# Physics 121, March 25, 2008.
# Rotational Motion and Angular Momentum.

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# Physics 121.
# March 25, 2008.

- Course Information
- Topics to be discussed today:
  - Review of Rotational Motion
  - Rolling Motion
  - Angular Momentum
  - Conservation of Angular Momentum

# Physics 121.
# March 25, 2008.

- Homework set # 7 is now available and is due on Saturday April 5 at 8.30 am.

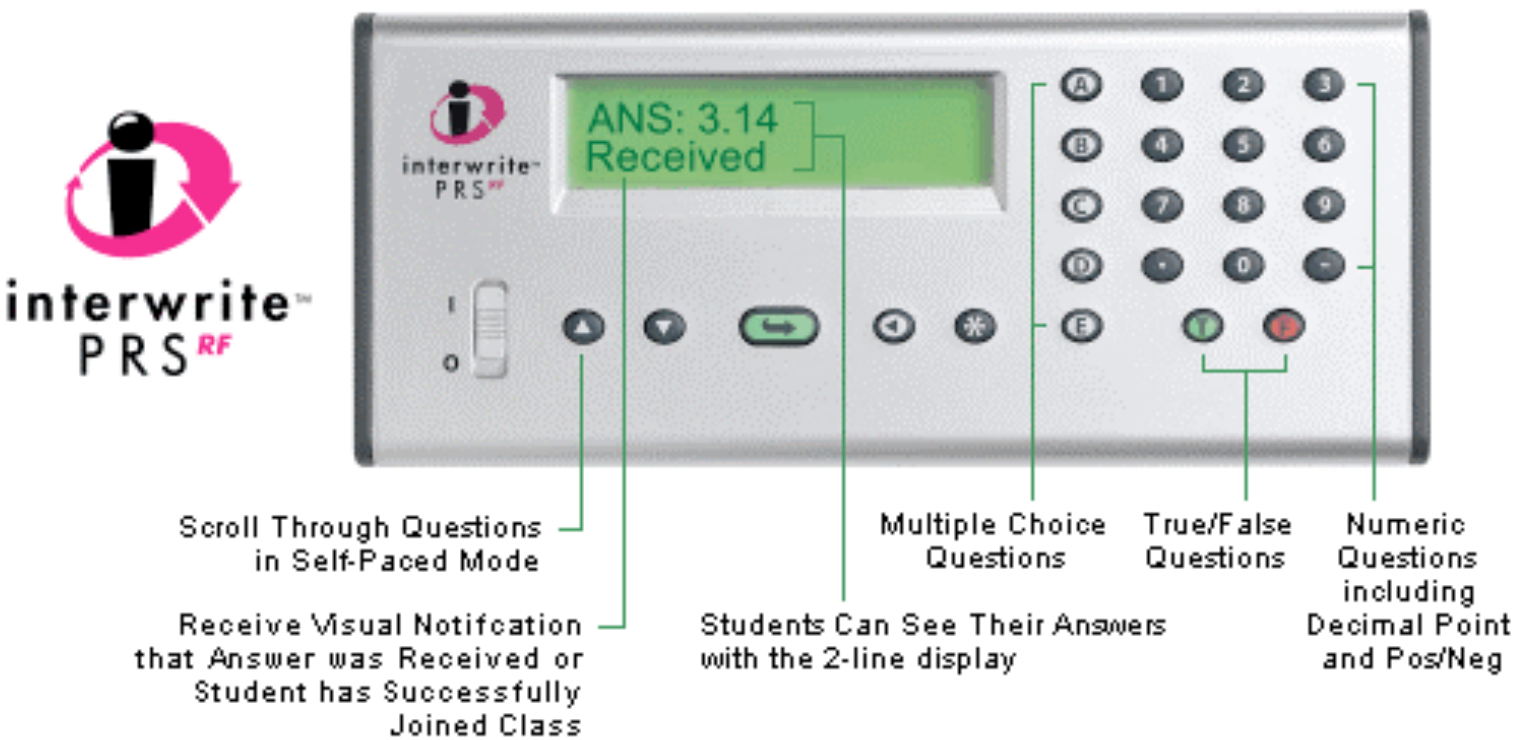
- There will be no workshops and office hours for the rest of the week. We will be busy grading exam # 2.

- The grades for exam # 2 will be distributed via email on Monday March 31.

