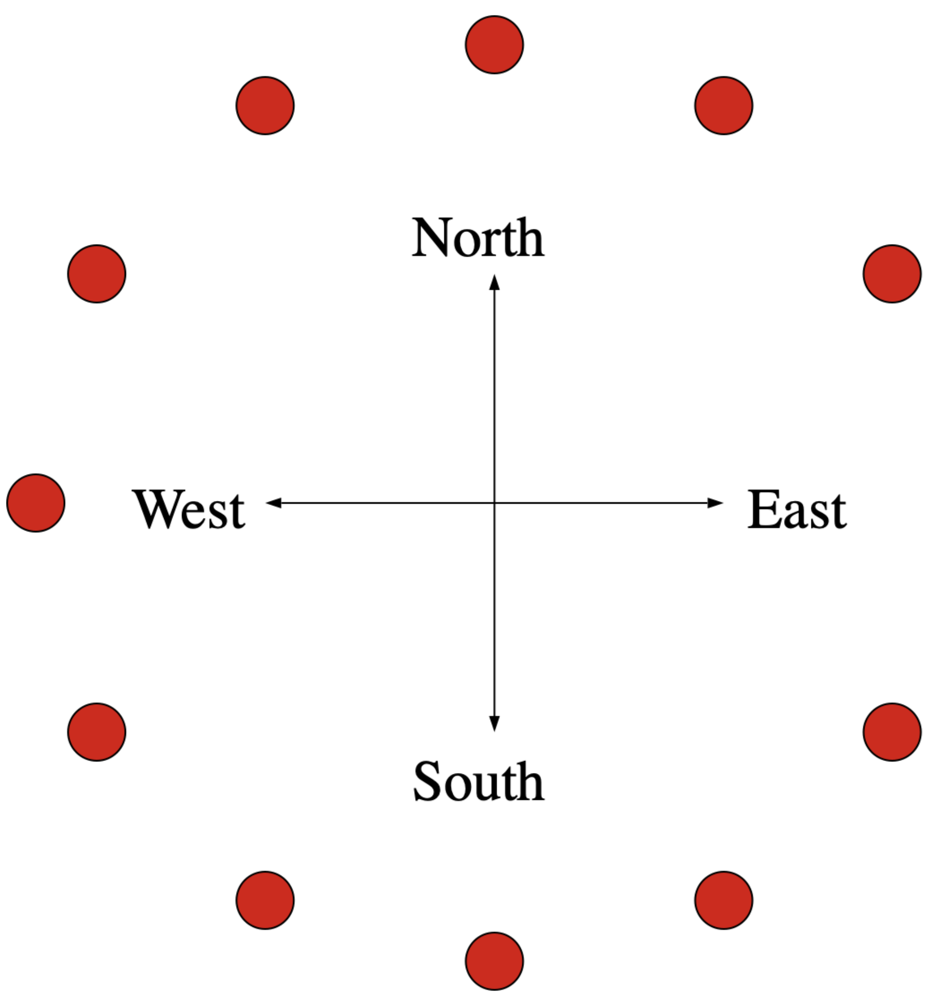

Figure 2: Mass distribution for Problem 3.

If we place another mass at the center of the circle, in what direction will the net gravitational force acting on this mass point?

1. Towards the East.
2. Towards the South.
3. **Towards the West.**
4. Towards the North.

Option 3 is the correct answer.

Problem 4 (2.5 points) **Answer on Scantron form**

The time dependence of a force, acting on a object of mass m, is shown in Fig. 3. The force is acting along the x axis of the coordinate system.

