# Physics 121, April 3, 2008.
# Equilibrium and Simple Harmonic Motion.

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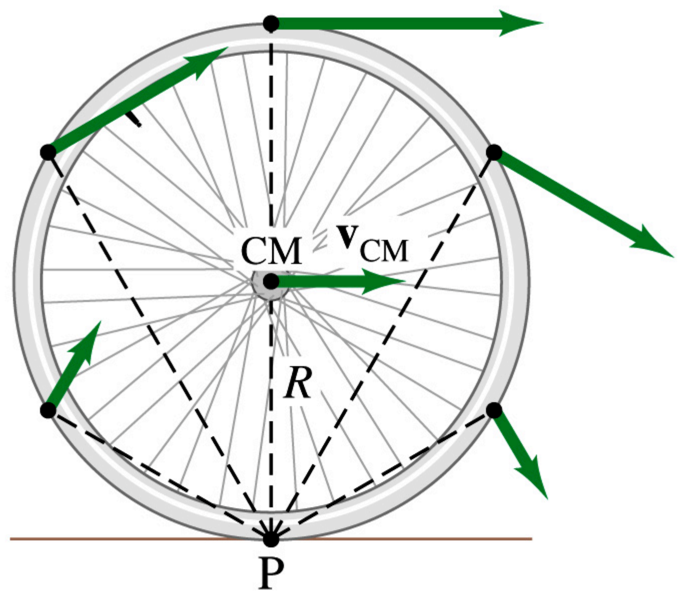
# Physics 121.
# April 3, 2008.

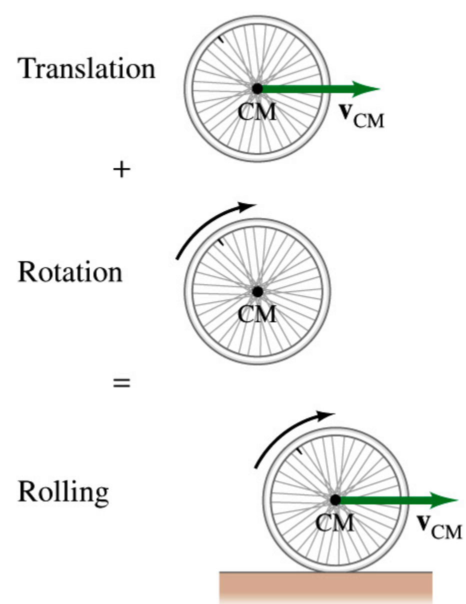
- Course Information
- Topics to be discussed today:
  - Requirements for Equilibrium (a brief review)
  - Stress and Strain
  - Introduction to Harmonic Motion

# Physics 121.
# April 3, 2008.

- Homework set # 7 is due on Saturday morning, April 5, at 8.30 am. This assignment has two components:
  - WeBWorK (75%)
  - Video analysis (25%): you can calculate the angular momentum quickly by using the expression for the vector product in terms of the components of the individual vectors (the x and y components of the position and velocity of the puck).
- Homework set # 8 is now available. This assignment contains only WeBWorK questions and will be due on Saturday morning, April 12, at 8.30 am.
- Exam # 3 will take place on Tuesday April 22.

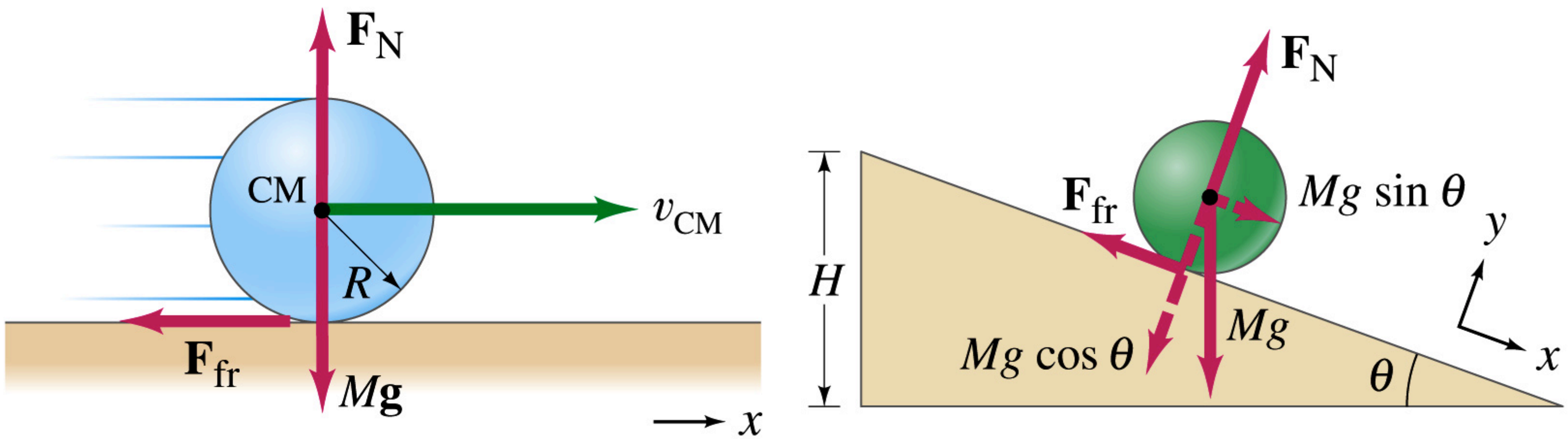
# Comments on Homework # 7.
# Rolling motion causes much confusion!




