Measuring Earth's Gravitational Constant with a Pendulum

Philippe Lewalle, Tony Dimino
PHY 141 Lab TA, Fall 2014, Prof. Frank Wolfs
University of Rochester

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Abstract

In this lab we aim to calculate Earth's gravitational constant by measuring the period of a pendulum. We obtain a value of $9.79 \pm 0.02 m/s^2$. This measurement agrees with the accepted value of $g = 9.81 m/s^2$ to within the precision limits of our procedure. Limitations of the techniques and assumptions used to calculate these values are discussed. The pedagogical context of this example report for PHY 141 is also discussed in the final remarks.

1 Theory

The gravitational acceleration g near the surface of the Earth is known to be approximately constant, disregarding small effects due to geological variations and altitude shifts. We aim to measure the value of that acceleration in our lab, by observing the motion of a pendulum, whose motion depends both on g and the length L of the pendulum.

It is a well known result that a pendulum consisting of a point mass and attached to a massless rod of length L obeys the relationship shown in eq. (1),

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta, \tag{1}$$

where θ is the angle from the vertical, as showng in Fig. 1. For small displacements (ie small θ), we make a small angle approximation such that $\sin\theta \approx \theta$, which yields eq. (2).

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \tag{2}$$

Eq. (2) admits sinusoidal solutions with an angular frequency ω. We relate this to the period T of oscillation, to obtain an expression for g in terms of the period and length of the pendulum, shown in eq. (3).

$$\omega = \sqrt{\frac{g}{L}} \quad \text{and} \quad T = \frac{2\pi}{\omega}, \quad \therefore \quad T = 2\pi\sqrt{\frac{L}{g}} \quad \to \quad T^2 = \frac{4\pi^2 L}{g} \quad \to \quad g = \frac{4\pi^2 L}{T^2} \tag{3}$$

We therefore see that by measuring the period of a pendulum of known length, we may calculate the gravitational acceleration near Earth's surface.

In deriving these equations, we have neglected air resistance and other forms of friction, and have assumed that we may model our pendulum as a point mass attached to a rigid, massless rod or string. We set up our experiment such that these assumptions are met as best we are able, noting that the use of a string that is long relative the size of our masses/mass hangers, and masses which are heavy relative to the mass of our string will match these assumptions best.

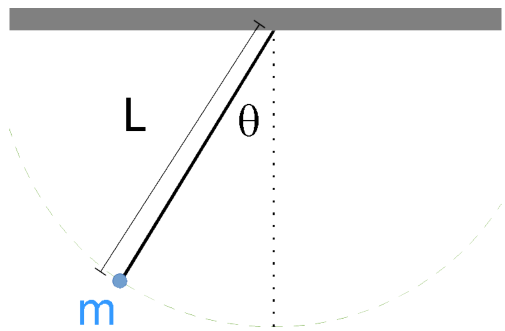

Fig. 1. A schematic of an idealized pedulum, as treated in our theory.

2 Experimental Apparatus and Procedures

Figure 2

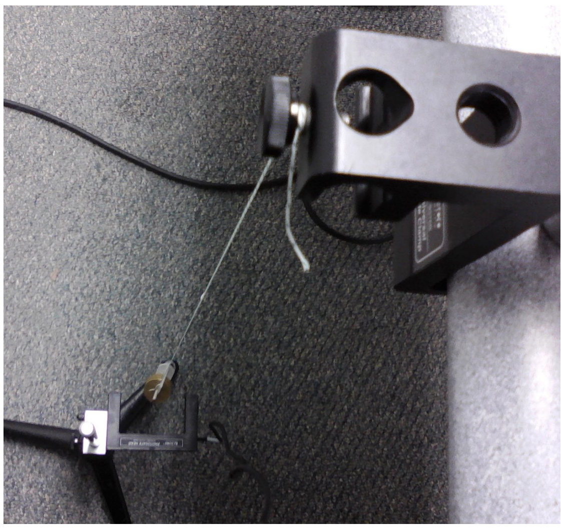

Figure 3

Figs. 2 and 3 show the experimental setup used. The string was tied to table clamp such that the mass and hanger could swing freely over small amplitudes in the photogate as shown. The photogate was mounted on an adjustable stand on the floor. Its height was adjusted for each different string length used, such that the bottom of the mass hanger would interrupt the photogate's beam each time it passed through the point $\theta = 0$.