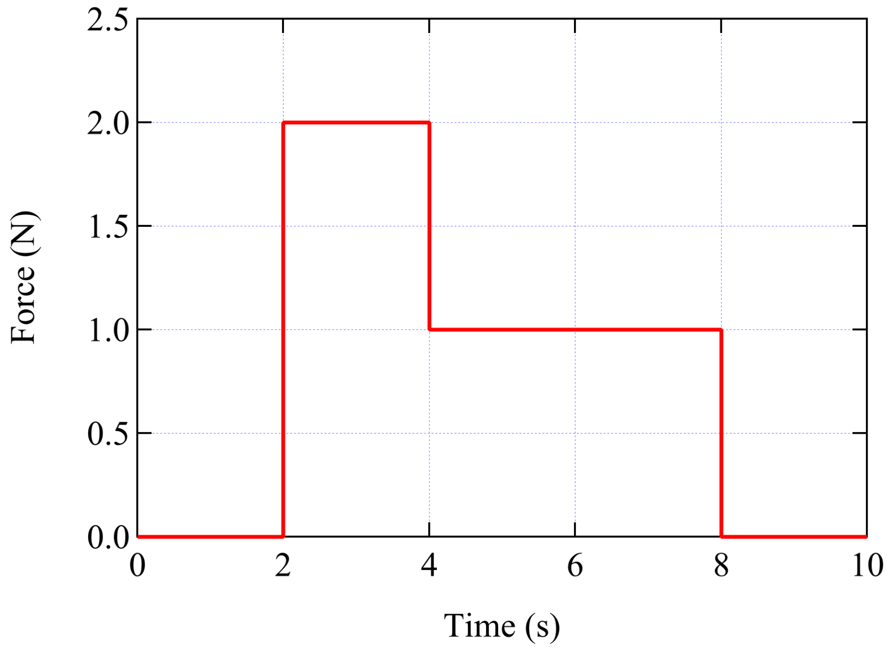

Figure 3: Force acting on mass m as function of time.

What is the change in the x component of the linear momentum of this object between t = 0 s and t = 10 s?

1. 2 kg m/s.
2. 4 kg m/s.
3. 5 kg m/s.
4. 6 kg m/s.
5. 7 kg m/s.
6. **8 kg m/s.**
7. 9 kg m/s.
8. 10 kg m/s.
9. 11 kg m/s.
10. 12 kg m/s.

Option 6 is the correct answer.

Problem 5 (2.5 points) **Answer on Scantron form**

Two balls are projected off a cliff. One is thrown horizontally while the other is released from rest and falls vertically. Which of the following statements is true?

1. The ball that falls vertically hits the ground first.
2. The ball that is projected horizontally hits the ground first.
3. **Both balls hit the ground at the same time.**
4. We can not determine which ball hits the ground first unless we know the speed at which the first ball was projected horizontally.

Option 3 is the correct answer.

Problem 6 (2.5 points) **Answer on Scantron form**

A ball is dropped from the edge of a cliff. Soon after this, a second ball is dropped. As a function of time, the separation between the two balls will

1. stay the same.
2. **increase.**
3. decrease.
4. depend on the time specified.

Option 6 is the correct answer.

Problem 7 (2.5 points) **Answer on Scantron form**

The measured velocity of a car, moving with a constant acceleration a, is shown in Fig. 4. Note: the indicated error bars are $\pm 1\sigma$. Assuming that the measured velocity v is proportional to the measured time t (that is $v = at$), which data point constrains the possible values of a the most?

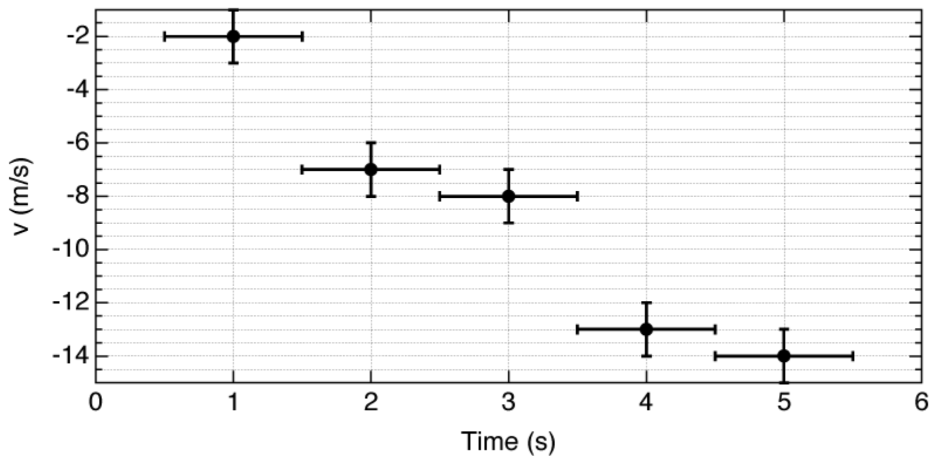

Figure 4: Measured velocity of a car as function of time.

