
Physics 121 Review Midterm Exam # 3

April 20, 2008

Review Physics 121

Midterm Exam # 3

- Main topics covered:
 - Rotational Motion:
 - Rotational Variables.
 - Angular Momentum.
 - Equilibrium:
 - Conditions for Equilibrium.
 - Harmonic Motion:
 - Properties of Simple Harmonic Motion
 - Requirements for Simple Harmonic Motion
 - Damped Harmonic Motion
 - Driven Harmonic Motion

Review Physics 121

Midterm Exam # 3

- Warning:
 - Use this review at your own risk.
 - I can not cover everything we discussed in 8 lectures in the period allocated for this review.
 - If I leave out certain topics in this review, this does not imply that these topics will not be covered on the exam!
 - The material covered on the exam is the material covered in Chapters 10, 11, 12, and 14 of our text book.

Review Physics 121

Midterm Exam # 3

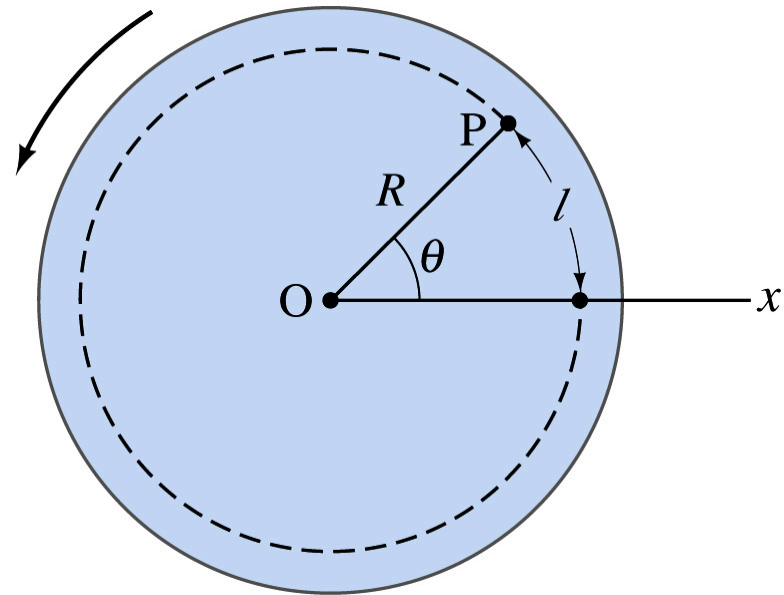
- Note:
 - I expect to be interrupted!
 - The TAs will see the exam for the first time at the same time you do (on Tuesday morning at 8 am). They are certainly telling you the truth if they state that they do not know what is being covered on the exam.

Rotational Motion

- There are many similarities between linear and rotational motion.
- If you really understand linear motion, understanding rotational motion should be easy.
- The concepts of moment of inertia, torque, and angular momentum are defined such as to preserve the similarities between linear and rotational motion.
- I will start this review with focusing on a detailed comparison between linear and rotational motion.

Rotational Motion. Variables

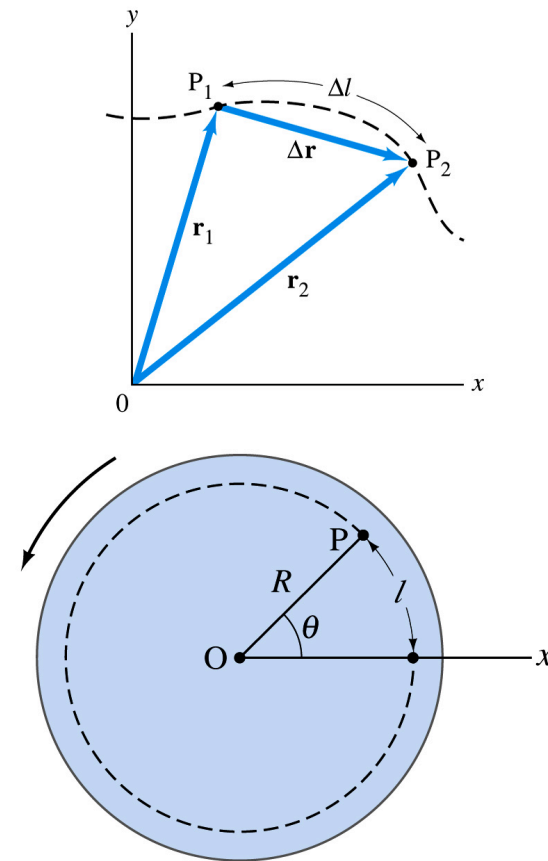
- In our discussion of rotational motion we will first focus on the rotation of **rigid** objects around a **fixed** axis.
- The variables that are used to describe this type of motion are similar to those we use to describe linear motion:
 - Angular position
 - Angular velocity
 - Angular acceleration



Rotational Variables.

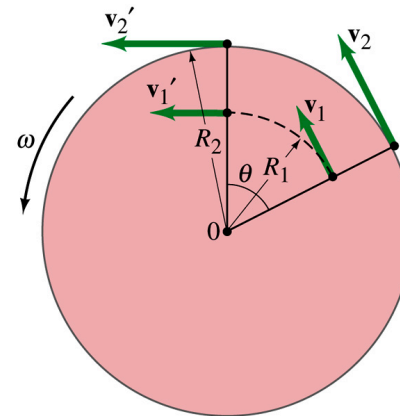
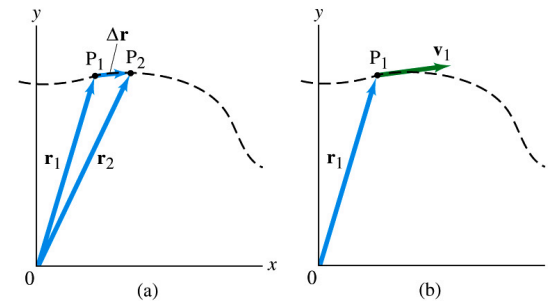
Position.

- In order to specify the position of a point we need to specify our reference point/axis.
- Linear position:
 - Specify the vector required to move from the origin of the coordinate system to point P .
 - Unit: m
- Angular position:
 - Specify the rotation angle required to rotate from the reference line to point P .
 - Unit: rad



Rotational Variables. Velocity.