Two views of rolling motion: 1) Pure rotation around the instantaneous axis or 2) rotation and translation.

# Comments on Homework # 7.
# Rolling motion causes much confusion!



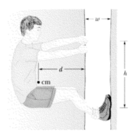

Note: friction provides the torque with respect to the center-of-mass.

# Homework # 8.
# Due: Saturday April 12, 2008.

**Frank Wolfs** — **Physics 121, Spring 2008**
**Homework Set 08** — **Due date: 04/12/2008 at 08:30am EDT**

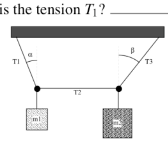
This assignment will be counted toward your final grade. You can attempt each problem 50 times; once you exceed this number of attempts, your solutions will not be recorded anymore. You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer. Note: to use scientific notion, use a notion like xxE+yy. It is important that you use a capital E; answers with a lower case e will be evaluated differently

**1.** (20 pts) library/type24/prob04.pg
A 66-$kg$ rock climber is in a lie-back climb along a fissure, with hands pulling on one side of the fissure and feet pressed against the opposite side. The fissure has a width $w$ = 0.16 $m$, and the center of mass of the climber is a horizontal distance $d$ = 0.26 $m$ from the fissure. The coefficient of static friction between the hands and the rock is 0.10. The coefficient of static friction between the boots and the rock is 1.31. What is the least horizontal pull by the hands that keeps the climber stable? __________



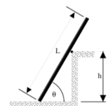
If the climber pulls with the minimum force (which was calculated in the previous step), what must be the vertical distance $h$ between his hands and his feet? __________

**2.** (20 pts) library/type24/prob03.pg
The system shown in the Figure is in equilibrium. The mass of block 1 is 1.2 $kg$ and the mass of block 2 is 6.4 $kg$. String 1 makes an angle $\alpha = 27°$ with the vertical and string 2 is exactly horizontal. What is the tension $T_1$? __________



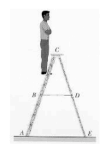

What is the tension $T_2$? __________
Find the angle β between string 3 and the vertical. __________
What is the tension $T_3$? __________

**3.** (20 pts) library/type24/prob09.pg
A plank, of length $L$ = 4.9 $m$ and mass $M$ = 9.0 $kg$, rests on the ground and on a frictionless roller at the top of a wall of height $h$ = 1.85 $m$ (see Figure). The center of gravity of the plank is at its center. The plank remains in equilibrium for any value of θ ¿ 75° but slips if θ ¡ 75°. Calculate the magnitude of the force exerted by the roller on the plank when θ = 75°. __________



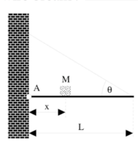

Calculate the magnitude of the normal force exerted by ground on the plank when θ = 75°. __________
Calculate the magnitude of the friction force between the ground and the plank when θ = 75°. __________

**4.** (20 pts) library/type24/prob05.pg
For the step ladder shown in the Figure, sides $AC$ and $CE$ are each 6.4 $ft$ long and hinged at $C$. $BD$ is a 3.3-$ft$ long tie rod, installed halfway up the ladder. A 54-$kg$ man climbs 3.7 $ft$ along the ladder. Assuming that the floor is frictionless and neglecting the weight of the ladder, find the tension in the tie rod. __________



What is the force exerted by the ladder on the floor at $A$? __________
What is the force exerted by the ladder on the floor at $E$? __________

**5.** (20 pts) library/type24/prob10.pg
A uniform horizontal bar of length $L$ = 5 $m$ and weight 284 $N$ is pinned to a vertical wall and supported by a thin wire that makes an angle of θ = 40° with the horizontal. A mass $M$, with a weight of 400 $N$, can be moved anywhere along the bar. The wire can withstand a maximum tension of 531 $N$. What is the maximum possible distance from the wall at which mass $M$ can be placed before the wire breaks? __________