In order to carry out this experiment, we use a $0.1kg$ mass and a 5-gram mass hanger, light string, a table clamp, and photogate relayed to a computer with DataStudio software installed. Our aim is to use the photogate to measure the period of oscillations. We fix our clamp over the side of the table, and attach a string bearing a mass to the clamp as shown in Figs. 2 and 3, such that the string is free to swing below the edge of the table. We tie a $0.005kg$ mass hanger to the end of the string, and place our $0.1kg$ mass on the hanger, as shown in Figs. 2, 3 and 4. Our total mass for the entire experiment is thereby taken to be $m = 0.105kg$. We then fix a photogate below the edge of the table, such that the string is free to swing through the photogate, and lowest-hanging point of the mass and hanger interrupts the photogate's beam when the string is vertical ($\theta = 0$). We choose the vertical for this due to the possibility that air resistance would change the amplitude of our oscillations; the effects on the period of our system due to air resistance would be negligible, but if the amplitude is changing and we measure at some angle off the vertical, we introduce additional error in how we *measure* our period, even if the period itself is not changing. Each time our pendulum passes the angle $\theta = 0$, the string interrupts the photogate's beam at a time recorded by DataStudio. This occurs twice per period (once when the pendulum is traveling to the right, and once as it travels to the left). We thereby take the period to be the difference in times between every other point recorded in DataStudio via the photogate. In practice, we measure the period by measuring the time $t = t_2 - t_1$ required for the pendulum to complete 10 complete oscillations and divide that time by

10 to obtain $T = t/10$. This, in conjunction with performing many trials, better allows us to average out fluctuations in the data collection process due to extraneous motion by the hanger, (it is, for instance, difficult to prevent it from rotating a bit as it swings), which could affect when the bottom of the hanger passes through the photogate. We take t_1 to be the time of the second event registered at the photogate and t_2 to be the twenty-second photogate event recorded in a given run. We set t_1 to be second event since there are variations in the first point depending on how the mass is released, despite our best efforts to do it the same way every time. In each run, the mass was displaced from the vertical towards the photogate (close to the edge of the gate, without touching it), and released as DataStudio was set to begin recording. The mass was allowed to oscillate for approximately 30 seconds in each run before it was stopped and reset to start another run.

We require that several different pendulum lengths be used, in order to confirm that the proportionality in eq. (3) is reasonable. We thereby use four different string lengths, $\ell_1 = 0.734m$, $\ell_2 = .619m$, $\ell_3 = .518m$, and $\ell_4 = .417m$. All trials were performed with the same string, which was retied/rewound on the mounting bracket to generate new lengths; each of these lengths ℓ were measured by placing a meter stick parallel to the hanging string, and measuring from the topmost point on the string able to swing freely, to the knot on the mass hanger. Ten runs were performed for each of these lengths. We additionally made a set of five measurements of a single ℓ value in order to estimate the error in this measurement. The same was done for the measurement of x (the distance from the lowest point of the mass hanger to the point of attachment, as shown in Fig. 4), and the measurement y from the bottom point to the center of the $0.1kg$ mass, which is taken as an estimate of the center of mass of the mass-hanger object. We take the total length of the pendulum to be the distance from the top of the string to the center of the weight hung on the bottom; for the i^{th} length, this is given by $L_i = \ell_i + \bar{x} - \bar{y}$, as shown in figure 4. The center point of the $0.1kg$ mass, neglecting the hanger, is taken to be the approximate position of the center of mass of the mass-hanger system attached to the spring.

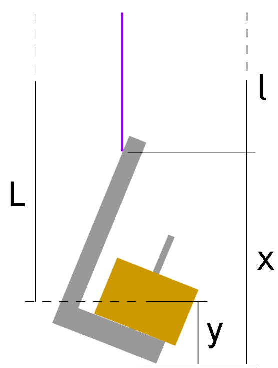

Fig. 4. A schematic of the $0.1kg$ mass (brass), and $5g$ mass hanger (grey). The parameters L, x, y, and ℓ are labeled to illustrate how they were measured using a meter stick. We see that $L = \ell + x - y$, where ℓ and L extend to the top attachment point of the string (purple), and L approximates the distance from the pivot point of the pendulum to the center of mass of the hanging weights.