- You should pick up your exam in workshop next week. Please check it carefully for any errors.

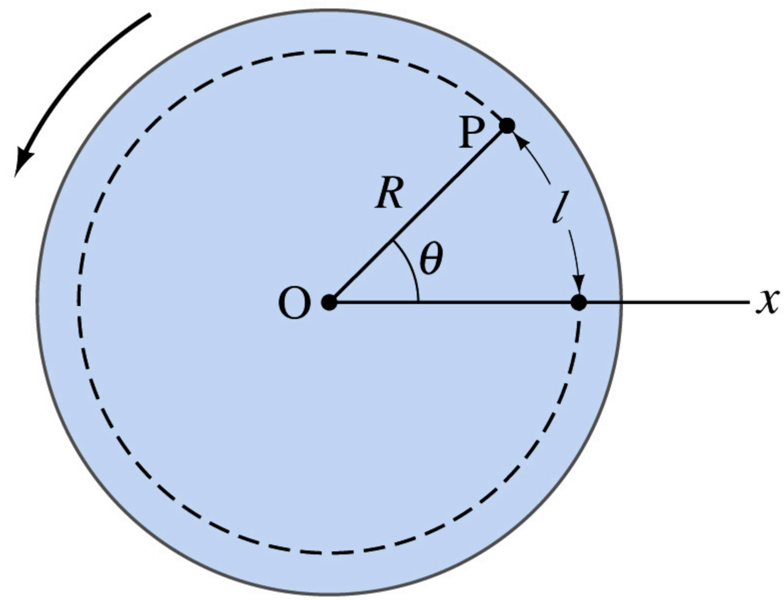
# Physics 121.
# Quiz lecture 17.

- The quiz today will have 4 questions!



# Rotational variables.
# A quick review.

- The variables that are used to describe rotational motion are:
  - Angular position $\theta$
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



# Rotational Kinetic Energy. A quick review.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i \left(\omega r_i\right)^2 = \frac{1}{2}\left\{\sum_i m_i r_i^2\right\}\omega^2 = \frac{1}{2} I \omega^2$$

- The kinetic energy is proportional to square of the rotational velocity ω. Note: the equation is similar to the translational kinetic energy (1/2 $mv^2$) except that instead of being proportional to the the mass m of the object, the rotational kinetic energy is proportional to the **moment of inertia I** of the object:

$$I = \sum_i m_i r_i^2 \quad \text{or} \quad I = \int r^2 dm$$

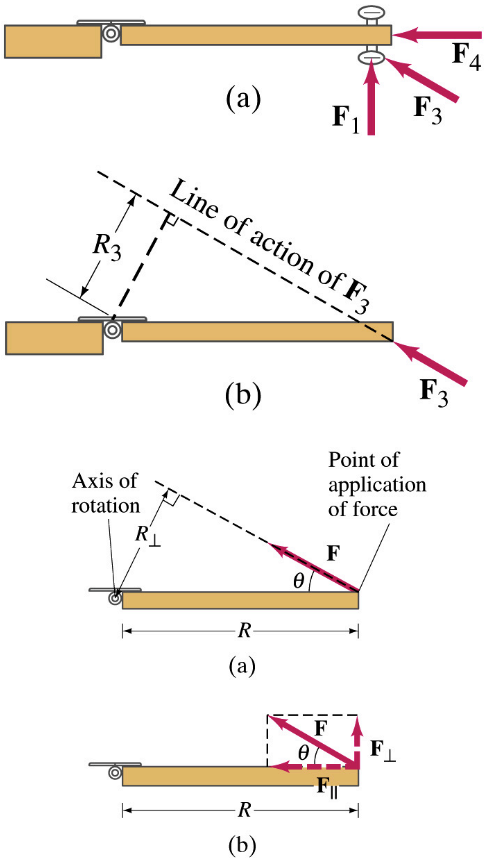
**Note: units of *I*: kg m$^2$**

# Torque.

- In general the torque associated with a force $F$ is equal to

$$\vec{\tau} = rF\sin\phi = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

- The arm of the force (also called the moment arm) is defined as $r\sin\phi$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle $\phi$ is 90°.