1. The data point at $t = 1$ s.
2. The data point at $t = 2$ s.
3. The data point at $t = 3$ s.
4. The data point at $t = 4$ s.
5. **The data point at $t = 5$ s.**

Option 5 is the correct answer.

Problem 8 (2.5 points) **Answer on Scantron form**

A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot shown in Fig. 5.

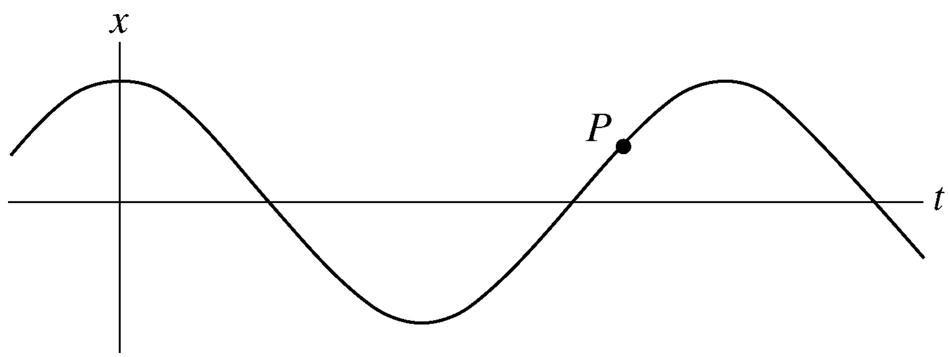

Figure 5: Position versus time distribution for a mass on a spring.

What is the velocity and the acceleration of the mass at point P?

1. Positive velocity and positive acceleration.
2. **Positive velocity and negative acceleration.**
3. Negative velocity and positive acceleration.
4. Negative velocity and negative acceleration.
5. Zero velocity and non-zero acceleration (positive or negative).
6. Zero velocity and zero acceleration.

Option 2 is the correct answer.

Problem 9 (2.5 points) **Answer on Scantron form**

You measure the length of a plate using a ruler, as shown in Fig. 6.

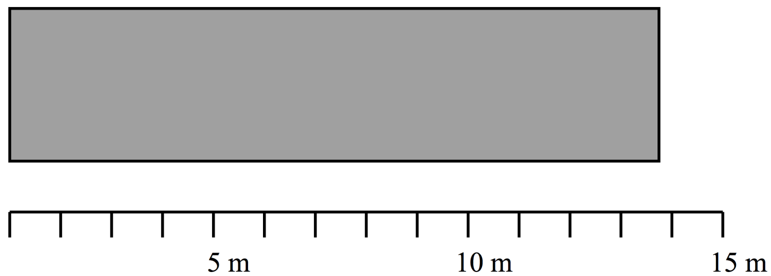

Figure 6: The measurement of the length of a plate.

What is your best estimate of the length of the plate (in units of meters)?

1. 10.0 - 10.5 m
2. 10.5 - 11.0 m
3. 11.0 - 11.5 m
4. 11.5 - 12.0 m
5. 12.0 - 12.5 m
6. **12.5 - 13.0 m**
7. 13.0 - 13.5 m
8. 13.5 - 14.0 m
9. 14.0 - 14.5 m
10. 14.5 - 15.0 m

Option 6 is the correct answer.

Problem 10 (2.5 points) **Answer on Scantron form**

Two satellites A and B of the same mass are going around Earth in concentric orbits. The distance of satellite B from Earth's center is twice that of satellite A. What is the ratio of the centripetal force acting on B to that acting on A?

