Physics 121, April 8, 2008.
Harmonic Motion.

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Physics 121.
April 8, 2008.

- Course Information
- Topics to be discussed today:
  - Simple Harmonic Motion (Review).
  - Simple Harmonic Motion: Example Systems.
  - Damped Harmonic Motion
  - Driven Harmonic Motion

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Physics 121.
April 8, 2008.

- Homework set # 8 is due on Saturday morning, April 12, at 8.30 am.
- Homework set # 9 will be available on Saturday morning at 8.30 am, and will be due on Saturday morning, April 19, at 8.30 am.
- Requests for regarding part of Exam # 1 and # 2 need to be given to me by April 17. You need to write down what I should look at and give me your written request and your blue exam booklet(s).

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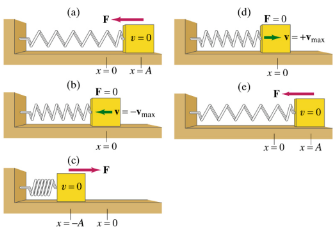
Harmonic motion (a quick review).
Motion that repeats itself at regular intervals.



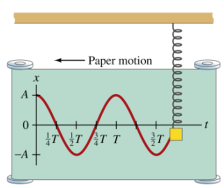
Simple Harmonic Motion (a quick review).

Phase Constant

Amplitude

$$x(t) = x_m \cos(\omega t + \phi)$$

Angular Frequency

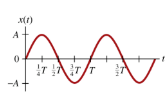


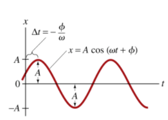
Simple Harmonic Motion (a quick review).

- Other variables frequently used to describe simple harmonic motion:
  - The period T: the time required to complete one oscillation. The period T is equal to $2\pi/\omega$.
  - The frequency of the oscillation is the number of oscillations carried out per second:

    $\nu = 1/T$

    The unit of frequency is the Hertz (Hz). Per definition, $1 \text{ Hz} = 1 \text{ s}^{-1}$.





Simple Harmonic Motion (a quick review). What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

  $F = ma = -m\omega^2 x$

- The total mechanical energy of a system carrying out simple harmonic motion is constant.

- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m:

  $\omega = \sqrt{(k/m)}$

Simple Harmonic Motion (SHM). The torsion pendulum.

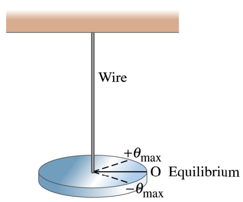
- What is the angular frequency of the SHM of a torsion pendulum:
  - When the base is rotated, it twists the wire and a the wire generated a torque which is proportional to the the angular twist:

    $\tau = -K\theta$

    The torque generates an angular acceleration α:

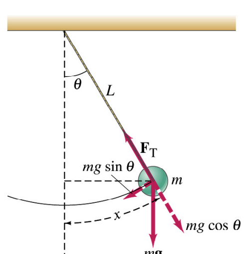
    $\alpha = d^2\theta/dt^2 = \tau/I = -(K/I)\ \theta$

    The resulting motion is harmonic motion with an angular frequency $\omega = \sqrt{(K/I)}$.



Simple Harmonic Motion (SHM). The simple pendulum.

- Calculate the angular frequency of the SHM of a simple pendulum.
  - A simple pendulum is a pendulum for which all the mass is located at a single point at the end of a massless string.
  - There are two forces acting on the mass: the tension T and the gravitational force mg.
  - The tension T cancels the radial component of the gravitational force.



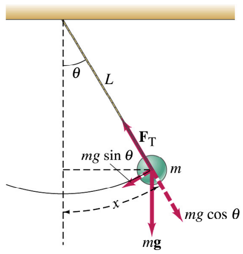
Simple Harmonic Motion (SHM). The simple pendulum.

- The net force acting on he mass is directed perpendicular to the string and is equal to

  $F = -\,mg \sin\theta$

  The minus sign indicates that the force is directed opposite to the angular displacement.
- When the angle θ is small, we can approximate $\sin\theta$ by θ:

  $F = -\,mg\theta = -\,mg\,x/L$
- Note: the force is again proportional to the displacement.