3 Data Analysis

We begin by estimating the errors in our length measurements. As all these length measurements involve holding a ruler up parallel to a hanging string there is some imprecision involved. Table 1 shows each of 5 sets of measurements of the same ℓ, x, and y. Variables with a bar denote the mean of the measurements of that variable. For N measurements, the arithmetic mean is given by:

$$\bar{x} = \sum_{k=1}^{N} \frac{x_k}{N} \quad \rightarrow \quad \text{Example: } \bar{x} = \frac{.080 + .081 + ...}{5} \approx 0.080m \tag{4}$$

The standard deviation in each value is calculated as follows:

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}(x_k - \bar{x})^2} \quad \rightarrow \quad \text{Example: } \sigma_x = \sqrt{\frac{(0.080 - 0.080)^2 + (0.081 - 0.080)^2 + ...}{4}} \approx 0.001m \tag{5}$$

Measurement No.	$\ell(m)$	$x(m)$	$y(m)$
1	0.778	0.080	0.017
2	0.778	0.081	0.020
3	0.779	0.079	0.018
4	0.776	0.082	0.017
5	0.777	0.080	0.018
Mean	0.778	0.080	0.018
Standard Dev.	0.001	0.001	0.001

Table 1. We show a set of measurements of ℓ, x, and y with the mean and standard deviation in each set. These values of ℓ were not used for oscillation; they are simply a repeated measurement used to estimate the error in the measurement process. The standard deviations σ_ℓ, σ_x, and σ_y are shown.

	$\ell_i(m)$	$L_i(m)$	$\sigma_L(m)$
$i=1$	0.734	0.796	0.002
$i=2$	0.619	0.681	0.002
$i=3$	0.518	0.580	0.002
$i=4$	0.417	0.479	0.002

Table 2. The final estimates of the distance L from the center of mass to the pivot point for each of the four pendulum lengths used is given, along with the estimated error in each of those measurements.

As discussed above and shown in Fig. 4, we have $L_i = \ell_i + \bar{x} - \bar{y}$. The error in each L can be determined using error propagation, on the basis of σ_ℓ, σ_x, and σ_y, all shown in Table 1. We propagate the error according to:

$$\sigma_L = \sqrt{\sigma_\ell^2 \left(\frac{\partial L}{\partial \ell}\right)^2 + \sigma_x^2 \left(\frac{\partial L}{\partial x}\right)^2 + \sigma_y^2 \left(\frac{\partial L}{\partial y}\right)^2} = \sqrt{\sigma_\ell^2 + \sigma_x^2 + \sigma_y^2} \approx 0.002m \tag{6}$$

We summarize our four values of L and ℓ as related above, and the error estimate in these is that calculated in eq. (6), in Table 2.

The data for each of the runs and string lengths are shown below in Tables 3-6. We expect that the period should be the same within each group of runs with the same length; we combine data within each data-set for a given length using the arithmetic mean for this reason (see eq. (7)). All of the sample calculations shown below are based on the data in Table 1. We use k to index the runs within a table, and let $N = 10$ be the number of runs.

$$\bar{T} = \sum_{k=1}^{N} \frac{T_k}{N} \quad \rightarrow \quad \text{Example: } \bar{T}_1 = \frac{1.78324 + 1.78196 + ...}{10} \approx 1.78206s \tag{7}$$

The error in each data point within the sets is taken the be the standard deviation σ of that set, calculated according to eq. (8):

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}(T_k - \bar{T})^2} \quad \rightarrow \quad \text{Example: } \sigma = \sqrt{\frac{1}{9}\left[(1.78324 - 1.78206)^2 + (1.78196 - 1.78206)^2 + ...\right]} \approx 0.00049s \tag{8}$$

The error in the mean for a given dataset is given by σ_i (see eq. (9)), where i corresponds to the i^{th} length tested:

$$\sigma_i = \frac{\sigma}{\sqrt{N}} \quad \rightarrow \quad \text{Example: } \sigma_1 = \frac{0.00049}{\sqrt{10}} \approx 0.00015s \quad \rightarrow \quad \bar{T}_1 = 1.78206 \pm 0.00015s \tag{9}$$

We proceed by combining our results with a weighted mean, in order to obtain a final estimate for g. We use the weighted mean for this since we now have an error in each L and $\bar{T}$, from which an overall g must be calculated. We calculate values of g for each length used (eq. (10)), and the error in each value of g (eq. (11)), denoted by σ_{gi}. The resulting values appear in Table 7.