# Torque.
# A quick review.

- The torque $\tau$ of the force $F$ is related to the angular acceleration $\alpha$:

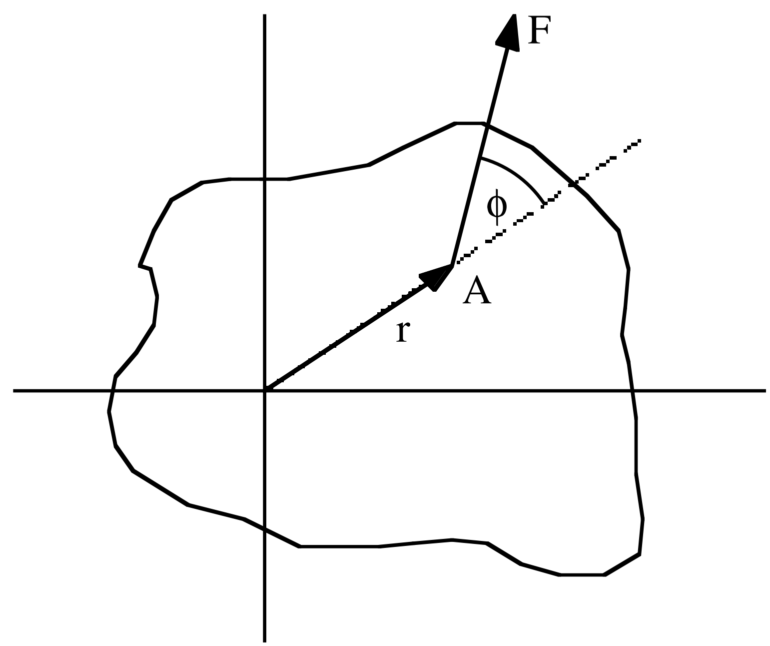
$$\tau = I\alpha$$

- This equation looks similar to Newton's second law for linear motion:

$$F = ma$$

- Note:

| linear | rotational |
| --- | --- |
| mass $m$ | moment $I$ |
| force $F$ | torque $\tau$ |

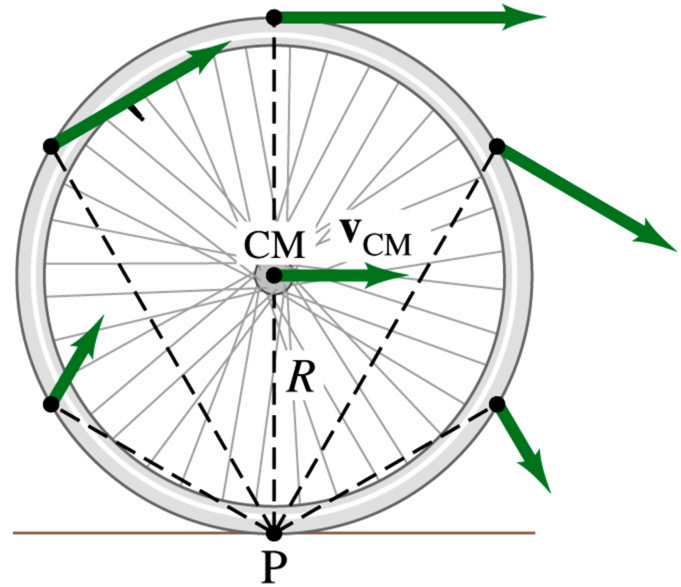


# Rolling motion.
# A quick review.

- Rolling motion is a combination of translational and rotational motion.

- The kinetic energy of rolling motion has thus two contributions:
    - Translational kinetic energy = (1/2) $M v_{cm}^2$.
    - Rotational kinetic energy = (1/2) $I_{cm} \omega^2$.

- Assuming the wheel does not slip: $\omega = v / R$.



# How different is a world with rotational motion? Sample problem.

- Consider the loop-to-loop. What height $h$ is required to make it to the top of the loop?
- First consider the case without rotation:
  - Initial mechanical energy = $mgh$.
  - Minimum velocity at the top of the loop is determine by requiring that