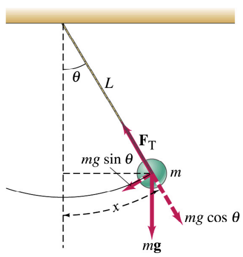
Simple Harmonic Motion (SHM). The Simple Pendulum.

- The equation of motion for the pendulum is thus

  $F = m\, d^2x/dt^2 = -(mg/L)\, x$

  or

  $d^2x/dt^2 = -\,(g/L)\, x$
- The equation of motion is the same as the equation of motion for a SHM, and the pendulum will thus carry out SHM with an angular frequency $\omega = \sqrt{(g/L)}$.
- The period of the pendulum is thus $2\pi/\omega = 2\pi\sqrt{(L/g)}$. Note: the period is independent of the mass of the pendulum.

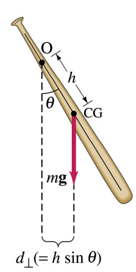


Simple Harmonic Motion (SHM). The physical pendulum.

- In a realistic pendulum, not all mass is located at a single point.
- The motion carried out by this realistic pendulum around its rotation point O can be determined by determining the total torque with respect to this point:

  $\tau = -mgh\sin\theta$
- If the angle θ is small, we can approximate the torque by

  $\tau = -mgh\theta$



Simple Harmonic Motion (SHM).
The physical pendulum.

- The angular acceleration α is related to the torque:

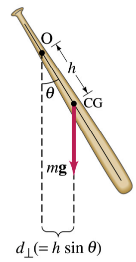
$$\tau = I\alpha$$

- The equation of motion for the angular acceleration α is given by

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{\tau}{I} = -\frac{mgh}{I}\theta$$

- This again is an equation for SHM with an angular frequency ω where

$$\omega^2 = \frac{mgh}{I}$$



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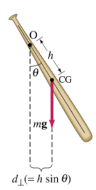
Simple Harmonic Motion (SHM).
The physical pendulum.

- The period of the physical pendulum is equal to

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgh}}$$

- We can double check our answer by requiring that the simple pendulum is a special case of the physical pendulum ($h = L$, $I = mL^2$):

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$



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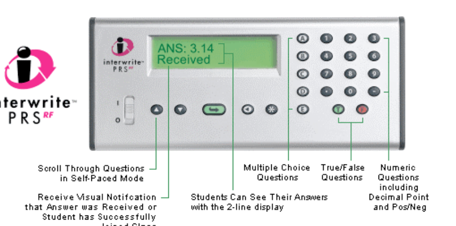
Physics 121.
Quiz lecture 21.

- The quiz today will have 3 questions!



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Simple Harmonic Motion (SHM).
The equation of motion.

- All examples of SHM were derived from he following equation of motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

- The most general solution to the equation is

$$x(t) = A\cos(\omega t + \alpha) + B\sin(\omega t + \beta)$$

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Simple Harmonic Motion (SHM).
The equation of motion.

- If $A = B$

$$\begin{aligned} x(t) &= A\cos(\omega t + \alpha) + B\sin(\omega t + \beta) = \\ &= A\left(\sin\left(\frac{1}{2}\pi - \omega t - \alpha\right) + \sin(\omega t + \beta)\right) = \\ &= 2A\sin\left(\frac{1}{4}\pi + \frac{\beta}{2} - \frac{\alpha}{2}\right)\cos\left(\frac{1}{4}\pi - \omega t - \frac{\beta}{2} - \frac{\alpha}{2}\right) \end{aligned}$$

which is SHM.

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Damped Harmonic Motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b\frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

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Damped Harmonic Motion.

- The general solution of this equation of motion is

$$x(t) = Ae^{i\omega t}$$

- If we substitute this solution in the equation of motion we find

$$-\omega^2 Ae^{i\omega t} + i\omega \frac{b}{m} Ae^{i\omega t} + \frac{k}{m} Ae^{i\omega t} = 0$$

- In order to satisfy the equation of motion, the angular frequency must satisfy the following condition:

$$\omega^2 - i\omega \frac{b}{m} - \frac{k}{m} = 0$$

Damped Harmonic Motion.

