Physics 141.
Lecture 14.

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Physics 141.
Lecture 14.

- Course Information:
  - Lab report # 3.
  - Exam # 2 and Exam # 3.
- Multi-Particle Systems (Chapter 9):
  - The center of mass of a multi-particle system.
  - The momentum principle for multi-particle systems.
  - The energy principle for multiple-particle systems.
  - The motion of the center of mass and variable mass systems.

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Course information.

- Labs:
  - Lab report # 3 will be due on Wednesday October 23 at noon.
  - Lab # 4 will take place on Monday October 28 in B&L 407.
- Exam # 2:
  - Solutions are posted on the PHY 141 webpages.
  - Exam # 2 will be returned during recitations this week.
  - The TAs cannot change the grade on your exam. If you feel you deserve more points, you will have to explain your reasoning in writing and submit this with your blue booklet(s) to me by Thursday October 31.
- Exam # 3:
  - Tuesday November 19.

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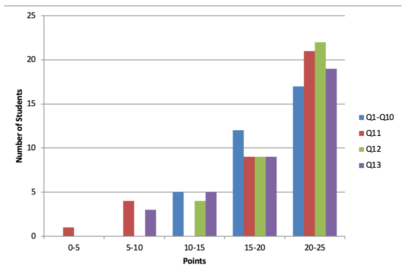
Results Exam 2.



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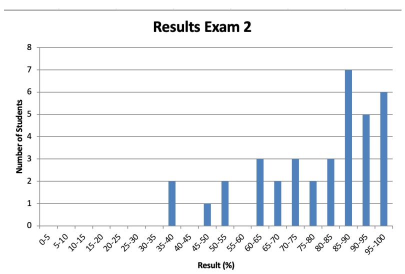
Results Exam 2.



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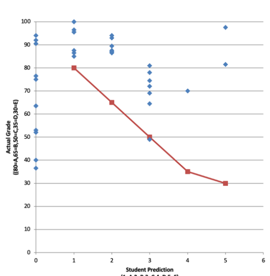
Results Exam 2.



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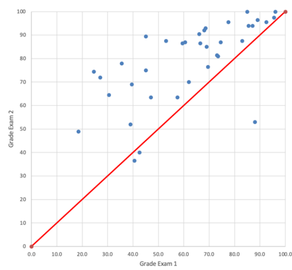
Results Exam 2.



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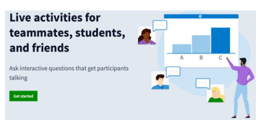
Quiz lecture 14.
PollEv.com/frankwolfs050

- The quiz today will have three questions.
- I will collect your answers electronically using the Poll Everywhere system.
- The answers for each question will be entered in sequence (first 60 s for question 1, followed by 60 s for question 2, etc.).



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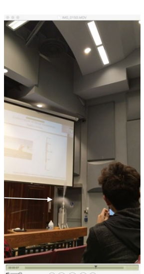
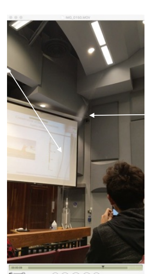
Rocket propulsion.
Observed in Hoyt.

The launch



Notice the shadow of the exhaust on the screen.



Projectile Motion.

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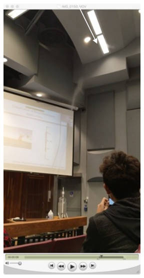
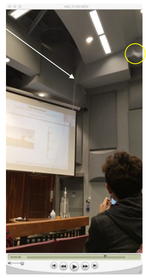
Rocket propulsion.
Observed in Hoyt.

The collision



Remaining exhaust.



The destroyed rocket.

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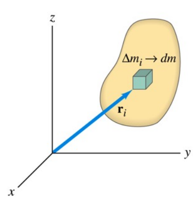
Location of the center of mass.

- In two or three dimensions the calculation of the center of mass is very similar, except that we need to use vectors.
- If we are not dealing with discreet point masses we need to replace the sum with an integral.

$$\vec{r}_{cm} = \frac{1}{M}\int_V \vec{r}\,dm$$



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Motion of the center of mass.
Non-relativistic limit.