1. 1/8
2. **1/4**
3. 1/2
4. $1/\sqrt{2}$
5. 1

Option 2 is the correct answer.

Problem 11 (25 points) **Answer in booklet 1**

A spherical hollow is made in two spheres of radius R such that its surface touches the outside surface of each sphere and passes through its center as shown in Fig. 7. The mass of each of the spheres before hollowing was M. What is the magnitude of the gravitational force between the two hollowed-out spheres when their centers are located a distance d apart?

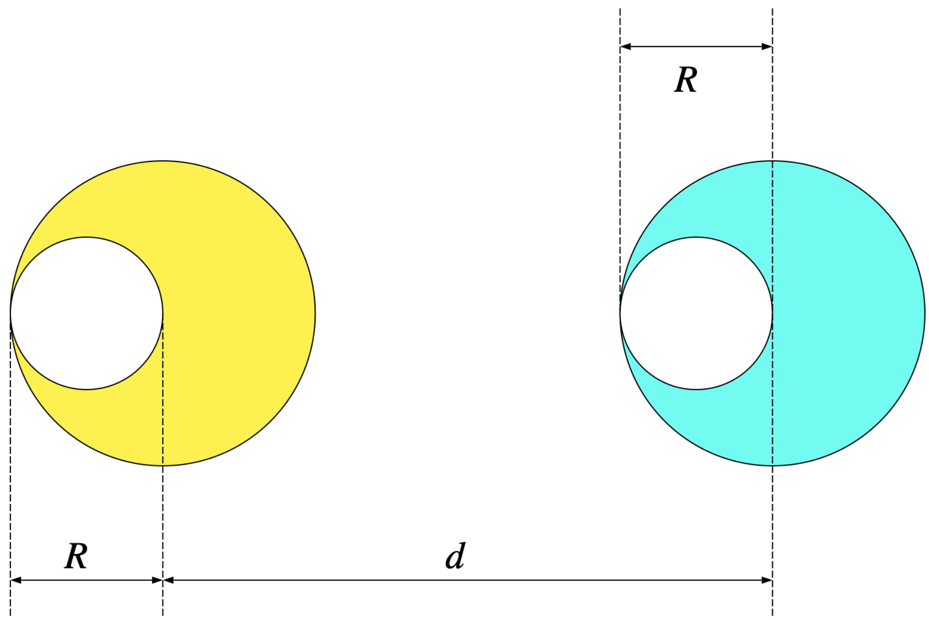

Figure 7: Mass distribution for Problem 11.

The density ρ of the spheres is equal to

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3}{4}\frac{M}{\pi R^3} \tag{1}$$

The mass of the volume that is removed from the spheres is equal to

$$M_{removed} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \frac{3}{4}\frac{M}{\pi R^3} = \frac{1}{8}M \tag{2}$$

The mass of the hollowed-out spheres is thus $7M/8$.
To calculate the gravitational force between the hollowed-out spheres we use the principle of superposition. Consider the three cases shown in Fig. 8.

