Physics 141.
Lecture 18.

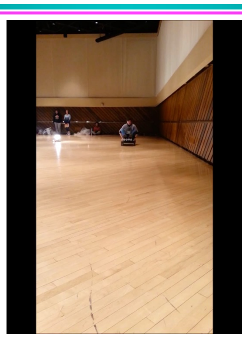
1

Physics 141.
Lecture 18.

- Concept Test
- Topics to be discussed today:
  - A quick review of rotational variables, kinetic energy, and torque.
  - Rolling motion.
  - Angular Momentum.

2

Physics 141.
Laboratory # 5.



3

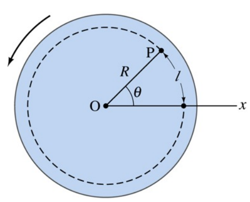
Physics 141. Course information.

- Homework # 7 is due on Friday November 10.
- Experiment # 5 will take place in Spurrier Gym on Monday November 13:
  - Please take a 12 pack if you did not take one on Tuesday.
  - Please remove the sparkling water.
  - Please rinse the cans.
  - Please bring all your cans to Spurrier Gym during your lab period on Monday November 13.

4

Rotational variables. A quick review.

- The variables that are used to describe rotational motion are:
  - Angular position θ
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



5

The moment of inertia. A quick review.

- The kinetic energy of a rotation body is equal to

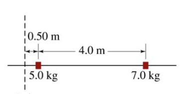
$$K = \frac{1}{2} I \omega^2$$

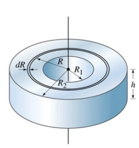
where I is the moment of inertia.
- For discrete mass distributions I is defined as

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions I is defined as

$$I = \int r^2 dm$$



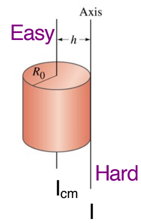


6

Parallel-axis theorem. A quick review.

- Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:

$$I = I_{cm} + Mh^2$$



7

Torque. A quick review.

- The torque τ of the force F is proportional to the angular acceleration of the rigid body:

$$\tau = I\alpha$$

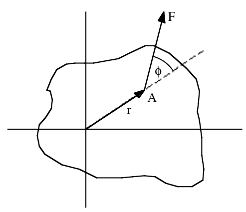
- This equation looks similar to Newton's second law for linear motion:

$$F = ma$$

- Note:

linear motion	rotational motion
mass m	moment I
force F	torque τ

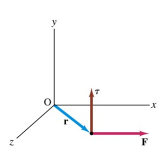
$$\vec{\tau} = \vec{r} \times \vec{F}$$

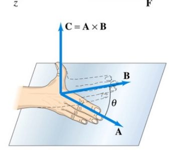


8

Torque. A quick review.

- The torque associated with a force is a vector. It has a magnitude and a direction.
- The direction of the torque can be found by using the right-hand rule to evaluate ***r*** x ***F***.
- The direction of the torque is the direction of the angular acceleration.
- For extended objects, the total torque is equal to the vector sum of the torque associated with each "component" of this object.





9

Rolling motion.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
  - Translational kinetic energy:

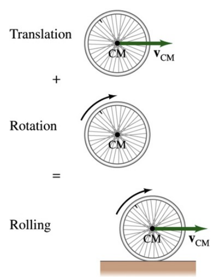
$$K_{translational} = \frac{1}{2} M v_{cm}^2$$

  - Rotational kinetic energy:

$$K_{rotational} = \frac{1}{2} I_{cm} \omega^2$$

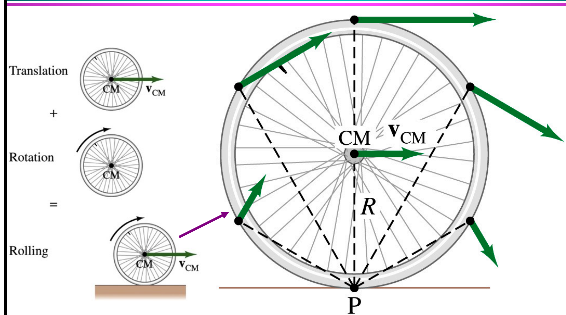
- Assuming that the wheel does not slip we know that

$$\omega = \frac{v_{cm}}{R}$$



10

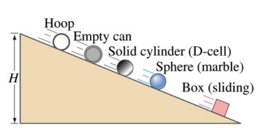
Rolling motion.



11

Rolling motion.

- Consider two objects of the same mass but different moments of inertia, released from rest from the top of an inclined plane:
  - Both objects have the same initial mechanical energy (assuming their CM is located at the same height).
  - At the bottom of the inclined plane they will have both rotational and translational kinetic energy.
  - Which object will reach the bottom first?



12

Rolling motion.