$$g_i = \frac{4\pi^2 L}{\bar{T}_i^2} \quad \rightarrow \quad \text{Example: } g_1 = \frac{4\pi^2 \cdot 0.796}{1.78206^2} \approx 9.89m/s^2 \tag{10}$$

Table 3 ($L = 0.796m$)

Run #	$T(s)$
1	1.78324
2	1.78196
3	1.78317
4	1.78299
5	1.78203
6	1.78272
7	1.78205
8	1.78226
9	1.78274
10	1.78287
Mean	1.78206
σ	0.00049
σ_1	0.00015

Table 4 ($L = 0.681m$)

Run #	$T(s)$
11	1.67352
12	1.67396
13	1.67395
14	1.67351
15	1.67387
16	1.67368
17	1.67436
18	1.67379
19	1.67441
20	1.67397
Mean	1.67390
σ	0.00031
σ_2	0.00010

Table 5 ($L = 0.580m$)

Run #	$T(s)$
21	1.52288
22	1.52543
23	1.52363
24	1.51641
25	1.51482
26	1.51611
27	1.52547
28	1.52408
29	1.51277
30	1.51724
Mean	1.51988
σ	0.00485
σ_3	0.00154

Table 6 ($L = 0.479m$)

Run #	$T(s)$
31	1.37951
32	1.37427
33	1.37880
34	1.37884
35	1.37064
36	1.37918
37	1.37853
38	1.37835
39	1.37890
40	1.37851
Mean	1.37755
σ	0.00284
σ_4	0.00090

Tables 3-6 display the measured periods in each of the runs performed for this experiment, along with the mean, standard deviation, and error in the mean for each period. We note that the standard deviations in many of the mean periods (which defines the error bars on those values), are small compared to the errors estimated in our length measurements.

	$g(m/s^2)$	$\sigma_g(m/s^2)$
$i=1$	9.89	0.0259
$i=2$	9.60	0.0288
$i=3$	9.91	0.0701
$i=4$	9.97	0.0552

Table 7. We tabulate the values of g and standard deviations in those values, calculated according to eqs. (10) and (11). It is the errors σ_{gi}, shown here, which are used to form weight factors in a weighted mean of the values g_i. These values, and the process of taking the weighted mean, are represented graphically in Fig. 5.

$$\begin{aligned} \sigma_{gi} &= \sqrt{\sigma_L^2\left(\frac{\partial g}{\partial L}\right)^2 + \sigma_i^2\left(\frac{\partial g}{\partial T}\right)^2} = \sqrt{\sigma_L^2\left(\frac{4\pi^2}{\bar{T}_i^2}\right)^2 + \sigma_i^2\left(-\frac{8\pi^2 L}{\bar{T}_i^3}\right)^2} \\ &\rightarrow \quad \text{Example: } \sigma_{g1} = \sqrt{0.002^2\left(\frac{4\pi^2}{1.78206^2}\right)^2 + 0.00015^2\left(\frac{8\pi^2\cdot 0.796}{1.78206^3}\right)^2} \approx 0.0259m/s^2 \end{aligned} \tag{11}$$

We continue by calculating the weighted mean of the values g_i to obtain a final estimate of g:

$$\bar{g} = \frac{\sum_{i=1}^{4} w_i g_i}{\sum_{i=1}^{4} w_i} \quad \text{for} \quad w_i = \frac{1}{\sigma_{gi}^2} \quad \rightarrow \quad \text{Example: } \bar{g} = \frac{\frac{9.89}{0.0259^2} + \frac{9.60}{0.0288^2} + \ldots}{\frac{1}{0.0259^2} + \frac{1}{0.0288^2} + \ldots} \approx 9.79m/s^2 \tag{12}$$

The error in the weighted mean $\sigma_{\bar{g}}$ is given by:

$$\sigma_{\bar{g}} = \sqrt{\frac{1}{\sum_{i=1}^{4} w_i}} = \frac{1}{\sqrt{\frac{1}{0.0259^2} + \frac{1}{0.0288^2} + \ldots}} \approx 0.0176m/s^2 \tag{13}$$

We thereby report a final value of

$$\bar{g} = 9.79 \pm 0.02m/s^2,$$

as our best estimate for g. We see that this value is in agreement with the accepted value of $g^* = 9.806 m/s^2$, as $\bar{g}$ sits approximately one standard deviation away from g^*. The percent error between $\bar{g}$ and the accepted value, given by