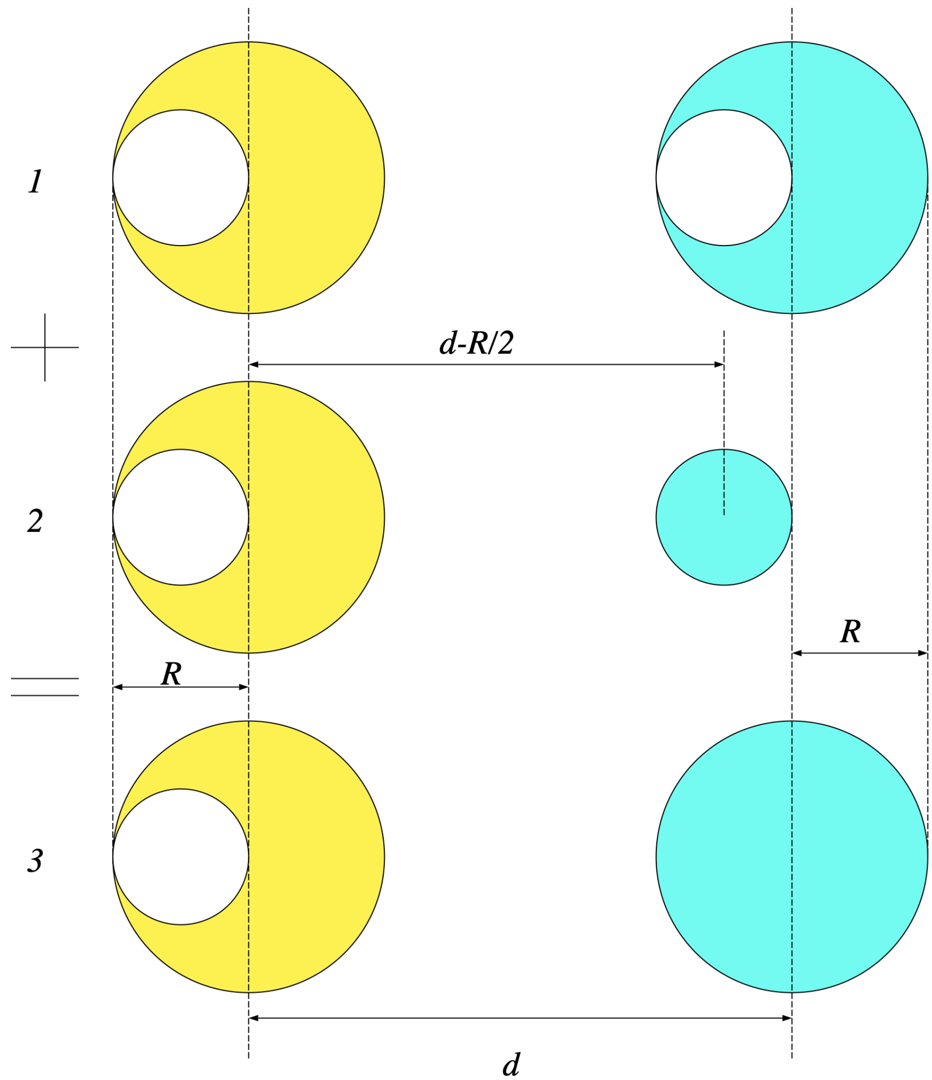

Figure 8: Using the principle of super position to determine the gravitational force between the masses shown in Fig. 7.

The problem asks us to determine the gravitational force for the mass configuration shown as configuration (1) in the Fig. 8. This force can be expressed in terms of the forces between configurations (2) and (3): $F_1 = F_3 - F_2$. The determination of the force for configurations (2) and (3) again relies on the principle of super position.
Consider the configurations shown in Fig. 9. We want to determine the force between the masses in configuration (1) of Fig. 9. Assume that the mass of the right sphere is m, and that the centers are separated by a distance r (we will substitute the proper values for m and r once we have determined the force for configuration (1)).

