Physics 141.
Lecture 17.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 17, Page 1

1

Physics 141.
Lecture 17.

- Course information.
- Topics to be discussed today (Chapter 11):
  - Rotational Variables
  - Rotational Kinetic Energy
  - Torque

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 17, Page 2

2

Physics 141.
Course information.

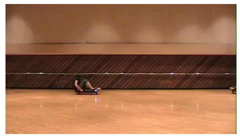
- Lab report # 4 is due on Wednesday 11/6 at noon.
- Homework set # 7 is due on Friday 11/1 at noon.
- Homework set # 8 is due on Friday 11/8 at noon.
- Homework set # 9 is due on Friday 11/15 at noon.
- Exam # 3 is scheduled for Tuesday 11/19 at 8 am in Hoyt. It covers the material contained in Chapters 8 – 11.

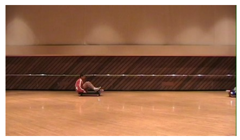
Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 17, Page 3

3

Experiment # 5.
Timeline (more details during next lectures).

- 11/11: collisions in Spurrier Gym
- 11/18: analysis files available.
- 11/25: each student has determined his/her best estimate of the velocities before and after the collisions (analysis during regular lab periods).
- 11/25: complete discussion and comparison of results with colliding partners and submit final results (velocities and errors) to professor Wolfs.
- 11/27: we will compile the results, determine momenta and kinetic energies, and distribute the results.
- 12/2: office hours by lab TA/TIs to help with analysis and conclusions.
- 12/6: students submit lab report # 5.





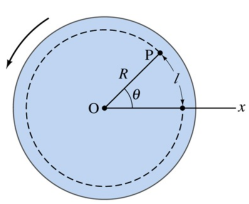
4

We need empty soda cans!
I will provide full ones on 11/5.

5

Rotational variables.

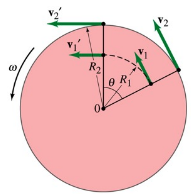
- The variables that are used to describe rotational motion are:
  - Angular position θ
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



6

Rotational variables.

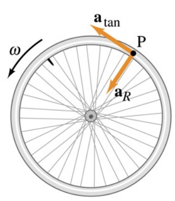
- Things to consider when looking at the rotation of rigid objects around a fixed axis:
  - Each part of the rigid object has the same angular velocity.
  - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
  - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



7

Rotational variables.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P.
- The two components of the acceleration of point P are:
  - The **radial component**: this component is always present since point P carried out circular motion around the axis of rotation.
  - The **tangential component**: this component is present only when the angular acceleration is not equal to 0 rad/s^2.



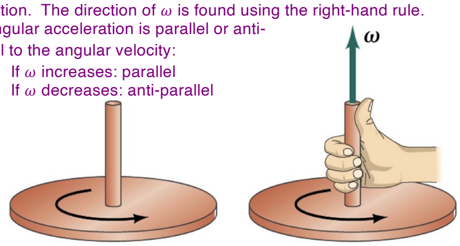
8

Rotational variables.

Angular velocity and acceleration are vectors! They have a magnitude and a direction. The direction of ω is found using the right-hand rule.

The angular acceleration is parallel or anti-parallel to the angular velocity:

If ω increases: parallel
If ω decreases: anti-parallel



(a) (b)

9

Rotational kinetic energy.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2}\sum_i m_i (\omega r_i)^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

- The kinetic energy is proportional to the rotational velocity ω. Note: the equation is similar to the translational kinetic energy except that instead of being proportional to the the mass m of the object, the rotational kinetic energy is proportional to the **moment of inertia *I*** of the object (unit of I is kg m^2):

$$I = \sum_i m_i r_i^2$$

10

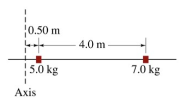
The moment of inertia *I*. Calculating *I*.

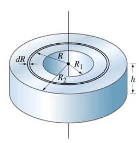
- The moment of inertia of an objects depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$





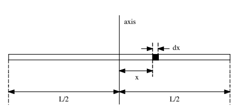
11

Calculating the moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its center of mass?

- Consider a slice of the rod, with width dx, located a distance x from the rotation axis. The mass dm of this slice is equal to

$$dm = \frac{m}{L}dx$$



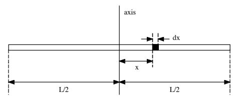
12

Calculating the moment of inertia. Sample problem.

- The moment of inertia dI of this slice is equal to

$$dI = x^2 dm = \frac{m}{L} x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:



$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{m}{L} x^2 dx =$$

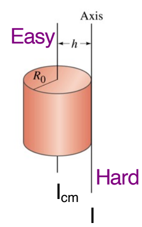
$$= \frac{m}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] = \frac{1}{12} mL^2$$

13

Calculating the moment of inertia. Parallel-axis theorem.

- Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:

$$I = I_{cm} + Mh^2$$

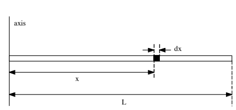


14

Calculating the moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its left corner?

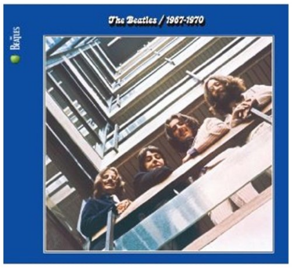
- We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:



$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} mL^2$$

15

3 Minute 2 Second Intermission.

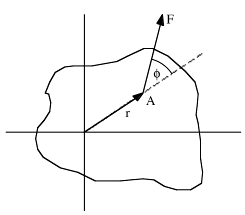


- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 2 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Solve a WeBWorK problem.

16

Torque.

- Consider a force F applied to an object that can only rotate.
- The force F can be decomposed into two two components:
  - A **radial component** directed along the direction of the position vector r. The magnitude of this component is $Fcos(\phi)$. This component will not produce any motion.
  - A **tangential component**, perpendicular to the direction of the position vector r. The magnitude of this component is $Fsin(\phi)$. This component will result in rotational motion.



17

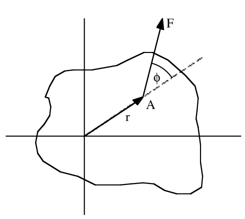
Torque.

- If a mass m is located at the position on which the force is acting (and we assume any other masses can be neglected), it will experience a linear acceleration equal to $Fsin(\phi)/m$.
- The corresponding angular acceleration α is equal to

$$\alpha = Fsin(\phi)/mr$$

- Since in rotational motion the moment of inertia plays an important role, we will rewrite the angular acceleration in terms of the moment of inertia:

$$\alpha = \frac{rFsin(\phi)}{mr^2} = \frac{rFsin(\phi)}{I}$$

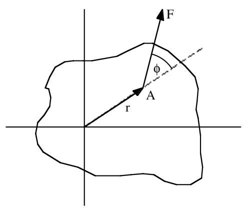


18

Torque.

- Consider rewriting the previous equation in the following way:
  $$rF\sin(\phi) = I\alpha$$
- The left-hand-side of this equation is called the torque τ of the force F:
  $$\tau = I\alpha$$
- This equation looks similar to Newton's second law for linear motion:
  $$F = ma$$
- Note:

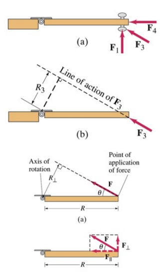
linear	rotational
mass m	moment I
force F	torque τ



19

Torque.

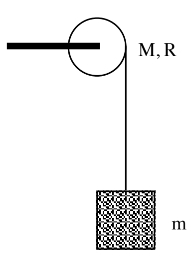
- In general, the torque associated with a force F is equal to
  $$|\vec{\tau}| = rF\sin(\theta) = |\vec{r}\times\vec{F}|$$
- The arm of the force (also called the moment arm) is defined as $r\sin(\theta)$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when $\theta = 90°$.



20

Rotational motion. Sample problem.

- Consider a uniform disk with mass M and radius R. The disk is mounted on a fixed axle. A block with mass m hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension of the cord.
- Expectations:
  - The linear acceleration should approach g when M approaches 0 kg.

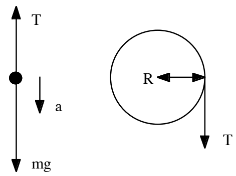


21

Rotational motion. Sample problem.

- Start with considering the forces and torques involved.
- Define the sign convention to be used.
- The block will move down and we choose the positive y axis in the direction of the linear acceleration.
- The net force on mass m is equal to

$$ma = mg - T$$



22

Rotational motion. Sample problem.

- The net torque on the pulley is equal to

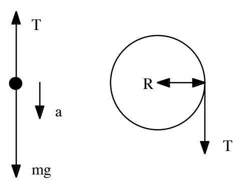
$$\tau = RT$$

- The resulting angular acceleration is equal to

$$\alpha = \frac{\tau}{I} = \frac{RT}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

- Assuming the cord is not slipping we can determine the linear acceleration:

$$a = \alpha R = 2\frac{T}{M}$$



23

Rotational motion. Sample problem.

- We now have two expressions for a:

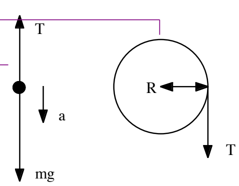
$$a = 2\frac{T}{M}$$

$$a = \frac{mg - T}{m} = g - \frac{T}{m}$$

- Solving these equations we find:

$$T = \frac{M}{M + 2m}mg$$

$$a = \frac{2m}{M + 2m}g$$



Note: $a = g$ when $M = 0$ kg!!!

24

Done for today!

Landing at Amsterdam Airport.

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Lecture 17, Page 25

25