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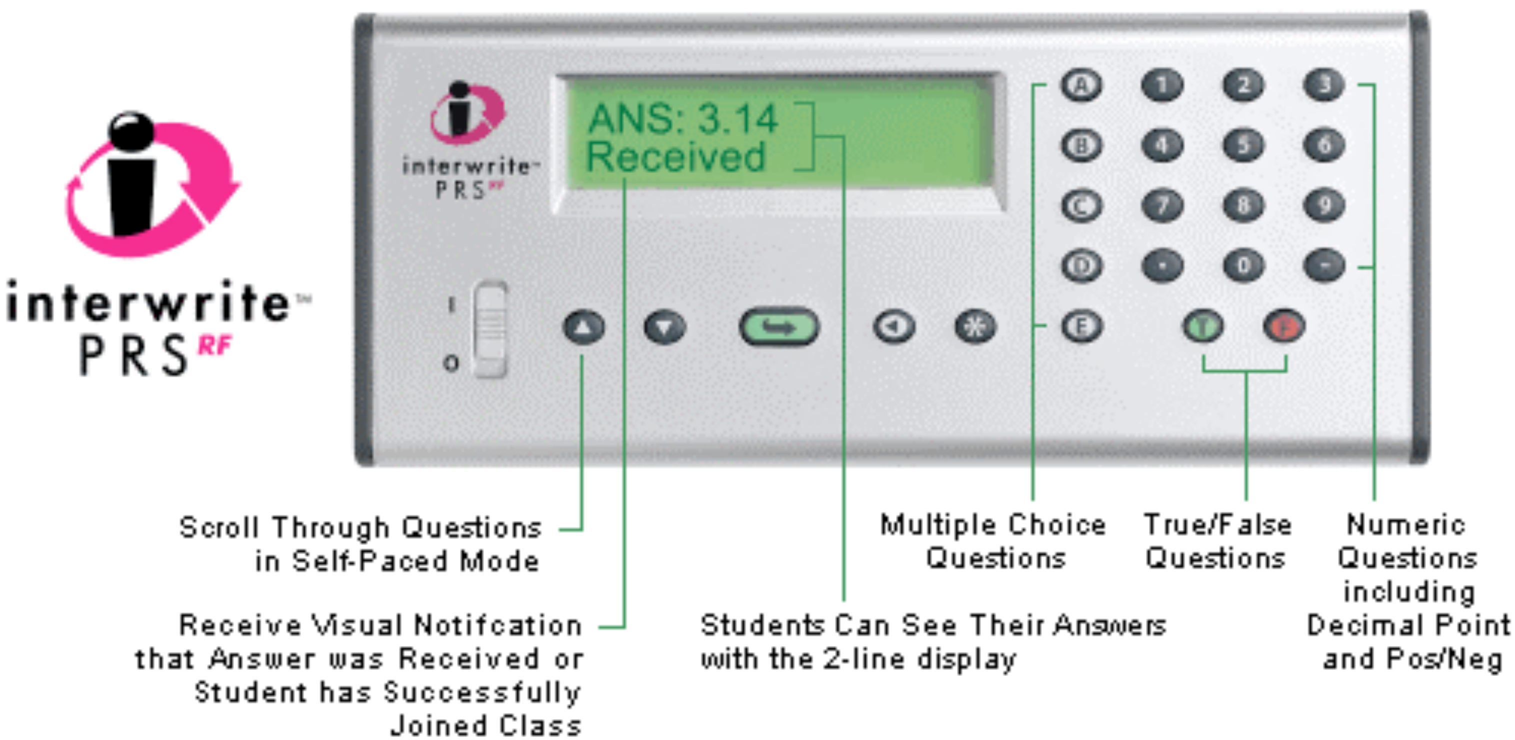
All problems in this assignment are related to equilibrium.

In all cases you need to identify all forces and torques that act on the system.

Remember to choose the reference point in a smart way!

# Physics 121.
# Quiz lecture 20.

- The quiz today will have 3 questions!



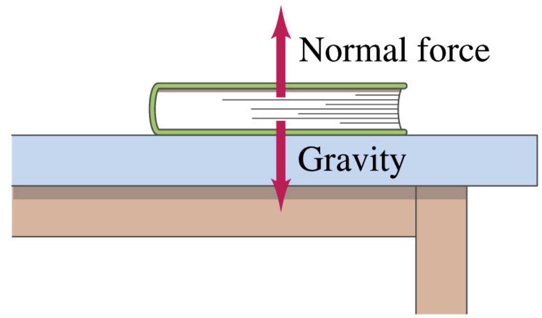
# Equilibrium (a quick review).

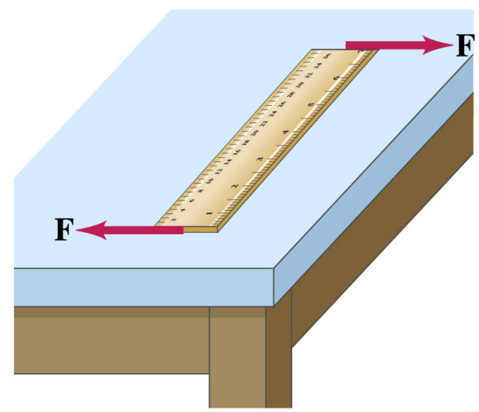
- An object is in equilibrium is the following conditions are met:

  Net force = 0 N (**first** condition for equilibrium) . This implies p = constant.

  and

  Net torque = 0 Nm (**second** condition for equilibrium). This implies L = constant.




- Conditions for **static** equilibrium:
  - p = 0 kg m/s
  - L = 0 kg $m^2$/s

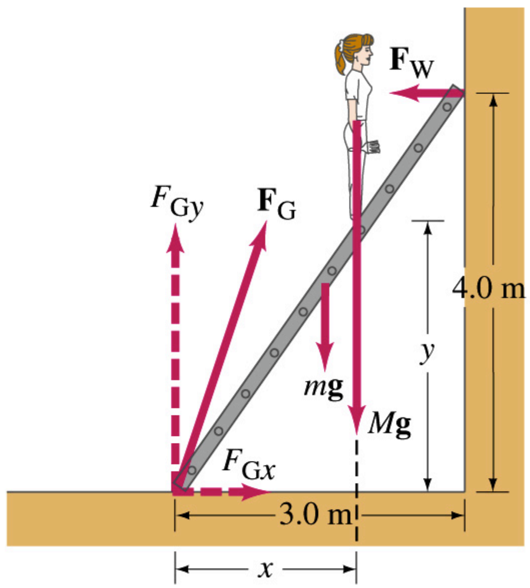
# Equilibrium.
# Summary of conditions (a quick review).