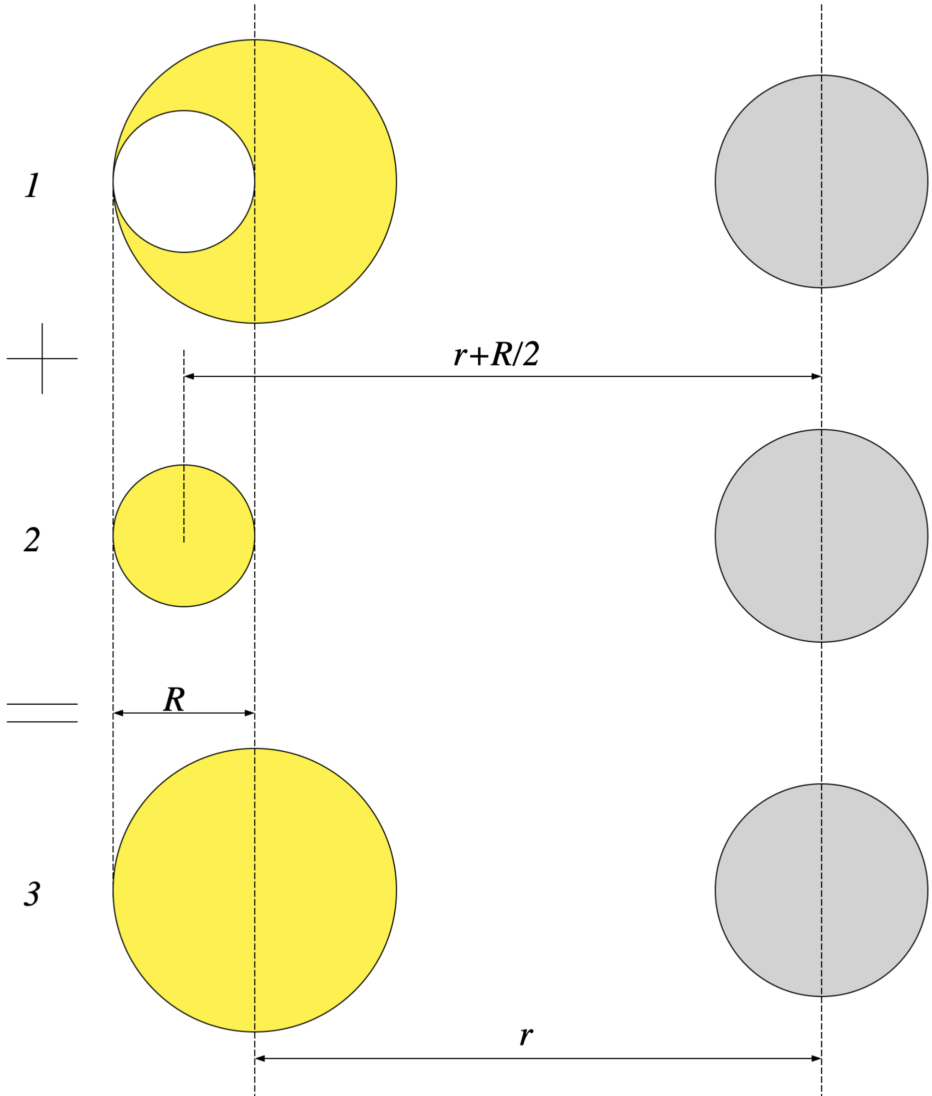

Figure 9: Using the principle of super position to determine the gravitational force between the masses shown in Fig. 8.

The force in configuration (1) of Fig. 9 is difficult to calculate directly. However, if we add a spherical mass of radius $R/2$ to fill the hole in the left sphere, we end up with configuration (3). The gravitational force in configuration (3) is easy to calculate and is equal to

$$F_3 = G\frac{mM}{r^2} \tag{3}$$

The force calculated in Eq. 3 is larger than the force in configuration (1) due to the extra component associated with configuration (2). The force in configuration (2) is also easy to calculate and is equal to

$$F_2 = G\frac{m\frac{M}{8}}{\left(r+\frac{R}{2}\right)^2} = \frac{1}{8}G\frac{mM}{\left(r+\frac{R}{2}\right)^2} \tag{4}$$

The force in configuration (1) of Fig. 9 is thus equal to the difference between F_3 and F_2.

$$F_1 = F_3 - F_2 = G\frac{mM}{r^2} - \frac{1}{8}G\frac{mM}{\left(r+\frac{R}{2}\right)^2} = G\frac{mM}{r^2}\left[1 - \frac{1}{8\left(1+\frac{1}{2}\frac{R}{r}\right)^2}\right] \tag{5}$$

Using this relation we can now go back to the configurations shown in Fig. 9 and determine the forces for configurations (2) and (3).