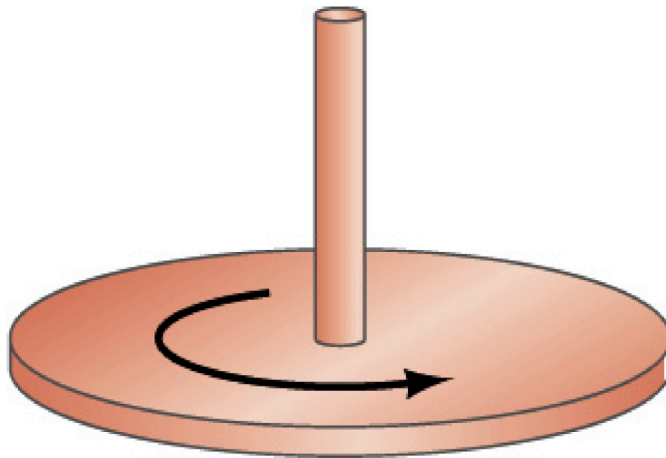
- Velocity is a measure of how quickly the position changes.
- Linear velocity:
 - Definition: $v = dr/dt$
 - Symbol: v
 - Units: m/s
- Angular velocity:
 - Definition: $\omega = d\theta/dt$
 - Symbol: ω
 - Units: rad/s



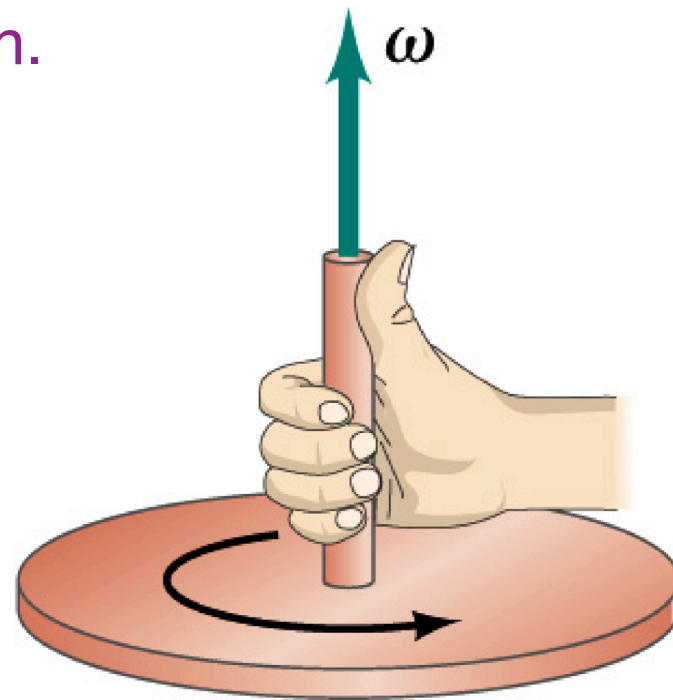
Direction of the Angular Velocity.

User your Right Hand!

Angular velocity is a vector!
It has a magnitude and a direction.



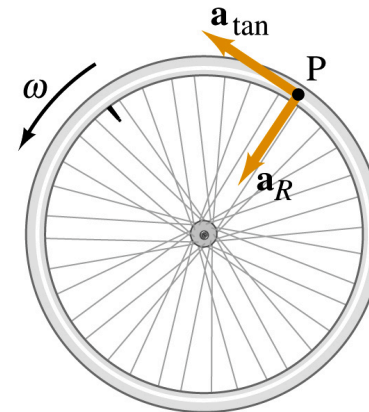
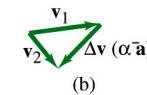
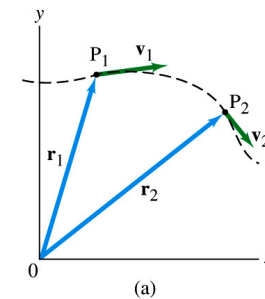
(a)



(b)

Rotational Variables. Acceleration.

- Acceleration is a measure of how quickly the velocity changes.
- Linear acceleration:
 - Definition: $a = dv/dt = d^2r/dt^2$
 - Symbol: a
 - Units: m/s^2
- Angular acceleration:
 - Definition: $\alpha = d\omega/dt = d^2\theta/dt^2$
 - Symbol: α
 - Units: rad/s^2

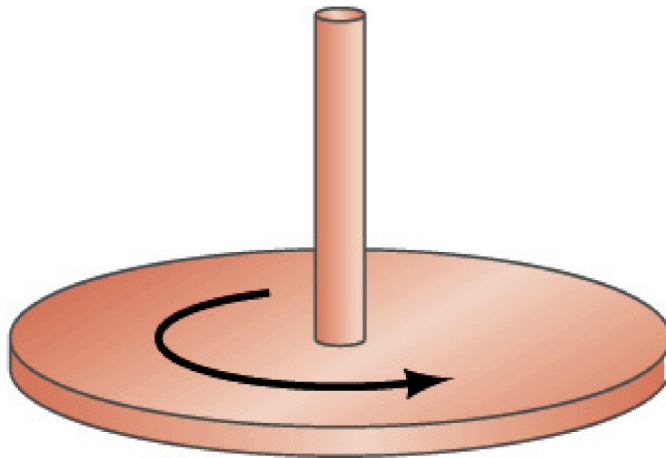


Rotational Variables. Acceleration.

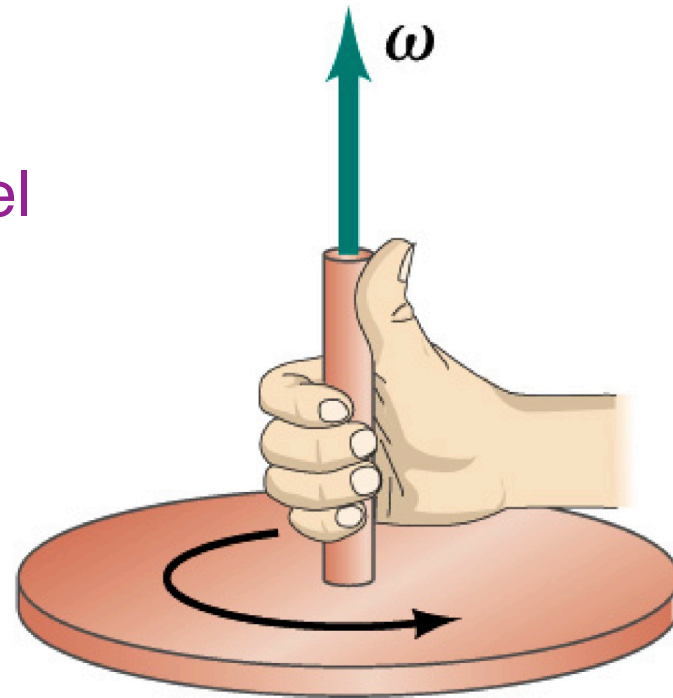
The angular acceleration is parallel or anti-parallel to the angular velocity:

If ω increases: parallel

If ω decreases: anti-parallel



(a)



(b)

Summary of Rotational Variables.