Figure 5: Values of g for a Given Length vs. Length Index (i)

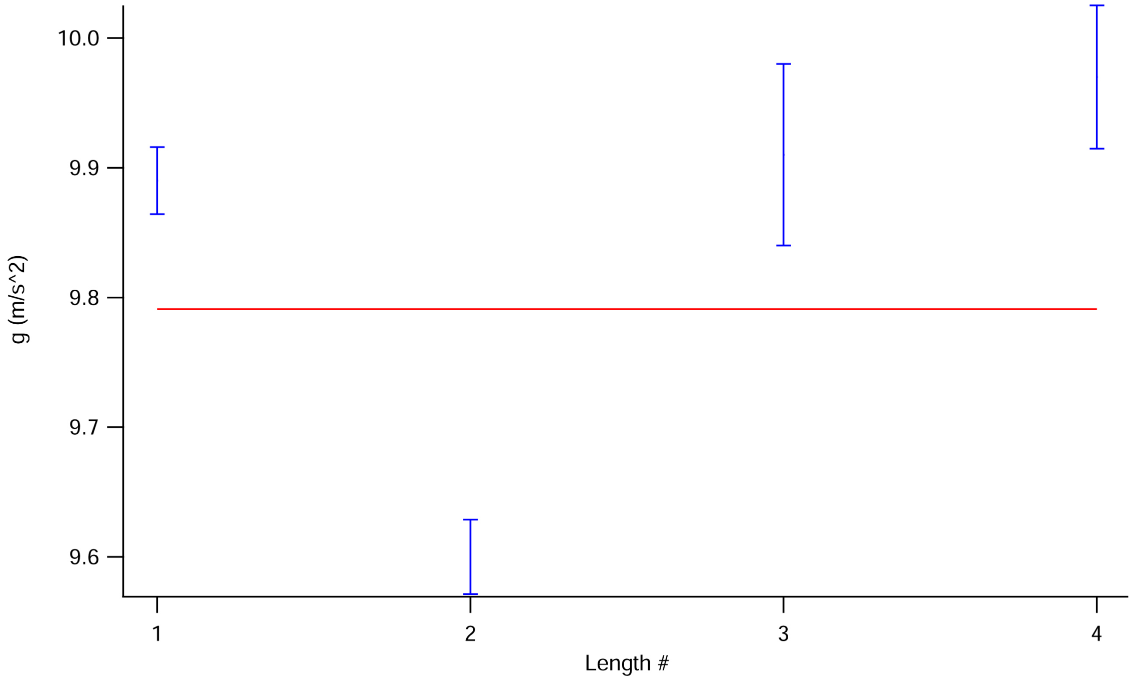

Fig. 5. We plot each g_i vs. i in order to visualize the weighted mean. The points g_i and their errors σ_{gi} are shown in blue. The line in red shows the mean, which can be obtained by doing a weighted fit of a constant value to the data points. We perform this process in Igor, which returns the line at $g = 9.79 \pm 0.02$, equivalent to our estimate of $\bar{g}$ calculated by hand above. We see that there is some variation of the data points around the mean (none are within one error bar of the mean itself), which is summarized by the value $\chi^2 = 72.1 \gg 4$. The Chi-Square value returned by Igor would be equal to 4, (the number of data points) if we had a perfect fit.

$100 \cdot \left(\frac{\bar{g}-g^*}{g^*}\right)$ is -0.204%. The negative sign on the percent error indicates that we have underestimated, rather than overestimated, g. This provides further indication that our value of g is reliable. Since our error bars are of the same order as the difference between $\bar{g}$ and g^*, the small percent error also helps us understand that our error bars are small relative to the value we are calculating, and that our procedure was consequently relatively precise. The weighted mean can be represented graphically by fitting a constant to the data points. The points are weighted according to their errors σ_{gi}. A plot illustrating this process can be found in Fig. 5, above. (This method is equivalent to the calculation of the weighted mean shown above, and returns the same value of, and error in, $\bar{g}$). Please see the appendices for a discussion of other possible approaches to the analysis.