- Ma_{cm} can be rewritten in terms of the forces on the individual components:

$$M\vec{a}_{cm} = \frac{d}{dt}(M\vec{v}_{cm}) = \frac{d\vec{P}_{cm}}{dt} = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- The motion of the center of mass is thus only determined by the external forces. Forces exerted by one part of the system on other parts of the system are internal forces and the sum of all internal forces cancels.

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Motion of the center of mass. Linear momentum.

- Now consider the special case where there are no external forces acting on the system:

$$\frac{d\vec{P}_{tot}}{dt} = 0$$

- This equations tells us that the total linear momentum of the system is constant.
- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = M\vec{v}_{cm} = \sum_i m_i \vec{v}_i = \sum_i \vec{p}_i$$

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Systems with variable mass.

- Rocket motion is an example of a system with a variable mass:

$$M(t + dt) = M(t) + dM$$

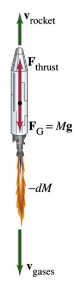
Mass of exhaust: $dM < 0$ kg

- As a result of dumping the exhaust, the rocket will increase its velocity:

$$v(t + dt) = v(t) + dv$$

- Since this is an isolated system, linear momentum must be conserved. The initial momentum is equal to

$$p_i = M(t)v(t)$$



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Systems with variable mass.

- The final linear momentum of the system is given by

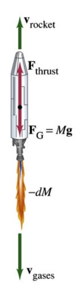
$$p_f = (M(t) + dM)(v(t) + dv) + (-dM)U$$

where U is the velocity of the exhaust.
- Conservation of linear momentum therefore requires that

$$M(t)v(t) = (M(t) + dM)(v(t) + dv) + (-dM)U$$

- The exhaust has a fixed velocity U_0 with respect to the engine. U_0 and U are related in the following way:

$$U - U_0 = v(t) + dv$$



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Systems with variable mass.

- Conservation of linear momentum can now be rewritten as

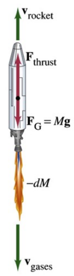
$$M(t)v(t)$$
$$= (M(t) + dM)(v(t) + dv) - dM(v(t) + dv + U_0)$$

or

$$M(t)v(t) = M(t)(v(t) + dv) - (dM)U_0$$

- We conclude

$$(dM)U_0 = M(t)dv$$



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Systems with variable mass.

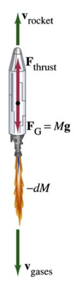
- The previous equation can be rewritten as

$$\frac{dM}{dt}U_0 = M(t)\,\frac{dv(t)}{dt}$$

In this equation:
- $dM/dt = -R$ where R is the rate of fuel consumption.
- $U_0 = -u$ where u is the (positive) velocity of the exhaust gasses relative to the rocket.
- dv/dt is the acceleration of the rocket.

- This equation can be rewritten as $RU_0 = Ma_{rocket}$ which is called the **first rocket equation**. This equation can be used to find the velocity of the rocket (**second rocket equation**):

$$v_f = v_i + u\ln\left(\frac{M_i}{M_f}\right)$$



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Rocket propulsion at the U or R. Introducing the Nimbus 4000.

A unique present. See Mike Culver for orders.

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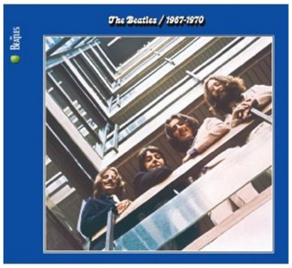
3 Minute 52 Second Intermission



- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 52 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

19

Energy of multi-particle systems. Kinetic and potential energy.

- In order to determine the (mechanical) energy of a multiple-particle system we need to determine both its kinetic energy and its potential energy:
  - The **kinetic energy** of the system will be the sum of the kinetic energy of the center-of-mass and the kinetic energy of the motion of the particles with respect to the center of mass.
  - The **potential energy** of the system may or may not depend on the position of the center of mass:
    - The gravitational potential energy can be expressed in terms of the position of the center of mass.
    - The electrostatic potential energy depends on the position of charges, not on the position of mass, and does not depend on the position of the center of mass.

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Energy of multi-particle systems. Kinetic energy.

- The kinetic energy of a multiple-particle system will have two components:
  - The **translational component**: the kinetic energy associated with the motion of the center of mass.
  - The **relative component**: the kinetic energy associated with the motion of the particles with respect to the center of mass. This type of motion an be vibrational, rotational, a combination of these two, etc.
- The decomposition of the kinetic energy into its components can be carried out with a bit of effort.