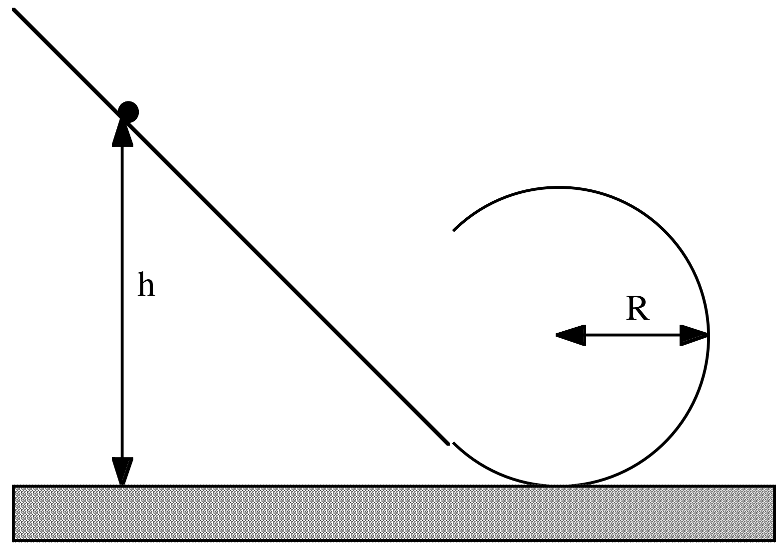
    $$mv^2/R > mg$$

    or

    $$v^2 > gR$$
  - The mechanical energy is thus equal to

    $$(1/2)mv^2 + 2mgR > (5/2)mgR$$
  - Conservation of energy requires

    $$h > (5/2)R$$

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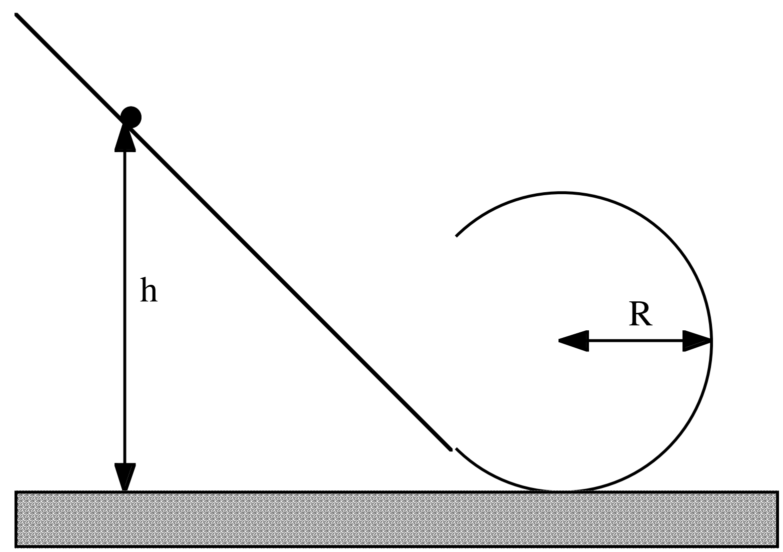
# How different is a world with rotational motion? Sample problem.

- What changes when the object rotates?
  - The minimum velocity at the top of the loop will not change.
  - The minimum translational kinetic energy at the top of the loop will not change.
  - But in addition to translational kinetic energy, there is now also rotational kinetic energy.
  - The minimum mechanical energy is at the top of the loop has thus increased.
  - The required minimum height must thus have increased.
- OK, let's now calculate by how much the minimum height has increased.



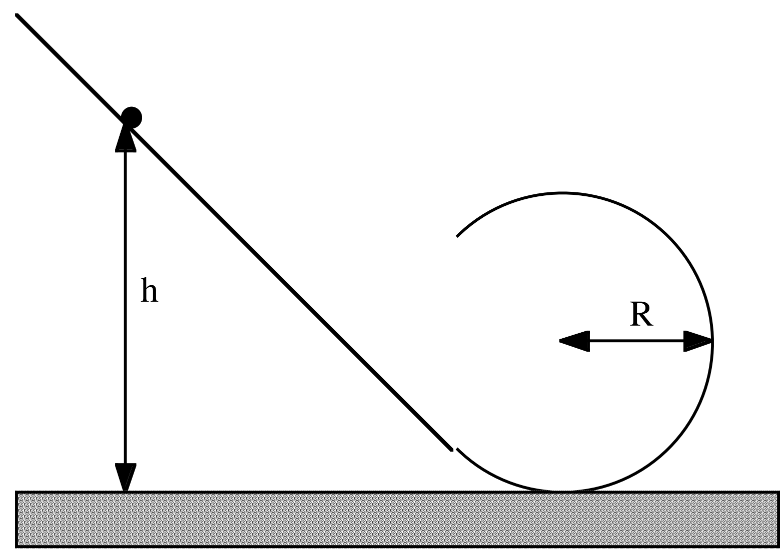
# How different is a world with rotational motion? Sample problem.