4 Conclusions

We measure the period of pendulums for 4 different lengths, and use the expression in eq. (3) to calculate the value of g, the gravitational acceleration near the surface of the Earth. The limiting precision factor in this experiment was the number of significant figures and error estimates we were able to obtain in our measurements of length. We repeat sample length measurements according to our measurement procedure to estimate the uncertainty in that procedure; these are found to be several orders of magnitude larger than the uncertainties in the extremely precise measurements of the pendulum's period we carry out. We combine the data for all four pendulums used with a weighted mean, which allows us to report a final value of $g = 9.79 \pm 0.02 m/s^2$. This is within one error bar of the accepted value of the constant on Earth's surface; we assume the actual gravitational acceleration at our lab in Rochester deviates negligibly from this accepted value, and that our results are therefore accurate.

4.1 Remarks

A more precise method for measuring the length of the pendulum would allow us to carry out a more precise experiment, since the measurements of the period were already very precise. This would have to come in tandem with a more complicated analysis (which includes the mass of the string, and specific shapes and densities of the masses) in order to make assumptions consistent with that higher level of precision. Although this could be done in principle, we do not see a practical way to implement these improvements with the lab equipment available for PHY 141.

4.2 Remarks for PHY 141 Students

This report is intended to be a model of a PHY 141 report, written slightly above the level of an ideal student report. Please note that the LaTeX formatting with numbered equations and such is not necessary for PHY 141; *neat* handwritten equations, or those written in the word processor/equation editor of your choice are acceptable, as are *neat* hand-drawn figures. This report is, however, intended to give students an example of the level of detail and style of scientific writing they can strive to master in their lab reports over the course of the year. Some things to pay attention to (pertaining to common errors or oversights I see in student reports):

- The theory section relates the assumptions implicit in the equations we use to the experimental setup. It is important to learn to make those connections; many students get comments on their reports to the effect of "what assumptions are made?", and I hope that the last, short paragraph of the theory section helps clarify what that means. It may be appropriate to discuss such assumptions in the conclusions (as part of a discussion of sources of error), rather than in the theory; either is fine, so long as there is some acknowledgment of the assumptions in the report, particularly if there is a disagreement between the expectations and results.

- Student experimental descriptions generally summarize the main points of the experiment effectively (either in paragraph form as shown here, or in a bulleted style – both are acceptable). Specific details such as those regarding the measurement procedure illustrated in Fig. 4 are often missing however. These sorts of details make the difference between an adequate experimental description and one that is truly complete.

- We try to encourage plot based analyses, as a visual presentation of the data is often more concise and easy to follow than a long list of tables and calculations (although both are necessary to varying degrees, depending on the experiment you are reporting on). Notice how Fig. 5 gives a much clearer impression of what is happening with the values and weighted mean (in terms of the bigger picture) than the details in Table 7 and the surrounding calculations can. A somewhat different, and more graphical approach without a weighted mean can be used in many PHY 141 activities; this method is described in the appendix below. Note also which details are present in the data analysis: there are enough example calculations and tables to summarize all of the data collected for this experiment, and I try to be very clear about where each error is coming from and what I am doing with it next.

- Notice how captions are used for all of the tables and plots. Supporting materials can appear in the text as done here, or in an appendix, but either way, numbered references from the text and insightful captions about the contents of a figure or table are not optional.

- Please note the style of the abstract and conclusions. There is no "new" information in the conclusion above; it is just a more concise summary of what happened in the preceding three sections. Some of the discussions in the Data Analysis section could be shifted here, but take care not to make the conclusions too long. For labs with questions in the manual, the conclusion will grow a bit as students summarize their responses to those questions. The conclusions would also have grown if there were a disagreement between the calculated result and the expected result, as that would warrant some additional discussion. The abstract contains similar information to the conclusions, but is distilled even further to provide a few sentences of context for a prospective reader. The abstract for a PHY 141 report should never need to be longer than the one shown here.

A Appendix: Alternate Data Analysis Methods and Discussion

We often like to do analyses by plotting two sides of an equation against each other, such that we can determine a value of interest by looking at the slope of a line of best fit through the data points. This approach is very effective in many situations in PHY 141 labs and beyond, but it runs into some problems in this particular data set as we will see below. These stem from the fact that we usually take measurements in our experiments assuming the controlled variable to be exact. (In the above experiment, that means we would generally treat the length as being measured without error. We added an estimate for the error in the length because in this case, the errors in the periods alone are miniscule, and consequently do not give us a very good error estimate by themselves.) We cannot set an overall weighting wave for our fit in Igor when we include our length errors, however, because we then have two separate error bars on each point (one in x and one in y); this gives us no correct/exact way to perform the error analysis with a line of best fit. We can still approximate the weighting by one of our two error bars, however, in order to illustrate how this kind of analysis might be carried out if we had error bars on only one parameter in eq. (3) which g depends on, rather than two. We also explore the limitations of approximating the weighting with the errors in only one variable, when one of them tends to dominate.