$$F_3 = G\frac{MM}{d^2}\left[1 - \frac{1}{8\left(1+\frac{1}{2}\frac{R}{d}\right)^2}\right] = G\frac{M^2}{d^2}\left[1 - \frac{1}{8}\left(\frac{2d}{2d+R}\right)^2\right] \tag{6}$$

$$F_2 = G\frac{\frac{M}{8}M}{\left(d-\frac{R}{2}\right)^2}\left[1 - \frac{1}{8\left(1+\frac{1}{2}\frac{R}{d-\frac{R}{2}}\right)^2}\right] = \frac{1}{8}G\frac{M^2}{\left(d-\frac{R}{2}\right)^2}\left[1 - \frac{1}{8}\left(\frac{2d}{2d-R}\right)^2\right] \tag{7}$$

The force on configuration (1) in Fig. 9 is thus equal to

$$F_1 = F_3 - F_2 = G\frac{M^2}{d^2}\left[1 - \frac{1}{8}\left(\frac{2d}{2d+R}\right)^2\right] - \frac{1}{8}G\frac{M^2}{\left(d-\frac{R}{2}\right)^2}\left[1 - \frac{1}{8}\left(\frac{2d}{2d-R}\right)^2\right] \tag{8}$$

Problem 12 (25 points) **Answer in booklet 1**

Moving objects left traces labelled $A-F$, shown in Fig. 10. The dots were deposited at equal time intervals (one dot each second). In each case, the object starts from the square.

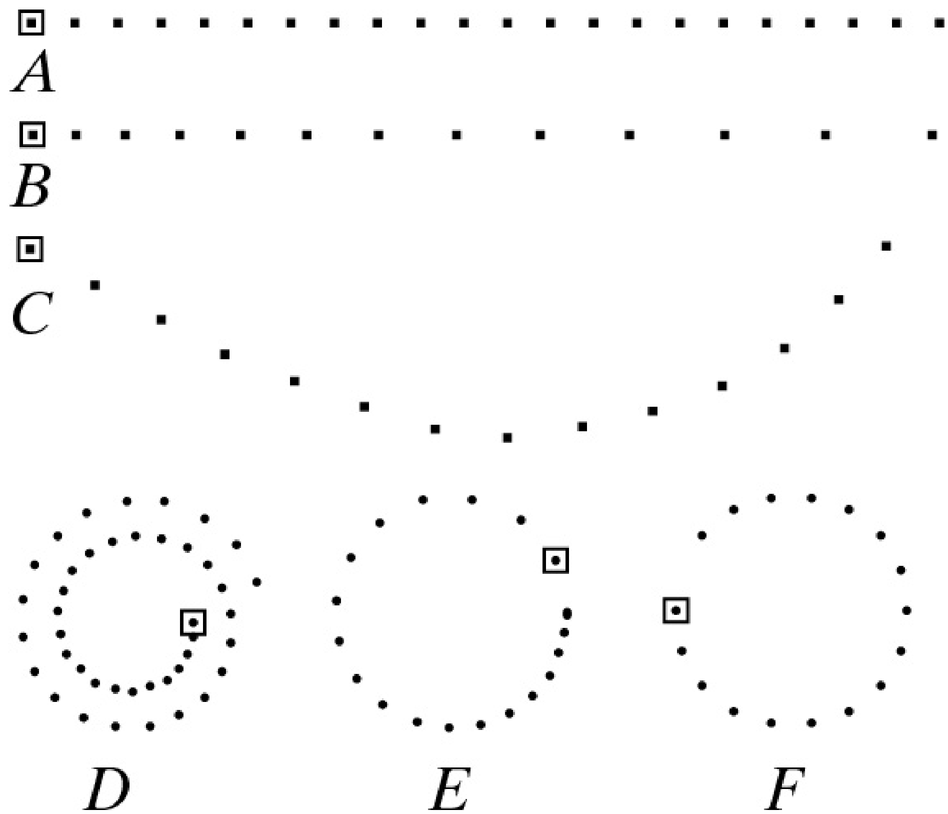

Figure 10: Traces left by moving objects.

(a) For trace A, determine if a force is acting on the object. If a force is acting on the object, describe its direction and its magnitude as function of time.

We observe that the object continues to move in the same direction with constant velocity since the dots are equally spaced. There is thus no force acting on the object in trace A.