- Equilibrium in 3D:

$$\sum F_x = 0 \qquad \sum \tau_x = 0$$
$$\sum F_y = 0 \quad \text{and} \quad \sum \tau_y = 0$$
$$\sum F_z = 0 \qquad \sum \tau_z = 0$$

- Equilibrium in 2D:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau_z = 0$$



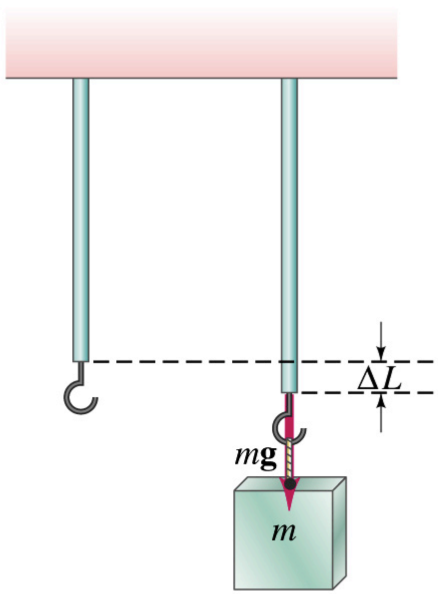

Torque condition must be satisfied with respect to any reference point.

# Stress and strain.
# The effect of applied forces.

- When we apply a force to an object that is kept fixed at one end, its dimensions can change.
- If the force is below a maximum value, the change in dimension is proportional to the applied force. This is called Hooke's law:

$$F = k\,\Delta L$$

- This force region is called the **elastic region**.

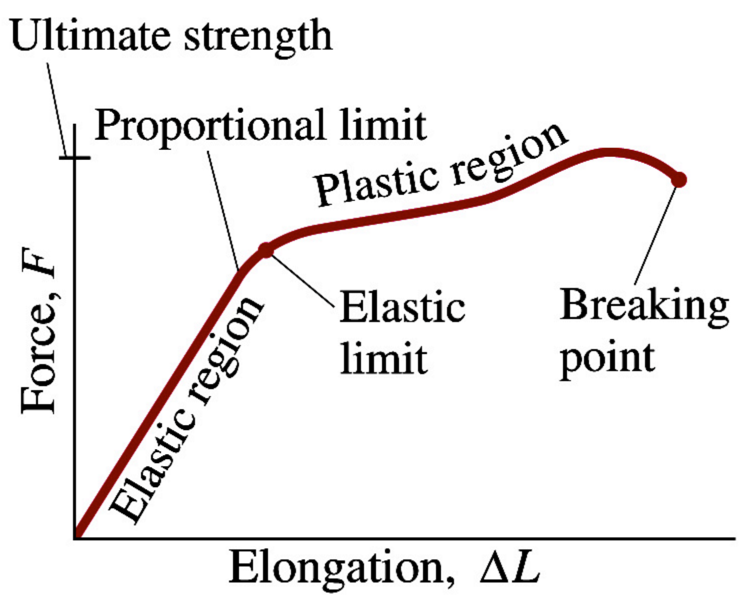


# Stress and strain.
# The effect of applied forces.

- When the applied force increases beyond the elastic limit, the material enters the **plastic region**.

- The elongation of the material depends not only on the applied force *F*, but also on the type of material, its length, and its cross-sectional area.

- In the plastic region, the material does not return to its original shape (length) when the applied force is removed.



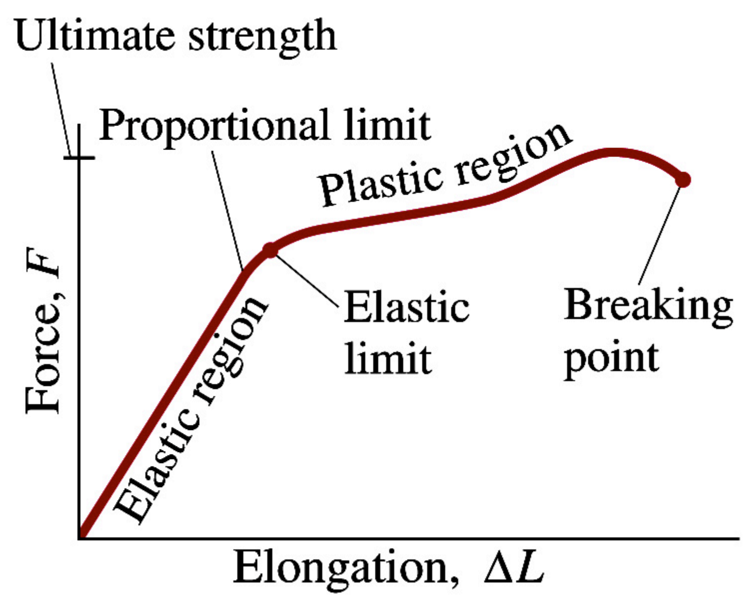
# Stress and strain.
# The effect of applied forces.