A.1 Line of Best Fit with σ_L

The gravitational acceleration g for each run is given by eq. (3). We see that we can rearrange that expression to read,

$$4\pi^2 L = gT^2, \tag{14}$$

which indicates that we may plot $4\pi^2 L$ vs. T^2 and expect to obtain a straight line with slope g. If we wish to do this, we must calculate the error in $\bar{T}^2$, and the error bars in $4\pi^2 L$. We let $f(T) = T^2$ and apply the usual error propagation formula, where σ_i^2 is the variance in $\bar{T}$ for the i^{th} length as calculated in Tables 3-6 above:

$$\sigma_f^2 = \sigma_i^2 \left(\frac{\partial f}{\partial T}\right)^2 = \sigma_i^2 (2T)^2 \quad \rightarrow \quad \sigma_{fi} = 2\bar{T}_i \sigma_i \quad \rightarrow \quad \text{Example: } \sigma_{f1} = 2 \cdot 1.78206 \cdot 0.00015 \approx 0.00055 s^2 \tag{15}$$

This tells us that σ_{fi} is the x-axis error bar in the i^{th} data point. The same process can be applied to $h(L) = 4\pi^2 L$ to determine the error bars $\sigma_h \approx 0.080m$.

We perform a weighted fit using Igor (shown in Fig. 6), using σ_h as the basis for the weight factors. Notice that σ_h is only the y-axis error bar, and that we have not factored the uncertainty in T into the weighting or error analysis by doing this. Table 8, shown below, allows us to consider the validity of this approximation, where σ_g is the overall error in g, as calculated in the body of the report, and σ_{gL} is the error in g neglecting the errors σ_i:

$$\sigma_{gi} = \sqrt{\sigma_L^2 \left(\frac{\partial g}{\partial L}\right)^2 + \sigma_i^2 \left(\frac{\partial g}{\partial T}\right)^2} = \sqrt{\sigma_L^2 \left(\frac{4\pi^2}{\bar{T}_i^2}\right)^2 + \sigma_i^2 \left(-\frac{8\pi^2 L}{\bar{T}_i^3}\right)^2} \quad \text{and} \quad \sigma_{gLi} = \frac{4\pi^2 \sigma_L}{\bar{T}_i^2} \quad \text{and} \quad \sigma_{gTi} = \frac{8\pi^2 L \sigma_i}{\bar{T}_i^3} \tag{16}$$

	$\sigma_g (m/s^2)$	$\sigma_{gL} (m/s^2)$
$i = 1$	0.0259	0.0252
$i = 2$	0.0288	0.0285
$i = 3$	0.0701	0.0346
$i = 4$	0.0552	0.0421

Table 8. We compare σ_g and σ_{gL}. We see that σ_{gL} is a reasonable approximation for σ_g for $i = 1, 2$, where the errors in T are the smallest. It is not, however, an adequate approximation for $i = 3, 4$, where the errors in T account for one third to one half of the total error. We can conversely infer that σ_{gT} will never be a good approximation for the total error; the errors in T are too small compared to those in length. This is discussed in section A.2.

The y-intercept of the fit is constrained to be zero as we do not anticipate any offset in our theory. The slope of this line, using only σ_h for weighting, gives us an estimate for g:

$$g_{final} = 9.82 \pm 0.02 m/s^2 \tag{17}$$

We see that we obtain a value that is within one error bar of the accepted value of $g^* = 9.806 m/s^2$ which is therefore in agreement with the accepted value. It is also within two error bars of the value we obtain with a weighted mean in the main body of the report, indicating that although our approximation for the weighting was not ideal, the effect on the final results is very small. This is an acceptable analysis in this case.