- The total kinetic energy at the top of the loop is equal to

$$K_f = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2 = \frac{1}{2}\left(\frac{I}{r^2} + M\right)v^2$$

- This expression can be rewritten as

$$K_f = \frac{1}{2}\left(\frac{2}{5}M + M\right)v^2 = \frac{7}{10}Mv^2$$




- We now know the minimum mechanical energy required to reach this point and thus the minimum height:

$$h \geq \frac{27}{10}R$$

**Note: without rotation $h \geq 25/10\, R$ !!!**

# Torque and rotational motion.

- Let’s test our understanding of the basic aspects of torque and rotational motion by working on the following concept problems:
  - Q17.1
  - Q17.2
  - Q17.3
  - Q17.4
  - Q17.5

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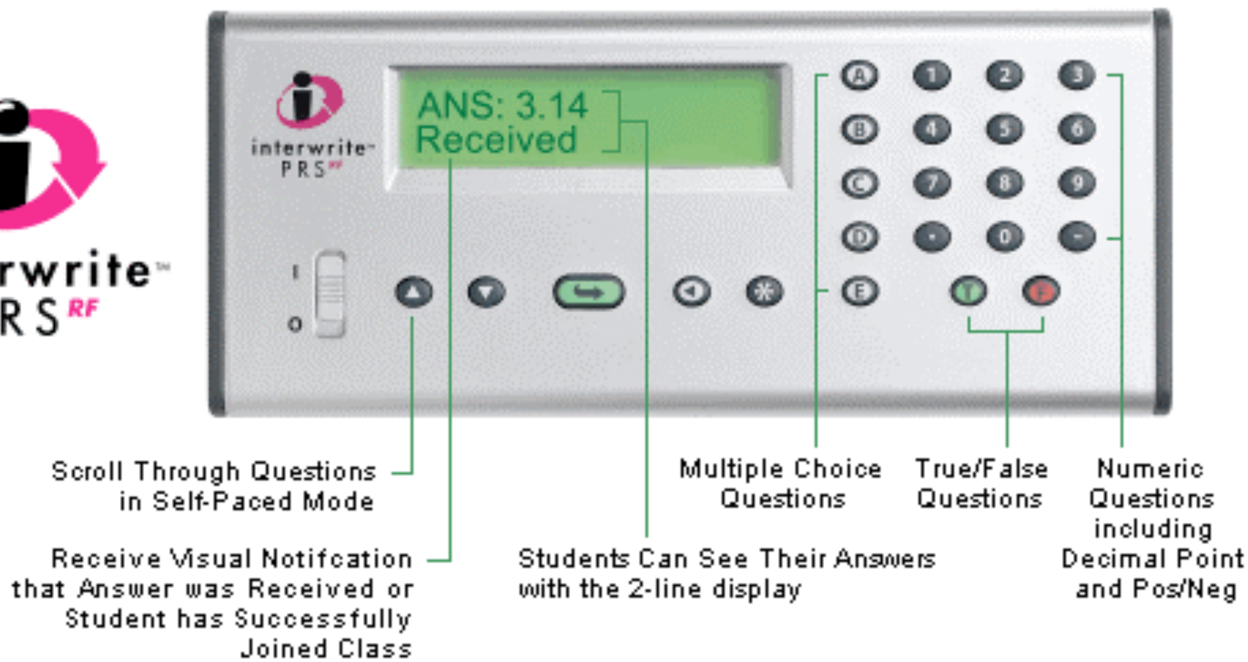
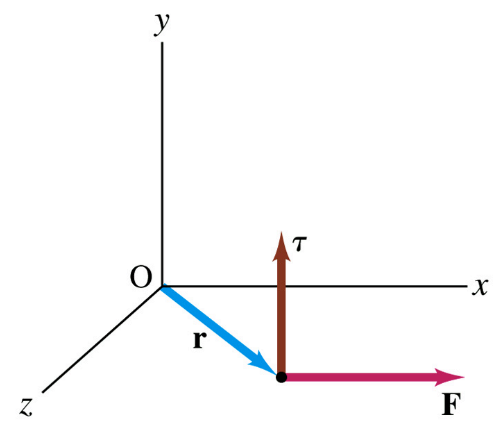
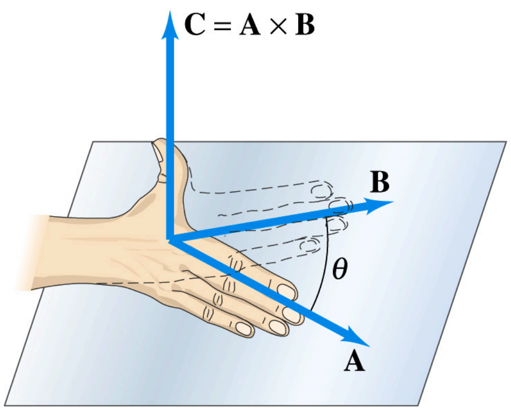