- The elongation $\Delta L$ can be specified as follows:

$$\Delta L = \frac{1}{E}\frac{F}{A}L_0$$

where

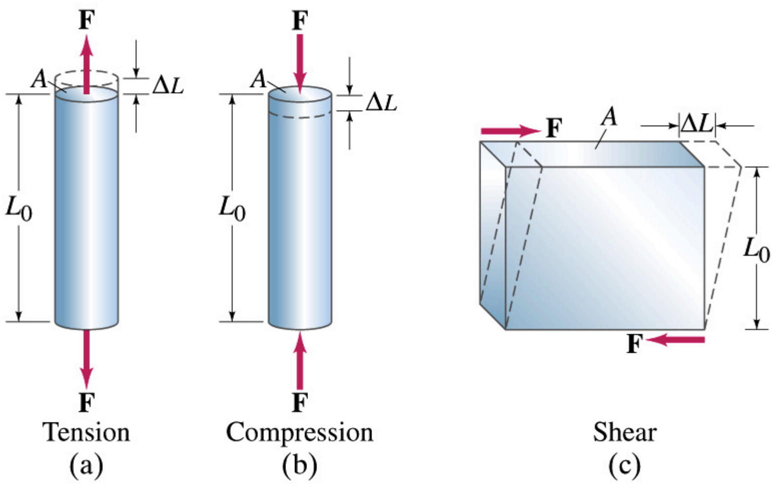
$L_0$ = original length
$A$ = cross sectional area
$E$ = Young's modulus

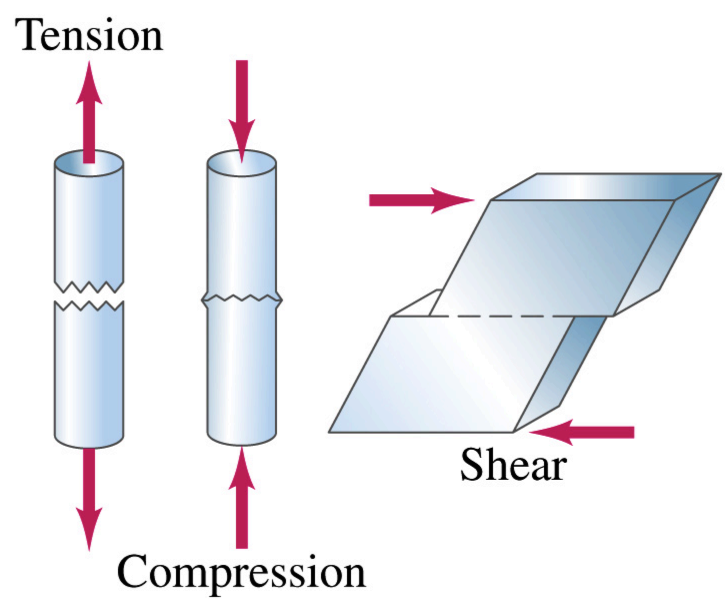



- **Stress** is defined as the force per unit area (= $F/A$).
- **Strain** is defined as the fractional change in length ($\Delta L_0/L_0$).

**Note: the ratio of stress to strain is equal to the Young's Modulus.**

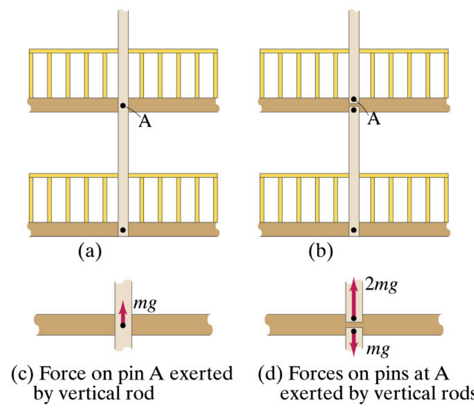
# Stress and strain.
# Direction matters.



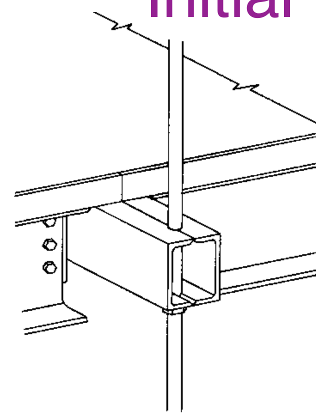



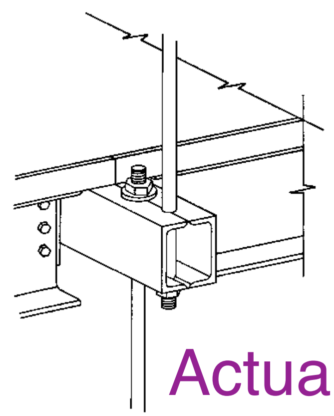
# Stress and strain. A simple calculation could have prevented the death of 114 people.