- We can solve this equation and determine the two possible values of the angular velocity:

$$\omega = \frac{1}{2}\left(i\frac{b}{m} \pm \sqrt{4\frac{k}{m} - \frac{b^2}{m^2}} \right) \simeq \frac{1}{2} i\frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

- The solution to the equation of motion is thus given by

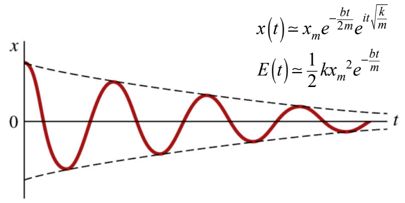
$$x(t) \simeq x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

Damping Term ↑ ↑ SHM Term

Damped Harmonic Motion.

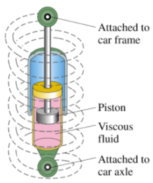
$$x(t) \simeq x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

$$E(t) \simeq \frac{1}{2} k x_m{}^2 e^{-\frac{bt}{m}}$$



The general solution contains a SHM term, with an amplitude that decreases as function of time

Damped Harmonic Motion has many practical applications.



Damping is not always a curse.

Driven Harmonic Motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0.
- Assume the external force $F(\text{t}) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0{}^2 x + F_0 \sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

Driven Harmonic Motion.

- Consider the general solution

$$x(t) = A\cos(\omega t + \phi)$$

- The parameters in this solution must be chosen such that the equation of motion is satisfied. This requires that

$$-\omega^2 A\cos(\omega t + \phi) + \omega_0{}^2 A\cos(\omega t + \phi) - F_0 \sin(\omega t) = 0$$

- This equation can be rewritten as

$$\left(\omega_0{}^2 - \omega^2\right) A\cos(\omega t)\cos(\phi) -$$
$$\left(\omega_0{}^2 - \omega^2\right) A\sin(\omega t)\sin(\phi) - F_0 \sin(\omega t) = 0$$

Driven Harmonic Motion.

- Our general solution must thus satisfy the following condition:

$$\left(\omega_0^2-\omega^2\right)A\cos\left(\omega t\right)\cos\left(\phi\right)-\left\{\left(\omega_0^2-\omega^2\right)A\sin\left(\phi\right)-F_0\right\}\sin\left(\omega t\right)=0$$

- Since this equation must be satisfied at all time, we must require that the coefficients of cos(ωt) and sin(ωt) are 0. This requires that

$$\left(\omega_0^2-\omega^2\right)A\cos\left(\phi\right)=0$$

and

$$\left(\omega_0^2-\omega^2\right)A\sin\left(\phi\right)-F_0=0$$

Driven Harmonic Motion.

- The interesting solutions are solutions where $A \neq 0$ and $\omega \neq \omega_0$. In this case, our general solution can only satisfy the equation of motion if

$$\cos\left(\phi\right)=0$$

and

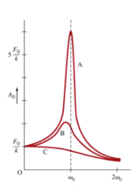
$$\left(\omega_0^2-\omega^2\right)A\sin\left(\phi\right)-F_0=\left(\omega_0^2-\omega^2\right)A-F_0=0$$

- The amplitude of the motion is thus equal to

$$A=\frac{F_0}{\left(\omega_0^2-\omega^2\right)}$$

Driven Harmonic Motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.



Driven Harmonic Motion.

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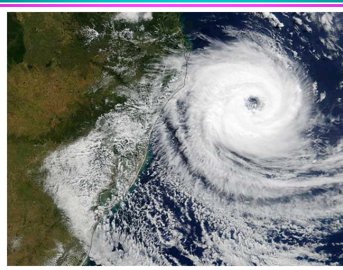
Done for today!
Thursday: Temperature and Heat!



Unusually Strong Cyclone Off the Brazilian Coast: A lot of Rotational Motion!
Credit: Jacques Descloitres, MODIS Land Rapid Response Team, GSFC, NASA

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