# Torque.

- The torque associated with a force is a vector. It has a magnitude and a direction.
- The direction of the torque can be found by using the right-hand rule to evaluate **r** x **F**.
- For extended objects, the total torque is equal to the vector sum of the torque associated with each "component" of this object.



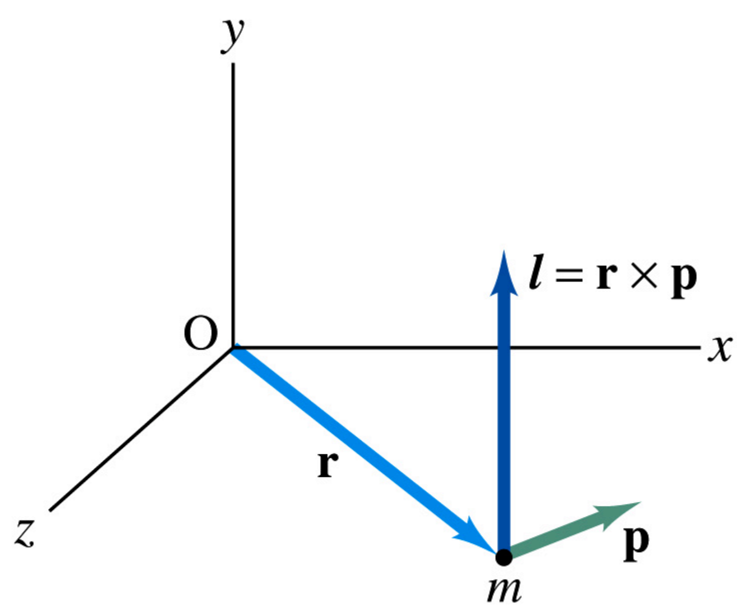



# Angular momentum.

- We have seen many similarities between the way in which we describe linear and rotational motion.
- Our treatment of these types of motion are similar if we recognize the following equivalence:

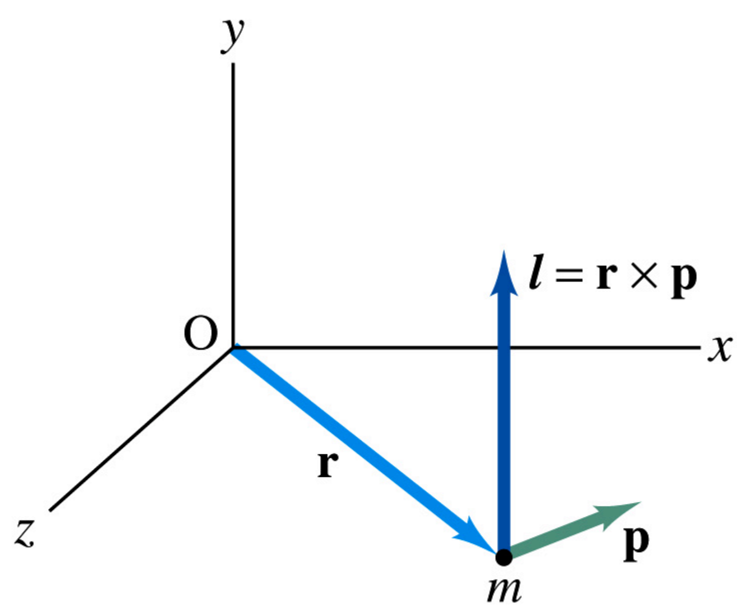
| linear | rotational |
| --- | --- |
| mass $m$ | moment $I$ |
| force $F$ | torque $\tau = r \text{ x } F$ |

- What is the equivalent to linear momentum? Answer: **angular momentum**.