	Definition	Linear Variable
Angular Position	θ	$l = R\theta$
Angular Velocity	$\omega = \frac{d\theta}{dt}$	$v = R\omega$
Angular Acceleration	$\alpha_{\text{tan}} = \frac{d^2\theta}{dt^2}$	$a_{\text{tan}} = R\alpha_{\text{tan}}$

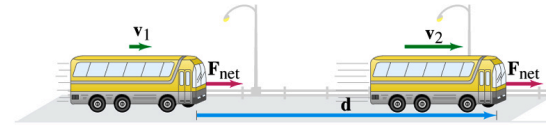
Equations of Rotational Motion. Constant Acceleration.

	Rotationional Motion	Linear Motion
Acceleration	$\alpha(t) = \alpha$	$a(t) = a$
Velocity	$\omega(t) = \omega_0 + \alpha t$	$v(t) = v_0 + at$
Position	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	$x(t) = x_0 + v_0 t + \frac{1}{2} at^2$

Rotational Variables.

Kinetic Energy.

- The kinetic energy of an object is proportional to the square of its velocity.

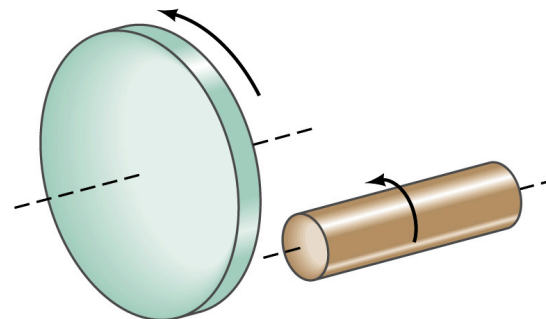


- Linear kinetic energy:

- Definition: $K = (1/2)mv^2$
- Unit: $\text{kg m}^2/\text{s}^2$ or J

- Rotational kinetic energy:

- Definition: $K = (1/2)I\omega^2$
- Unit: $\text{kg m}^2/\text{s}^2$ or J
- I is the moment of inertia of the mass distribution.



The Moment of Inertia.

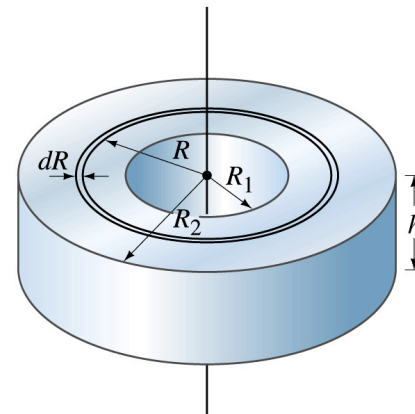
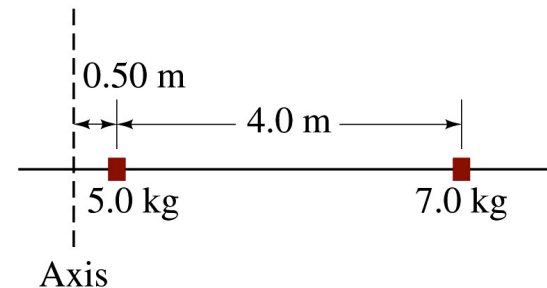
Calculating I.

- The moment of inertia of an object depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$



Rotational Variables.

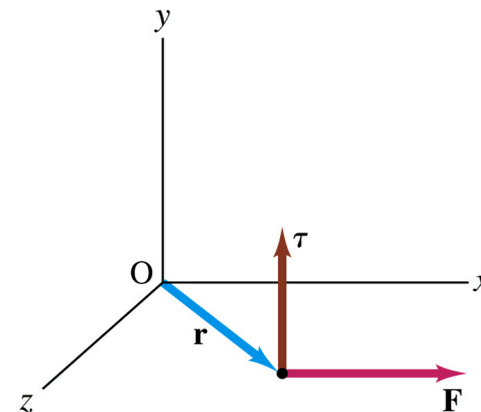
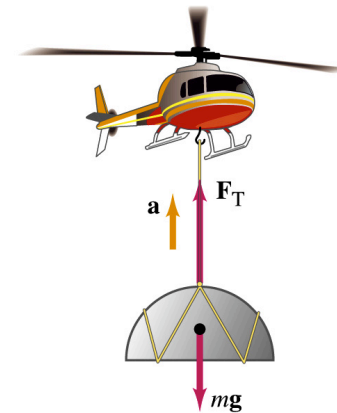
Force and Torque.

- Linear motion:

- The linear acceleration is proportional to the applied force.
- $F = ma$
- Unit: N

- Rotational motion:

- The angular acceleration is proportional to the torque.
- The torque is defined as the vector products $\mathbf{r} \times \mathbf{F}$.
- Unit: Nm
- $\tau = I\alpha$ for a rigid object, where I is the moment of inertia of the mass distribution.

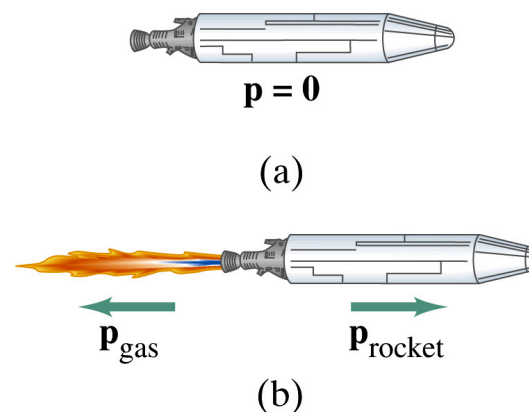


Rotational Variables.

Linear and Angular Momentum.

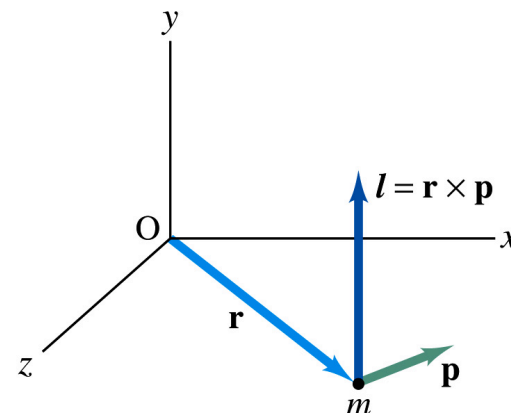
- Linear Momentum:

- Defined: $p = mv$
- Units: kg m/s
- Total linear momentum is conserved if the net external force is 0 N.