(b) For trace B, determine if a force is acting on the object. If a force is acting on the object, describe its direction and its magnitude as function of time.

We observe that the object continues to move in the same direction with increasing velocity since the spacing between the dots increases. There is thus a force acting on the object, directed along the trace to the right. The force appears to be constant as function of time.

(c) For trace C, determine if a force is acting on the object. If a force is acting on the object, describe its direction and its magnitude as function of time.

We observe that the object moves with constant speed since the distance between the dots appear to be constant. However, the direction of motion

changes and the velocity of the object thus changes. This requires a non-zero force that points in a direction perpendicular to the direction of motion. Based on the trajectory shown, the force must point towards the left of the motion of the object. The magnitude of the force is not constant, otherwise we would have observed circular motion.

(d) For trace D, determine if a force is acting on the object. If a force is acting on the object, describe its direction and its magnitude as function of time.

For trace D we observe an increase in the speed of the object and spiral motion. This type of motion can be produced by a force that is directed towards the center of the spriral whose magnitude changes as function of time or a net force that has a radial component and a non-zero tangential component, pointing in the direction of motion.

(e) For trace E, determine if a force is acting on the object. If a force is acting on the object, describe its direction and its magnitude as function of time.

Trace E shows motion with decreasing speed and constant radius. The circular motion requires a radial component of the force. Since we see a reduction in the speed, the force must be decreasing. To produce a reduction in the speed of the object, a tangential component must be present, pointing in a direction opposite the direction of motion.

(f) For trace F, determine if a force is acting on the object. If a force is acting on the object, describe its direction and its magnitude as function of time.

Trace F shows motion with a constant radius and a constant speed. This is uniform circular motion which requires a radial force of constant magnitude.

Problem 13 (25 points) **Answer in booklet 2**

Two blocks with masses M_1 and M_3, connected by a rod of mass M_2, are sitting on a frictionless surface, as shown in Fig. 11. A constant external force F, directed towards the left, is applied to the right-hand side of the right block. The motion of the system is such that you can treat the system non-relativistically.

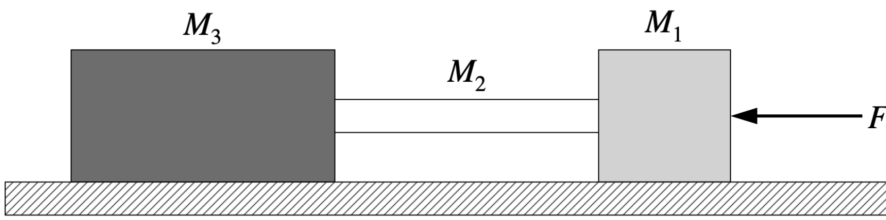

Figure 11: Mass distribution for Problem 12.

(a) What is the acceleration of the system of masses?

The total mass of the system is equal to $M_1 + M_2 + M_3$. The only external force acting on the system in the horizontal direction is the applied force F. The resulting acceleration of the system can be obtained using Newton's second law since the system can be treated non-relativistically:

$$a = \frac{F}{M_1 + M_2 + M_3} \tag{9}$$

(b) What is the compression force in the rod at its right end?

The net force that must be acting on the rod and on mass M_3 can be determined using Newton's second law (using the acceleration we calculated in Eq. 9):

$$F_{2,3} = (M_2 + M_3)a = \frac{M_2 + M_3}{M_1 + M_2 + M_3}F \tag{10}$$

This force must be provided by the right block pressing against the right-end of the rod and will thus be the compression force at the right end of the rod.

(c) What is the compression force in the rod at its left end?

The net force that must be acting on mass M_3 can be determined using Newton's second law (using the acceleration we calculated in part a, Eq. 9):

$$F_3 = M_3 a = \frac{M_3}{M_1 + M_2 + M_3}F \tag{11}$$

This force must be provided by the rod pressing against the right-hand side of the left block and will thus be equal in magnitude, but opposite in direction, to the compression force at the left end of the rod.