# Angular momentum.

- The **angular momentum** is defined as the vector product between the position vector and the linear momentum.
- Note:
  - Compare this definition with the definition of the torque.
  - Angular momentum is a vector.
  - The unit of angular momentum is kg m$^2$/s.
  - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
  - Under certain circumstances the angular momentum of a system is conserved!

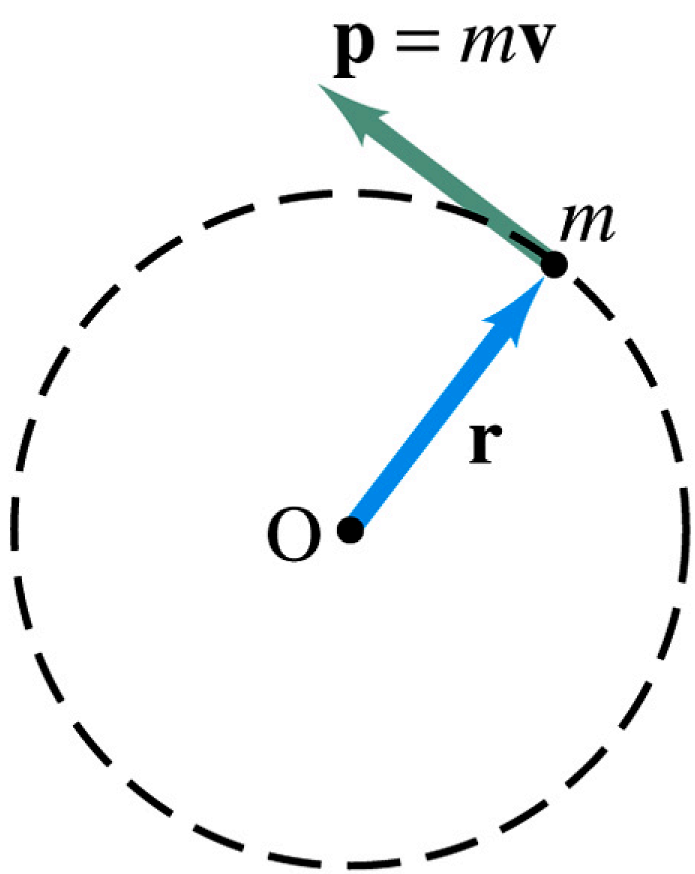


# Angular momentum.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius $r$ and the linear momentum $p$:

$$L = mvr = mr^2(v/r) = I\omega$$

- Note: compare this with $p = mv$!



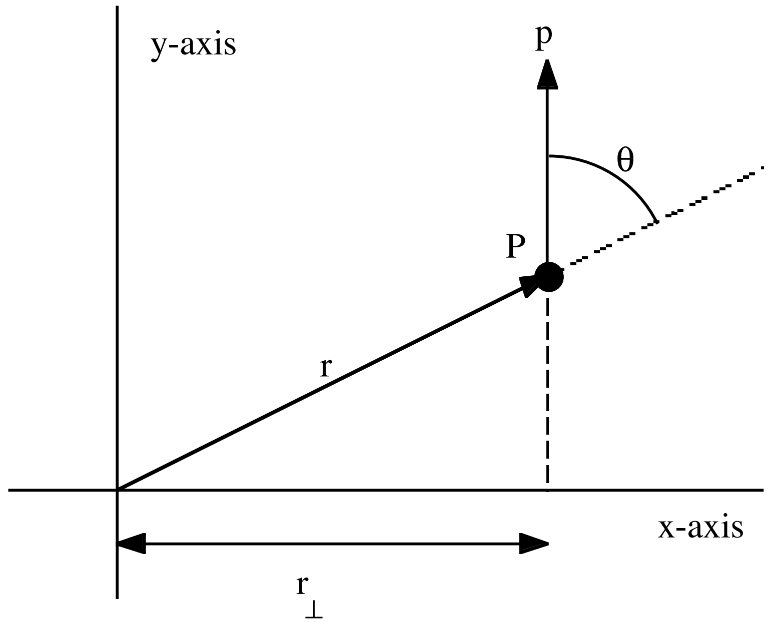
# Angular momentum.