- Angular Momentum:

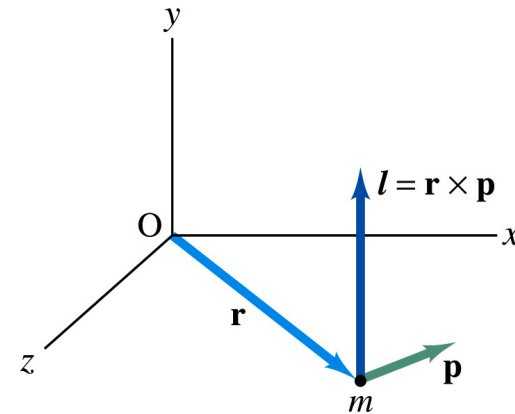
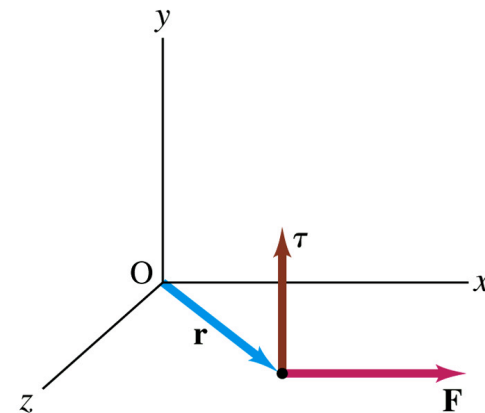
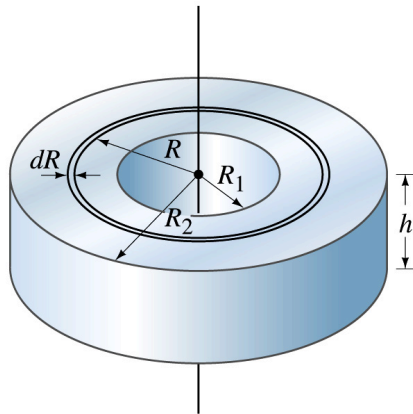
- Defined: $\mathbf{L} = \mathbf{r} \times \mathbf{p}$
- For rigid object: $L = I\omega$
- Unit: kg m²/s
- Total angular momentum is conserved if the net external torque is 0 Nm.



Rotational Variables.

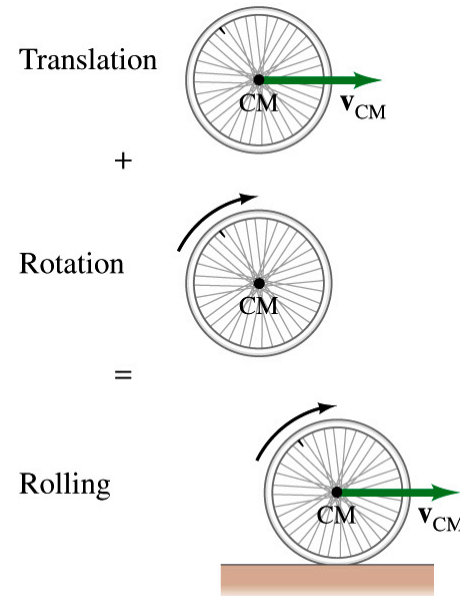
Make sure you know your references!

- The moment of inertia is calculated with respect to a specific rotation axis.
- The torque and angular momentum are calculated with respect to a specific reference point.



Rolling Motion.

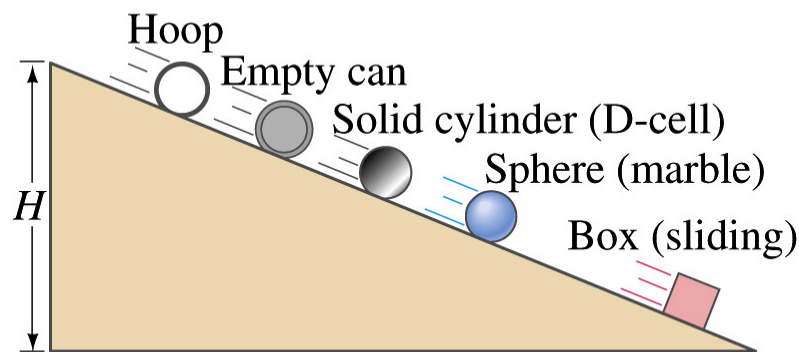
- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
 - Translational kinetic energy = $(1/2) M v_{\text{cm}}^2$.
 - Rotational kinetic energy = $(1/2) I_{\text{cm}} \omega^2$.
- Assuming the wheel does not slip: $\omega = v / R$.



Rolling Motion.

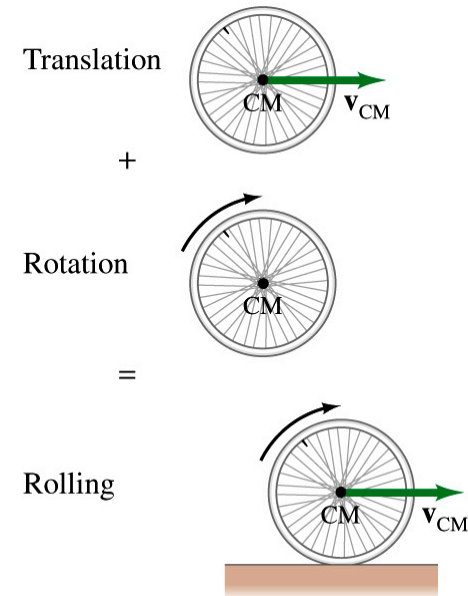
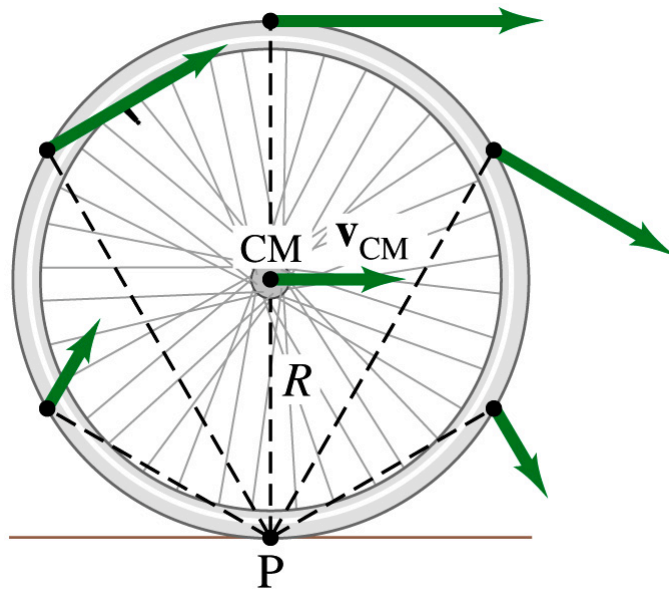
Effect of Moment of Inertia on Motion.