# Stress and strain. A simple calculation could have prevented the death of 114 people.




Initial Design





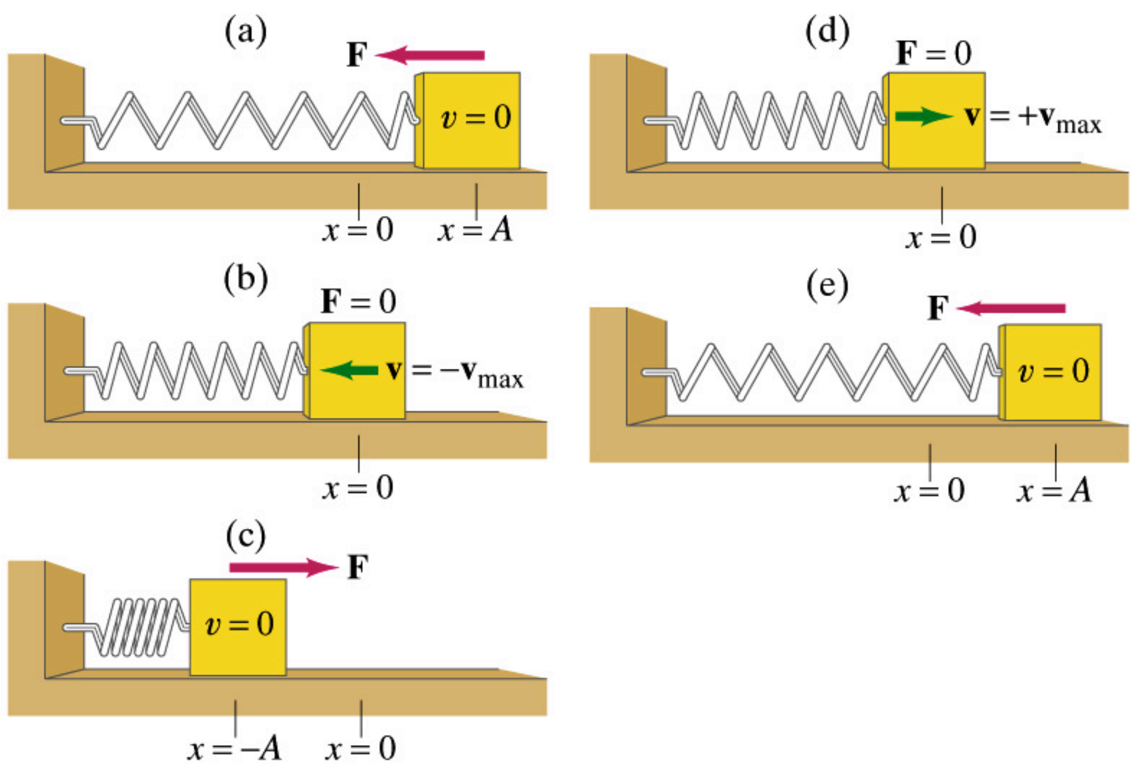
Actual Design

Credit: http://www.glendale-h.schools.nsw.edu.au/faculty_pages/ind_arts_web/bridgeweb/Hyatt_page.htm

# And now something completely different! Harmonic motion.

- We will continue our discussion of mechanics with the discussion of harmonic motion (simple and complex). This material is covered in Chapter 14 of our text book.
- Chapter 14 will be the last Chapter included in the material covered on Exam # 3 (which will cover Chapters 10, 11, 12, and 14).
- Note: We will not discuss the material discussed in Chapter 13 of the book, dealing with fluids, and this material will not be covered on our exams.

# Harmonic motion.
# Motion that repeats itself at regular intervals.

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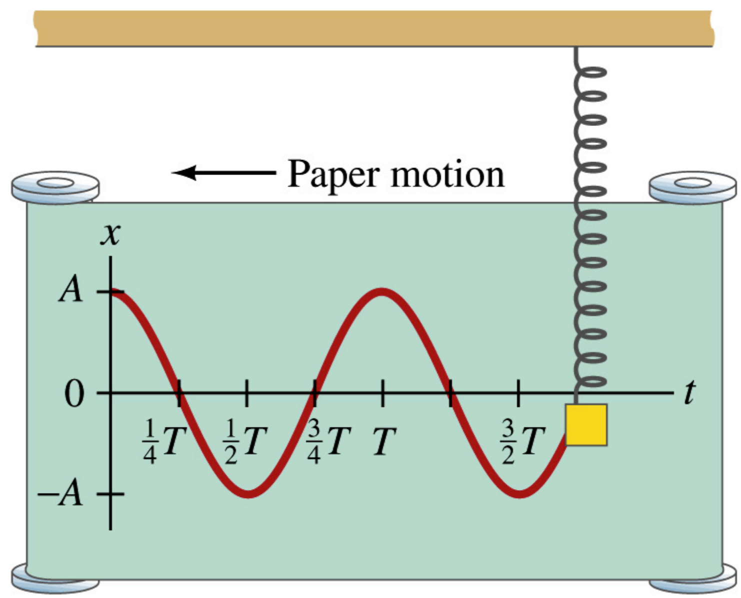
# Simple harmonic motion.

Phase Constant

Amplitude

$$x(t) = A\cos(\omega t + \phi)$$

Angular Frequency



# Simple harmonic motion.

- Instead of the angular frequency $\omega$ the motion can also be described in terms of its period $T$ or its frequency $\nu$.
- The period $T$ is the time required to complete one oscillation:

$$x(t) = x(t + T)$$

or

$$A\cos(\omega t + \phi) = A\cos(\omega t + \omega T + \phi)$$

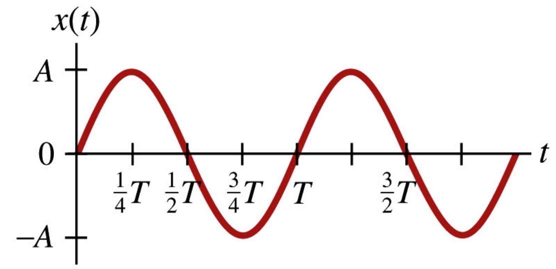
- In order for this to be true we must require $\omega T = 2\pi$. The period $T$ is thus equal to $2\pi/\omega$.
- The frequency $\nu$ is the number of oscillations carried out per second ($\nu = 1/T$). The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 $s^{-1}$.