Figure 6: $4\pi^2 L_i$ vs. $\bar{T}_i^2$

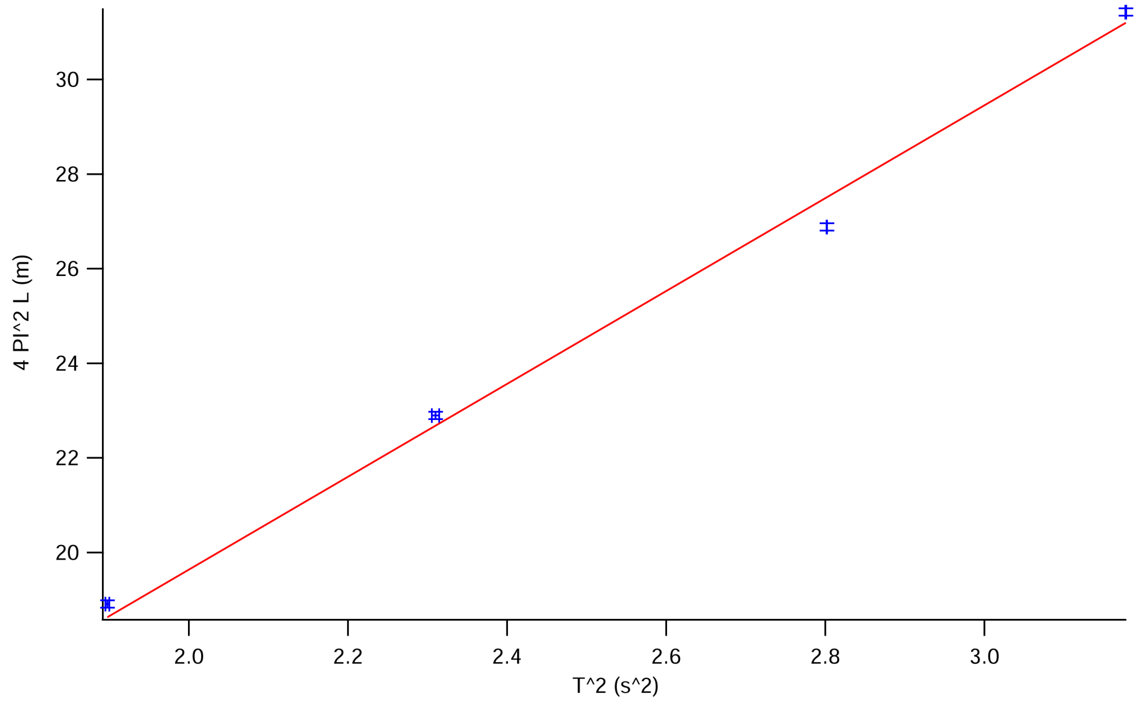

Fig. 6. We plot $4\pi^2 L$ against T^2, where each data point is $\bar{T}_i$ from Tables 3-6, with the error bars σ_g in the y direction, and the error bars σ_{fi} in the x direction (points and errors shown in blue). The linear fit shown (red) was performed through these points, using the error bars as the basis for weights, with the y-intercept constrained to be zero as expected in our theory. The slope of the line is $g = 9.82 \pm 0.02 m/s^2$. The Chi-Square value for the fit returned by Igor is $\chi^2 = 88.5 \gg 4$ (where 4 is the number of data points; since Igor does not return the reduced χ^2 we expect $\chi^2 \approx 4$ for a near perfect fit). Our χ^2 indicates that our points still sit several error bars away from the fit line, even though the points look approximately linear, and are well correlated as indicated by Pearson's Correlation Coefficient $V_{pr} = 0.997$. From the correlation values we conclude that despite the variation of the data points around the fit, it is still reasonable to have assumed the direct proportionality in eq. (3).

A.2 Line of Best Fit with σ_T

Suppose we use σ_{fi} instead of σ_h to weight the linear fit. We noted above that the errors are dominated by σ_h, and that we obtained a value very similar to the weighted mean used in the body of the report by using only the error related to σ_h. It is thus unsurprising that using only the T errors for the fit weighting, with the exact same process as in the preceding section, we obtain a value of $g = 9.68 \pm 9.5 \times 10^{-5} m/s^2$. The value of g from this weighting is further from the expected value, and the error is virtually negligible, since it sits beyond the last significant figure we are able to report for the value itself. This is consequently not a terribly good approach for the analysis for this particular data set, even though for most data sets in a PHY 141 lab this procedure would actually be the expected one, and would provide good results. This agrees neither with the weighted mean, any of the other fits, nor the accepted value of g, since it sits many more than three standard deviations from all of them.