- Initial mechanical energy = mgH .
- Final mechanical energy = $(1/2) m v_{cm}^2 + (1/2) I_{cm} \omega^2$.
- Assuming no slipping, we can rewrite the final mechanical energy as $(1/2)\{m + I_{cm} / R^2\} v_{cm}^2$.
- Conservation of energy implies:
 $(1/2)\{m + I_{cm} / R^2\} v_{cm}^2 = mgH$
or
 $(1/2)\{1 + I_{cm} / mR^2\} v_{cm}^2 = gH \rightarrow$



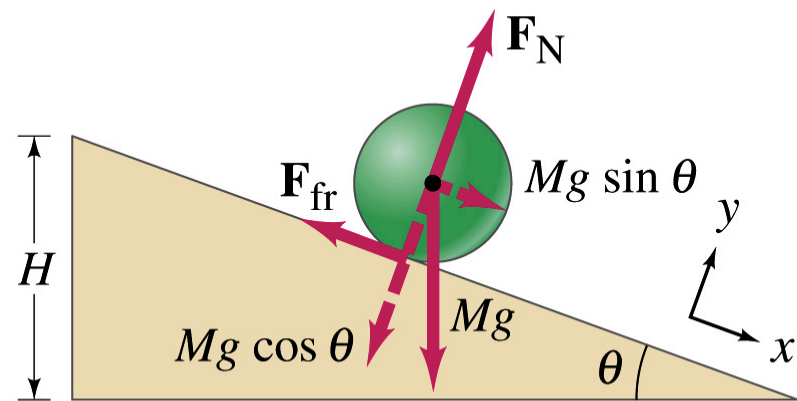
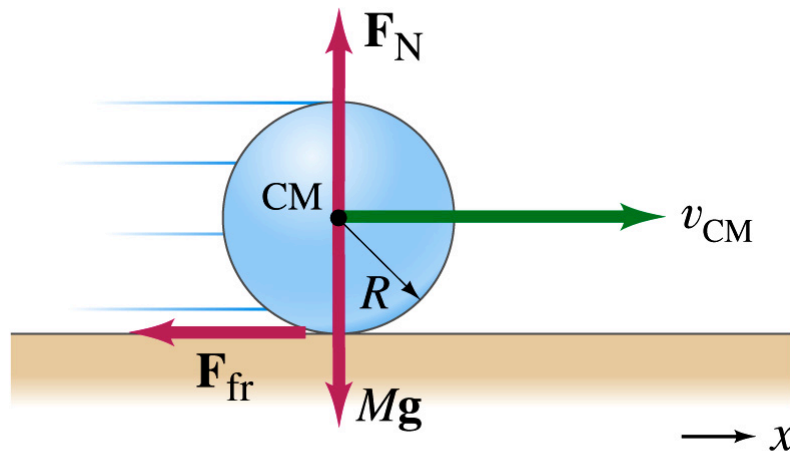
The smaller I_{cm} , the larger v_{cm} at the bottom of the incline.

Rolling motion causes much confusion!



Two views of rolling motion: 1) Pure rotation around the instantaneous axis or 2) rotation and translation.

Rolling motion causes much confusion!



Note: friction provides the torque with respect to the center-of-mass.

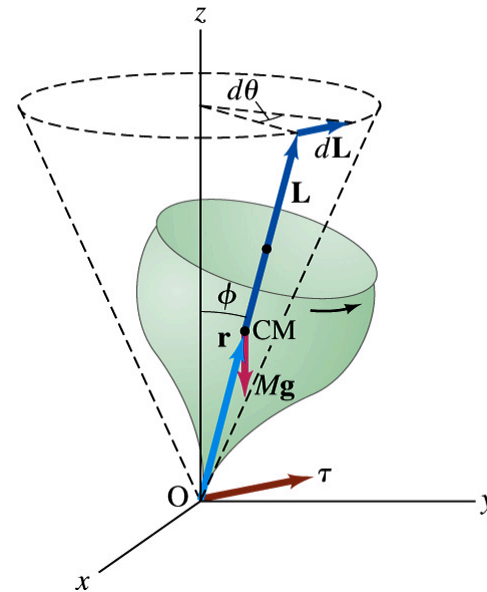
A Final Note about Angular Momentum.

- The connection between the angular momentum L and the torque τ

$$\sum \bar{\tau} = \frac{d\bar{L}}{dt}$$

is only true if L and τ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).

- The relation is also true if L and τ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).



Equilibrium.

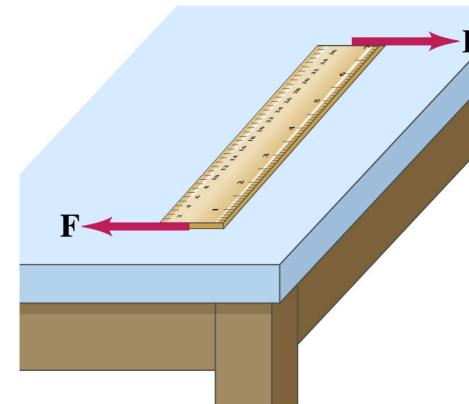
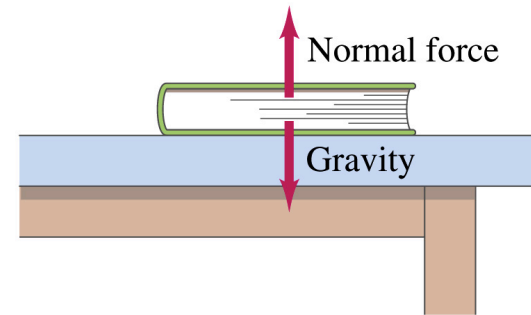
- An object is in equilibrium if the following conditions are met:

Net force = 0 N (first condition for equilibrium)

and

Net torque = 0 Nm (second condition for equilibrium)

- Note: both conditions must be satisfied. Even if the net force is 0 N, the system can start to rotate if the net torque is not equal to 0 Nm.