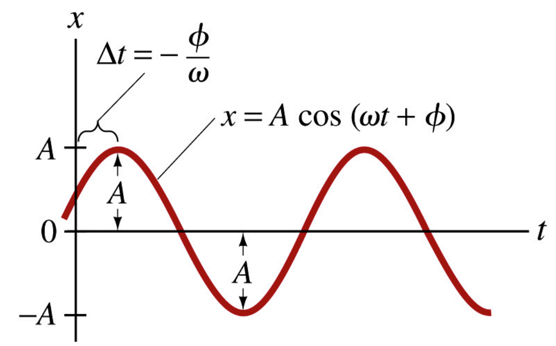
# Simple harmonic motion.

- The frequency of the oscillation is the number of oscillations carried out per second:

$$\nu = 1/T$$

- The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 $s^{-1}$.



# Simple harmonic motion.
# What forces are required?

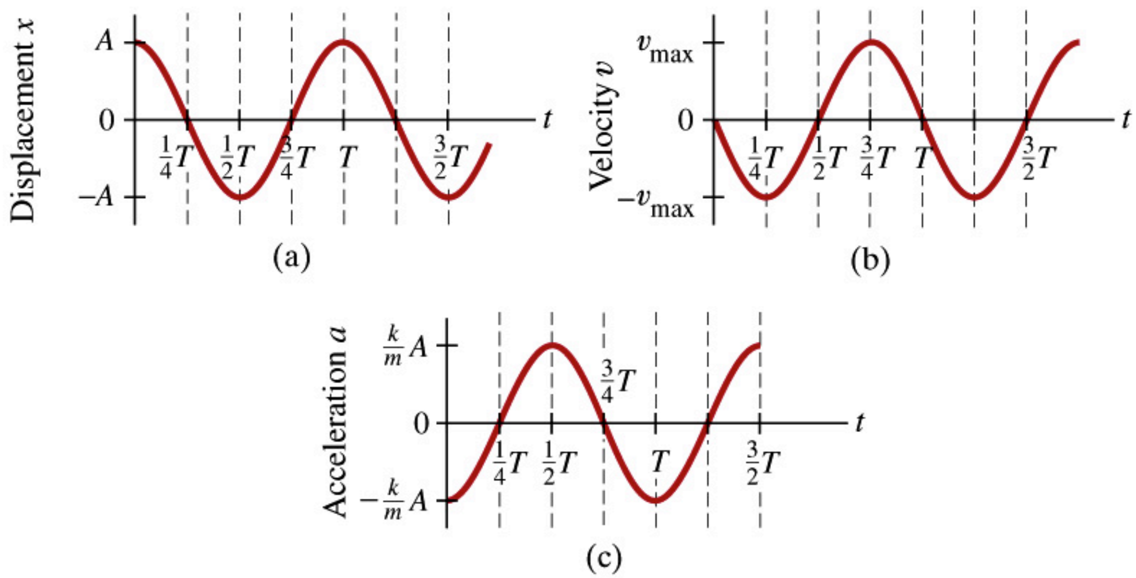
- Consider we observe simple harmonic motion. The observation of the motion can be used to determine the nature of the force that generates this type of motion. In order to do this, we need to determine the acceleration of the object carrying out the harmonic motion:

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(A\cos(\omega t + \phi)\right) = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(-\omega A\sin(\omega t + \phi)\right) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$$

# Simple harmonic motion.
# What forces are required?




Note: maxima in displacement correlate with minima in acceleration.

# Simple harmonic motion. What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant $k$ and the mass $m$:

$$\omega = \sqrt{(k/m)}$$

# Simple harmonic motion. What forces are required?

- We conclude:

  ***Simple harmonic motion is the motion executed by a particle of mass m, subject to a force F that is proportional to the displacement of the particle, but opposite in sign.***

- Any force that satisfies this criterion **can** produce simple harmonic motion. If more than one force is present, you need to examine the net force, and make sure that the net force is proportional to the displacements, but opposite in sign.

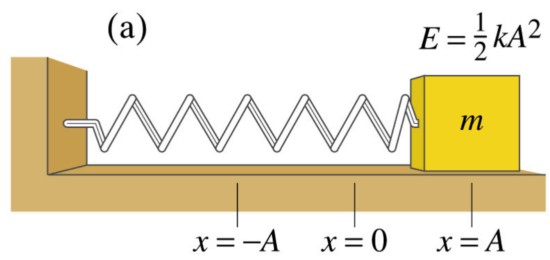
# Simple harmonic motion. Total energy.