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Energy of multi-particle systems. Kinetic energy.

- Consider a multi-particle system for which we have specified the position of the center of mass r_{cm}, and the position of the particle with respect to the center of mass r_i.
- The kinetic energy of particle i is equal to

$$K_i = \frac{1}{2} m_i \left|\frac{d}{dt}(\vec{r}_{cm} + \vec{r}_i)\right|^2 = \frac{1}{2} m_i |(\vec{v}_{cm} + \vec{v}_i)|^2 =$$
$$\frac{1}{2} m_i (\vec{v}_{cm} + \vec{v}_i) \cdot (\vec{v}_{cm} + \vec{v}_i) = \frac{1}{2} m_i \{v_{cm}^2 + 2\vec{v}_{cm} \cdot \vec{v}_i + v_i^2\}$$

- The total kinetic energy of the system is thus equal to

$$K = \frac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 + \vec{v}_{cm} \cdot \sum_i m_i \vec{v}_i + \frac{1}{2}\sum_i m_i v_i^2$$

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Energy of multi-particle systems. Kinetic energy.

$$K = \frac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 + \vec{v}_{cm} \cdot \sum_i m_i \vec{v}_i + \frac{1}{2}\sum_i m_i v_i^2$$

$$\vec{v}_{cm} \cdot \sum_i m_i \vec{v}_i = \vec{v}_{cm} \cdot \frac{d}{dt} \sum_i m_i \vec{r}_i = 0$$

$$\frac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 = \frac{1}{2} M v_{cm}^2 = K_{translational}$$

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Energy of multi-particle systems. Kinetic energy.

- We thus see that

$$K = \frac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 + \frac{1}{2}\sum_i m_i v_i^2 =$$
$$= K_{translational} + K_{relative}$$

- The relative kinetic energy (relative to the center of mass of the system) can be equal to
  - The vibrational kinetic energy (e.g. the vibrational motion of atoms in an element).
  - The rotational kinetic energy (e.g. energy associated with the rotation of a wheel).
  - ……….

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Energy of multi-particle systems. Potential energy.

- Consider a multi-particle system located close to the surface of the earth.
- The gravitational potential energy of this system is equal to

$$U = \sum_i m_i g y_i = g \sum_i m_i y_i = gMy_{cm}$$

- The gravitational potential energy thus depends on the vertical position of the center of mass of the system.

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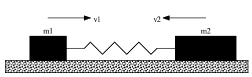
Conservation of linear momentum. An example.

- Two blocks with mass m_1 and mass m_2 are connected by a spring and are free to slide on a frictionless horizontal surface. The blocks are pulled apart and then released from rest. What fraction of the total kinetic energy will each block have at any later time?



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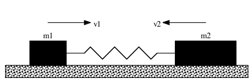
Conservation of linear momentum. An example.

- The system of the blocks and the spring is a closed system, and the horizontal component of the external force is 0 N. The horizontal component of the linear momentum is thus conserved.
- Initially the masses are at rest, and the total linear momentum is thus 0 kg m/s.
- At any point in time, the velocities of block 1 and block 2 are related:

$$v_2 = -\frac{m_1}{m_2} v_1$$



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Conservation of linear momentum. An example.

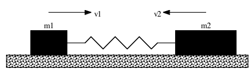
- The kinetic energies of mass m_1 and m_2 are thus equal to

$$K_1 = \frac{1}{2} m_1 v_2^2$$

$$K_2 = \frac{1}{2}\frac{p_2^2}{m_2} = \frac{1}{2}\frac{m_1}{m_2}\frac{p_1^2}{m_1} = \frac{m_1}{m_2} K_1$$

- The fraction of the total kinetic energy carried away by block 1 is equal to

$$f_1 = \frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + \frac{m_1}{m_2} K_1}$$
$$= \frac{m_2}{m_1 + m_2}$$



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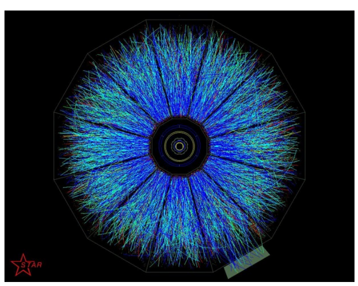
Next lecture: collisions.



Conservation of Linear Momentum at RHIC (STAR Experiment).

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