Static Equilibrium.

- What happens when the net force is equal to 0 N?
 - $P = \text{constant}$
- What happens when the net torque is equal to 0 Nm?
 - $L = \text{constant}$
- We conclude that an object in equilibrium can still move (with constant linear velocity) and rotate (with constant angular velocity).
- Conditions for **static** equilibrium:
 - $P = 0 \text{ kg m/s}$
 - $L = 0 \text{ kg m}^2/\text{s}$

Equilibrium.

Summary of Conditions.

- Equilibrium in 3D:

$$\sum F_x = 0$$

$$\sum \tau_x = 0$$

$$\sum F_y = 0 \quad \text{and}$$

$$\sum \tau_y = 0$$

$$\sum F_z = 0$$

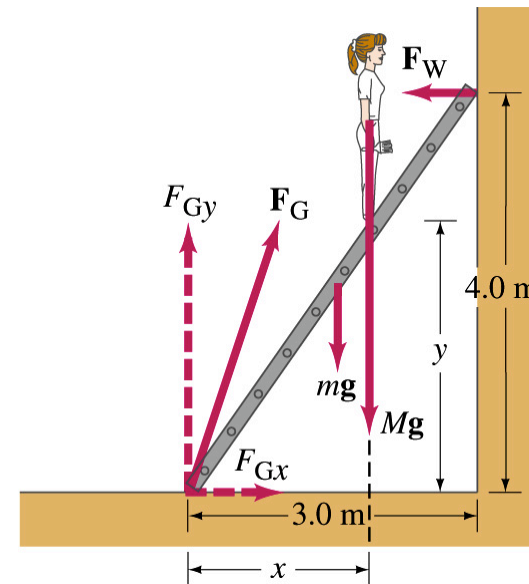
$$\sum \tau_z = 0$$

- Equilibrium in 2D:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

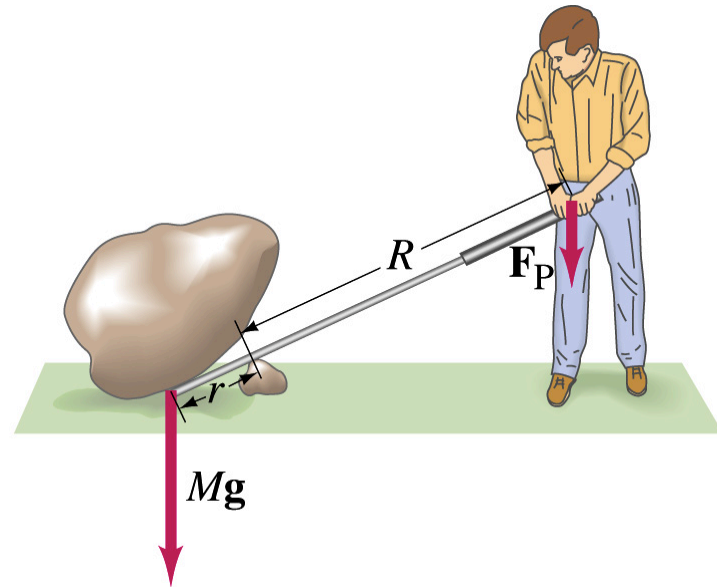
$$\sum \tau_z = 0$$



Equilibrium.

Be sure to include all forces!!!

- When evaluating conditions for equilibrium, you need to make sure to include all forces acting on the system.
- In the system shown in the Figure, there are more forces acting on the system than the forces indicated. For example, there should be an upward force to balance the downward forces.
- Of course, the problem is how to apply the equilibrium conditions correctly.



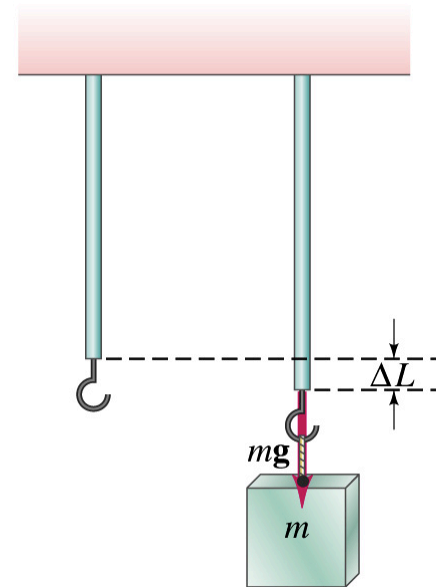
Stress and Strain.

The Effect of Applied Forces.

- When we apply a force to an object that is kept fixed at one end, its dimensions can change.
- If the force is below a maximum value, the change in dimension is proportional to the applied force. This is called Hooke's law:

$$F = k \Delta L$$

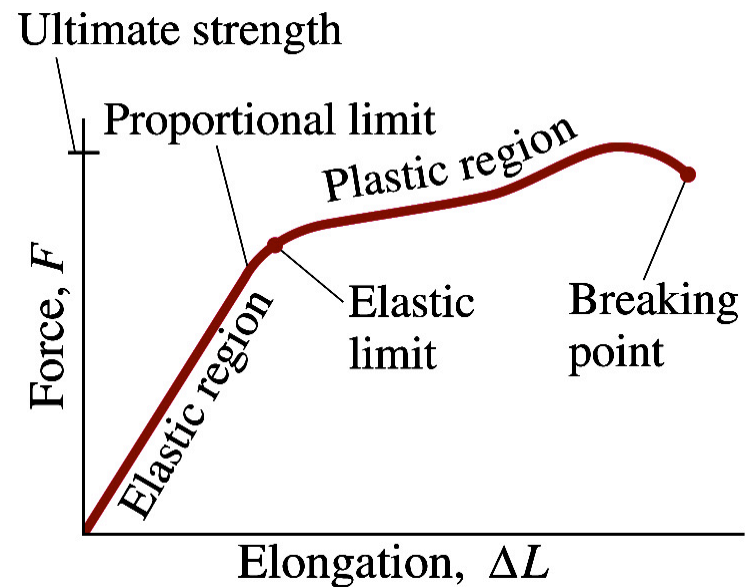
- This force region is called the elastic region.



Stress and Strain.

The Effect of Applied Forces.