- Initial mechanical energy:

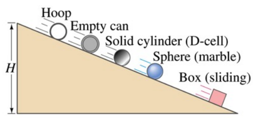
$$E_i = mgH$$

- Final mechanical energy:

$$E_f = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

- Assuming no slipping, we can rewrite the final mechanical energy as

$$E_f = \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right) v_{cm}^2$$



- Conservation of energy implies:

$$\frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right) v_{cm}^2 = mgH$$

or

$$\frac{1}{2}\left(1 + \frac{I_{cm}}{mR^2}\right) v_{cm}^2 = gH$$

→ *The smaller I_{cm}, the larger v_{cm} at the bottom of the incline.*

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 18, Page 13

13

2 Minute 19 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 19 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Solve a WeBWorK problem.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 18, Page 14

14

Concept test lecture 18.
PollEv.com/frankwolfs050

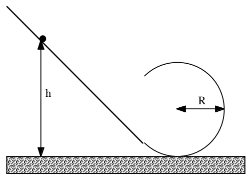
- The concept test today will have five questions.
- I will collect your answers electronically using the Poll Everywhere system.
- After submitting your answer, I will give you time to discuss the question with your neighbor(s) before submitting a new answer.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 18, Page 15

15

How different is a world with rotational motion?

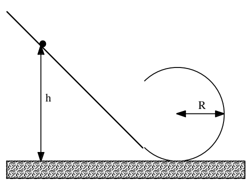
- Consider the loop-to-loop. What height h is required to make it to the top of the loop?
- First consider the case without rotation:
  - Initial mechanical energy = mgh.
  - Minimum velocity at the top of the loop is determined by requiring that
    $mv^2/R > mg$
    or
    $v^2 > gR$
  - The mechanical energy is satisfy the following condition:
    $(1/2)mv^2 + 2mgR > (5/2)mgR$
  - Conservation of energy requires
    $h > (5/2)R$



16

How different is a world with rotational motion?

- What changes when the object rotates?
  - The minimum velocity at the top of the loop will not change.
  - The minimum translational kinetic energy at the top of the loop will not change.
  - But in addition to translational kinetic energy, there is now also rotational kinetic energy.
  - The minimum mechanical energy is at the top of the loop has thus increased.
  - The required minimum height must thus have increased.
- OK, let's now calculate by how much the minimum height has increased.



17

How different is a world with rotational motion?

- The total kinetic energy at the top of the loop is equal to

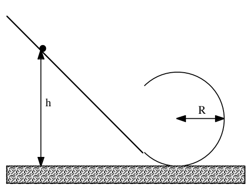
$$K_f = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{I}{r^2}+m\right)v^2$$

- This expression can be rewritten as

$$K_f = \frac{1}{2}\left(\frac{2}{5}m+m\right)v^2 = \frac{7}{10}mv^2$$

- We now know the minimum mechanical energy required to reach this point and thus the minimum height:

$$h \ge \frac{27}{10}R$$



Note: without rotation $h \ge 25/10\ R$!!!

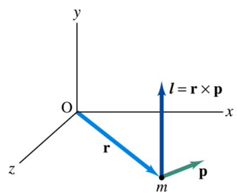
18

Angular momentum. Definition.

- We have seen many similarities between the way in which we describe linear and rotational motion.
- Rotational motion can be treated in similar fashion as linear motion:

linear motion	rotational motion
mass m	moment I
force F	torque $\tau = r \times F$

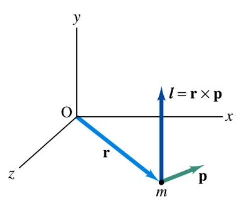
- What is the equivalent to linear momentum? Answer: **angular momentum $L = r \times p$.**



19

Angular momentum. Definition.

- The angular momentum is defined as the vector product between the position vector and the linear momentum.
- Note:
  - Compare this definition with the definition of the torque.
  - Angular momentum is a vector.
  - The unit of angular momentum is kg m^2/s.
  - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
  - Under certain circumstances the angular momentum of a system is conserved!



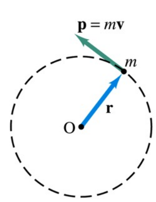
20

Angular momentum. Circular motion.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius r and the linear momentum p:

$$L = mvr = mr^2(v/r) = I\omega$$

- Note: compare this with $p = mv$!



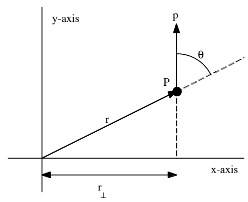
21

Angular momentum. Linear motion.

- An object does not need to carry out rotational motion to have an angular moment.
- Consider a particle P carrying out linear motion in the xy plane.
- The angular momentum of P (with respect to the origin) is equal to

$$\vec{L} = \vec{r} \times \vec{p} = mrv\sin\theta\hat{z} = = mvr_{\perp}\hat{z} = pr_{\perp}\hat{z}$$

and will be constant (if the linear momentum is constant).



22

Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\vec{r} \times \vec{p}\right) = m\left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v}\right) = m\left(\vec{r} \times \vec{a} + \vec{v} \times \vec{v}\right) = = \vec{r} \times m\vec{a} = \vec{r} \times \sum \vec{F} = \sum \vec{\tau}$$

- When the net torque is equal to 0 Nm:

$$\sum \vec{\tau} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \text{constant}$$

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.

23

Done for today!

24