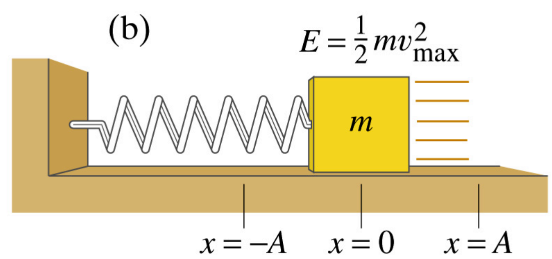
- During the motion of a block on a spring, there is a continuous conversion of energy:

  - **The potential energy:**
    $U(t) = (1/2)kx_m{}^2 \cos^2(\omega t+\phi)$

  - **The kinetic energy**:
    $K(t) = (1/2)m(-x_m\omega)^2 \sin^2(\omega t+\phi)$
    or
    $K(t) = (1/2)\ kx_m{}^2 \sin^2(\omega t+\phi)$

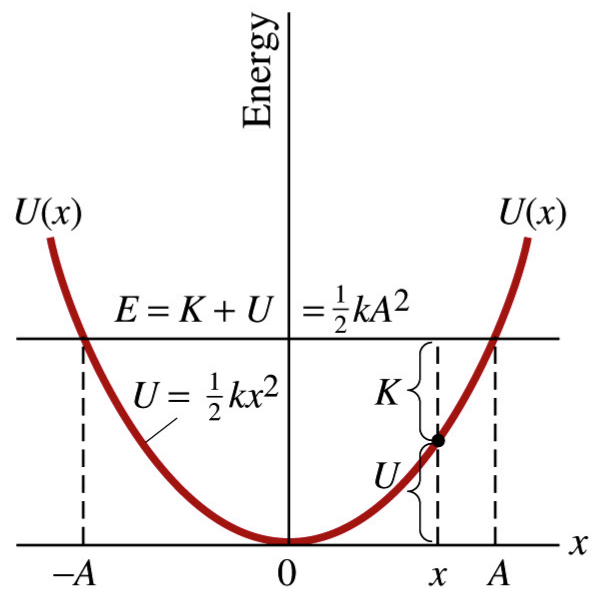
  - **The mechanical energy**:
    $E(t) = U(t) + K(t) = (1/2)kx_m$



# Simple harmonic motion.
# Total energy.

- Since the mechanical energy is independent of time, we conclude that the mechanical energy of the system is constant!
- If we know the total mechanical energy of the system, we can determine the region of oscillation. This region is constraint by the fact that the kinetic energy is always positive, and the potential energy is thus constraint by the mechanical energy ($U \leq E$).

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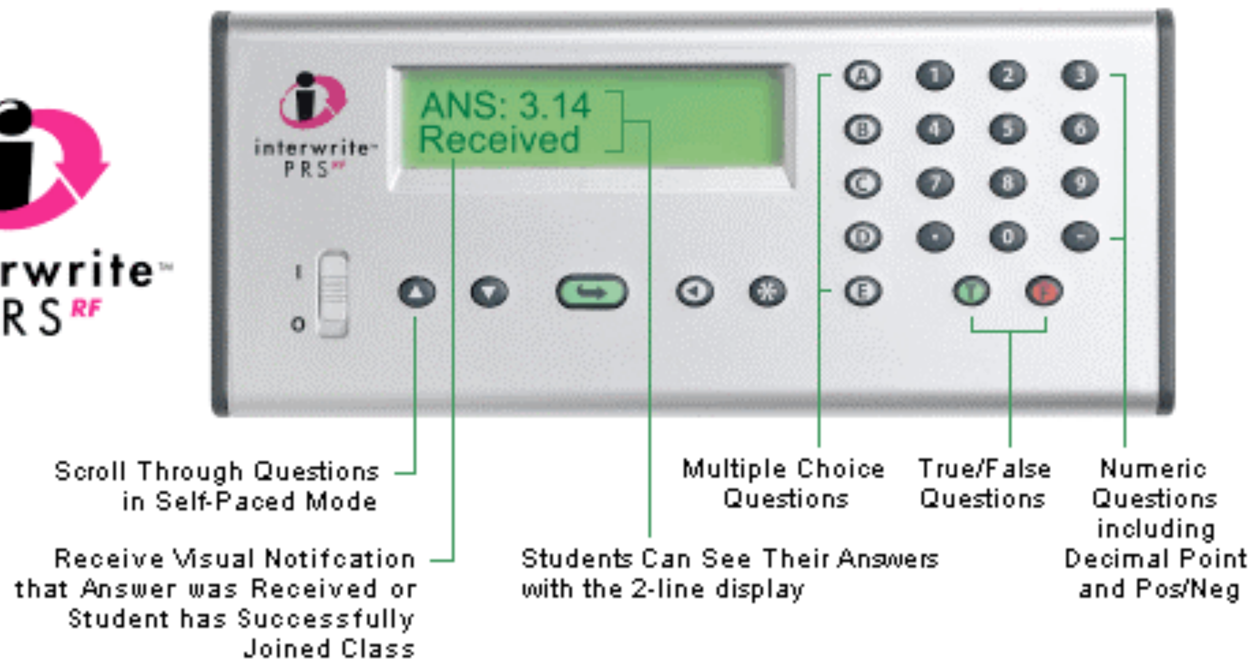
# Harmonic motion.

- Let's test our understanding of the basic aspects of harmonic motion by working on the following concept problems:
  - Q20.1
  - Q20.2
  - Q20.3



# Done for today!
# Next week: more harmonic motion!

**Mir Dreams** Credit: STS-76 Crew, NASA