- When the applied force increases beyond the elastic limit, the material enters the plastic region.
- The elongation of the material depends not only on the applied force F , but also on the type of material, its length, and its cross-sectional area.
- In the plastic region, the material does not return to its original shape (length) when the applied force is removed.



Stress and Strain.

The Effect of Applied Forces.

- The elongation ΔL can be specified as follows:

$$\Delta L = \frac{1}{E} \frac{F}{A} L_0$$

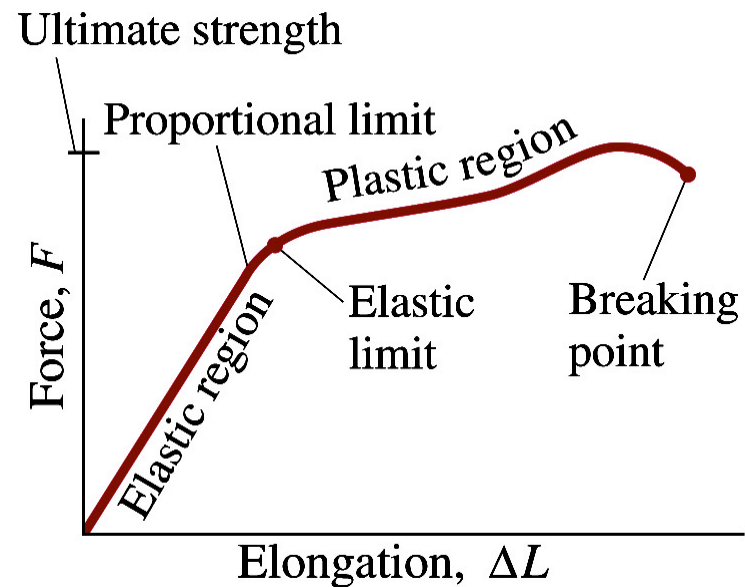
where

L_0 = original length

A = cross sectional area

E = Young's modulus

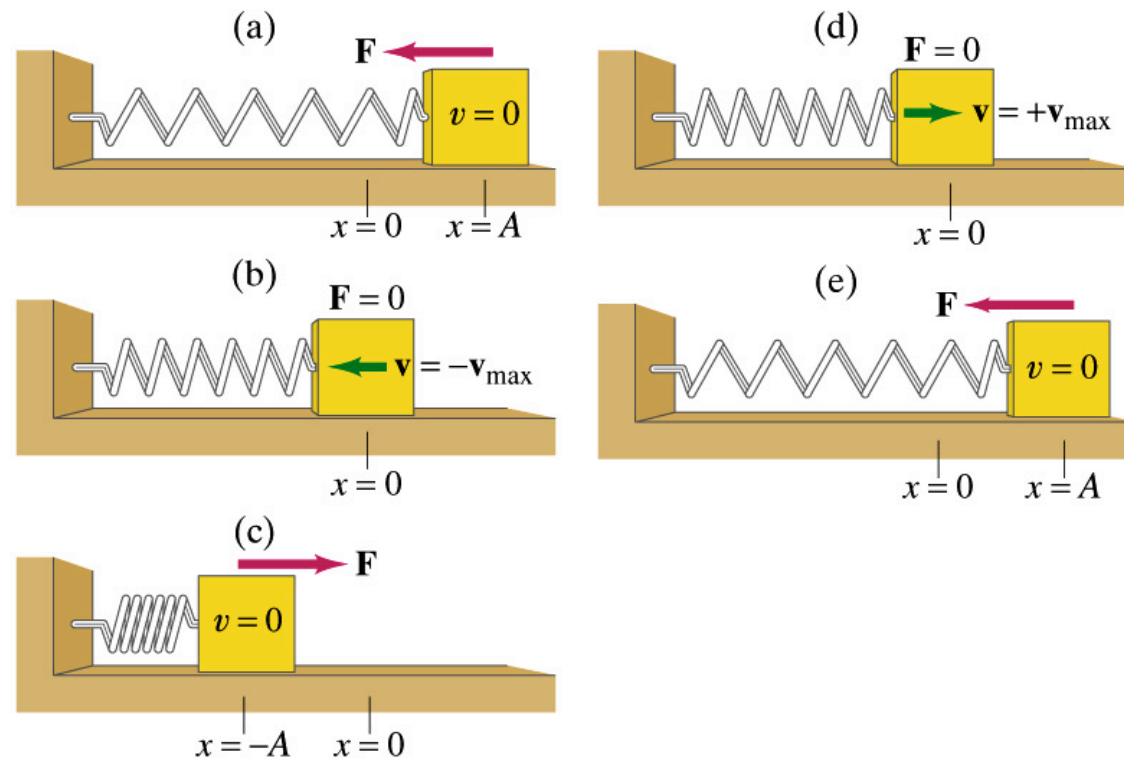
- Stress is defined as the force per unit area ($= F/A$).
- Strain is defined as the fractional change in length ($\Delta L_0/L_0$).



Note: the ratio of stress to strain is equal to the Young's Modulus.

Harmonic Motion.

Motion that repeats itself at regular intervals.



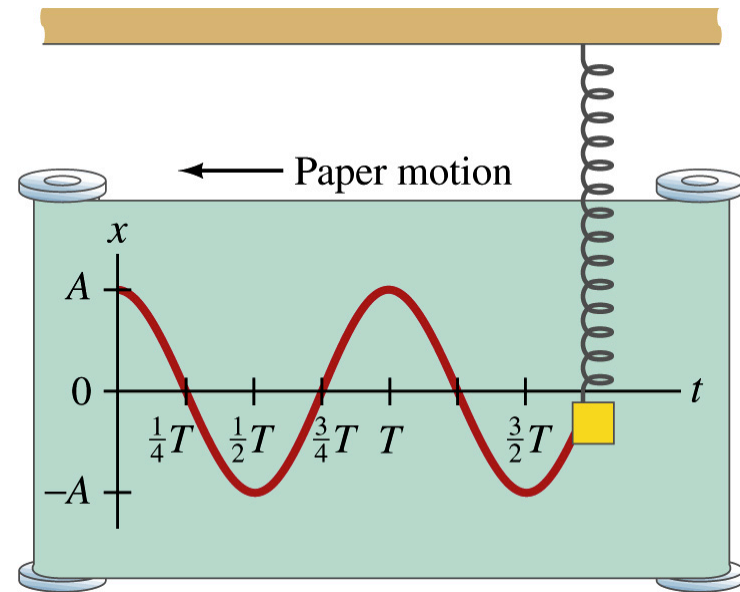
Simple Harmonic Motion.

Phase Constant

Amplitude

$$x(t) = x_m \cos(\omega t + \phi)$$

Angular Frequency



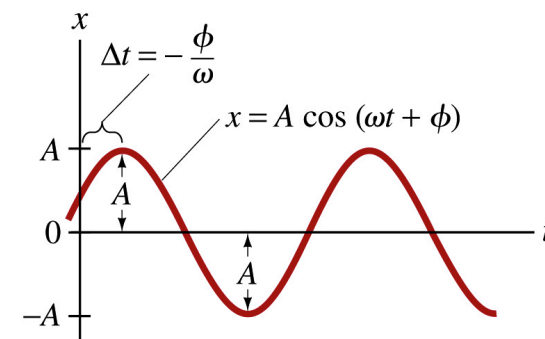
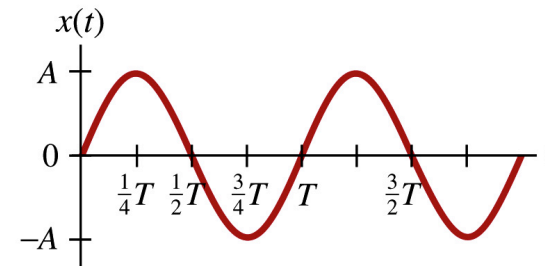
Simple Harmonic Motion.

- Other variables frequently used to describe simple harmonic motion:

- The period T : the time required to complete one oscillation. The period T is equal to $2\pi/\omega$.
- The frequency of the oscillation is the number of oscillations carried out per second:

$$\nu = 1/T$$

The unit of frequency is the Hertz (Hz). Per definition, $1 \text{ Hz} = 1 \text{ s}^{-1}$.



Simple Harmonic Motion.

What Forces are Required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2x$$

- The total mechanical energy of a system carrying out simple harmonic motion is constant.
- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m :

$$\omega = \sqrt{k/m}$$

Simple Harmonic Motion (SHM).

The Equation of Motion.

- All examples of SHM were derived from the following equation of motion:

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

- The most general solution to the equation is

$$x(t) = A \cos(\omega t + \alpha) + B \sin(\omega t + \beta)$$

- SHM will occur when $A = B$.

Damped Harmonic Motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b \frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

Damped Harmonic Motion.

- The general solution of this equation of motion is

$$x(t) = A \exp(i \omega t)$$

- There are two possible values of the angular velocity:

$$\omega = \frac{1}{2} \left(i \frac{b}{m} \pm \sqrt{4 \frac{k}{m} - \frac{b^2}{m^2}} \right) \approx \frac{1}{2} i \frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

- The solution to the equation of motion is thus given by

$$x(t) \approx x_m \exp\left(-\frac{b t}{2 m}\right) \exp\left(i t \sqrt{\frac{k}{m}}\right)$$

Damping Term

SHM Term

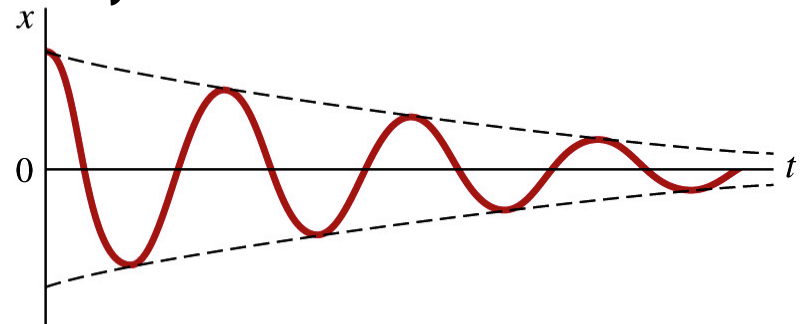
Damped Harmonic Motion.

- Let's examine the general solution in more detail:

$$x(t) \approx x_m \exp\left(-\frac{b t}{2 m}\right) \exp\left(i t \sqrt{\frac{k}{m}}\right)$$

- The general solution contains a SHM term, with an amplitude that decreases as function of time.
- The mechanical energy associated with the damped HM will decrease as function time:

$$E(t) = \frac{1}{2} k x_m^2 \exp\left(-\frac{b t}{m}\right)$$



Driven Harmonic Motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0 .
- Assume the external force $F(t) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + F_0 \sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

Driven Harmonic Motion.

- Consider the general solution

$$x(t) = A \cos(\omega t + \phi)$$

- The amplitude is equal to

$$A = \frac{F_0}{(\omega_0^2 - \omega^2)}$$

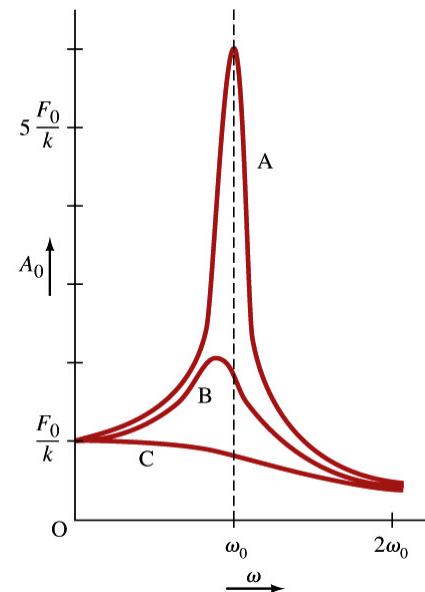
- The phase angle must satisfy the following relation:

$$\cos(\phi) = 0$$

This requires that $\phi = 90^\circ$ or 270° .

Driven Harmonic Motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.



Final Remarks

- The hardest part of each problem is recognizing the approach to take. Different approaches may lead to the same answer, but can differ greatly in difficulty.
- A suggestion:
 - Look at the end of chapter problems. There is only a limited number of types of question one can ask.
 - But Since the questions are grouped by section, you know already what approach to use based on the section to which the problems are assigned.
 - Some students benefit from copying the questions, cutting them out, writing the chapter/section numbers on the back, mixing them up, and then reading through them and determining what approach you would take if you would see that question on the exam (compare it with the focus of the section to which the problem was assigned).

Final Remarks

- You will only need your pen, a pencil, and an eraser. Being awake might also help!
- The TAs will have extra office hours on Monday. Please go and see them if you need to resolve any last-minute questions.
- The exam will start at 8 am and end at 9.30 am. If you show up late you will just have less time to finish. Over the years I have heard every excuse possible for being late, but I have never heard one that I accepted.

Good luck preparing for the exam.