- An object does not need to carry out rotational motion to have an angular moment.
- Consider a particle *P* carrying out linear motion in the *xy* plane.
- The angular momentum of *P* (with respect to the origin) is equal to

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = mrv\sin\theta\,\hat{\mathbf{k}} =$$

$$= mvr_{\perp}\hat{\mathbf{k}} = pr_{\perp}\hat{\mathbf{k}}$$

and will be constant (if the linear momentum is constant).



# Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{\mathbf{L}}}{dt} = \frac{d}{dt}\left(\vec{\mathbf{r}} \times \vec{\mathbf{p}}\right) = m\left(\vec{\mathbf{r}} \times \frac{d\vec{\mathbf{v}}}{dt} + \frac{d\vec{\mathbf{r}}}{dt} \times \vec{\mathbf{v}}\right) =$$

$$= m\left(\vec{\mathbf{r}} \times \vec{\mathbf{a}} + \vec{\mathbf{v}} \times \vec{\mathbf{v}}\right) = \vec{\mathbf{r}} \times \sum \vec{\mathbf{F}} = \sum \vec{\tau}$$

- Consider what happens when the net torque is equal to 0:

$\mathrm{d}L/\mathrm{d}t = 0 \ \rightarrow \ L = \text{constant}$ (conservation of angular momentum)

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.
- Note: notice again the similarities between linear and rotational motion.

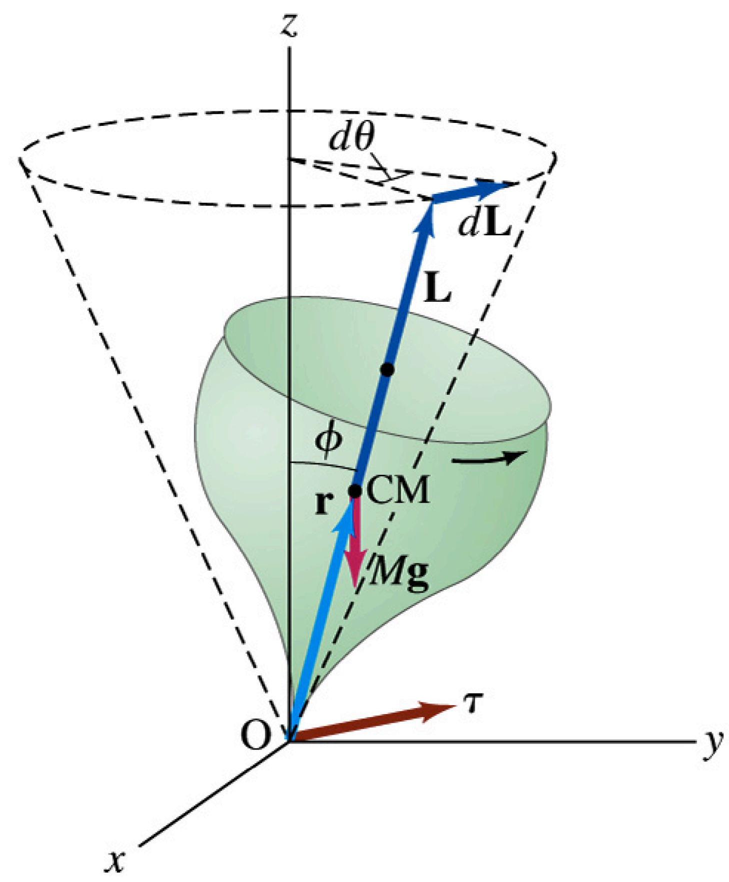
# Conservation of angular momentum.

- The connection between the angular momentum $L$ and the torque $\tau$

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

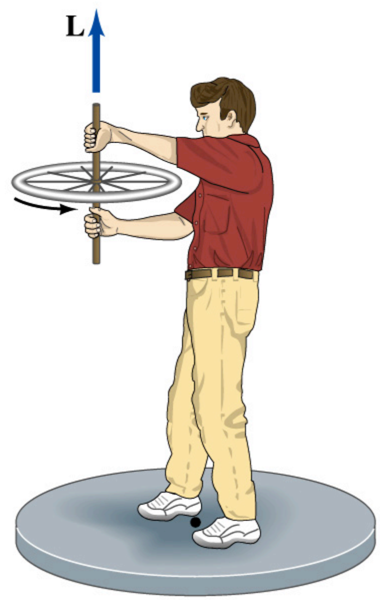
is only true if $L$ and $\tau$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).

- The relation is also true if $L$ and $\tau$ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).

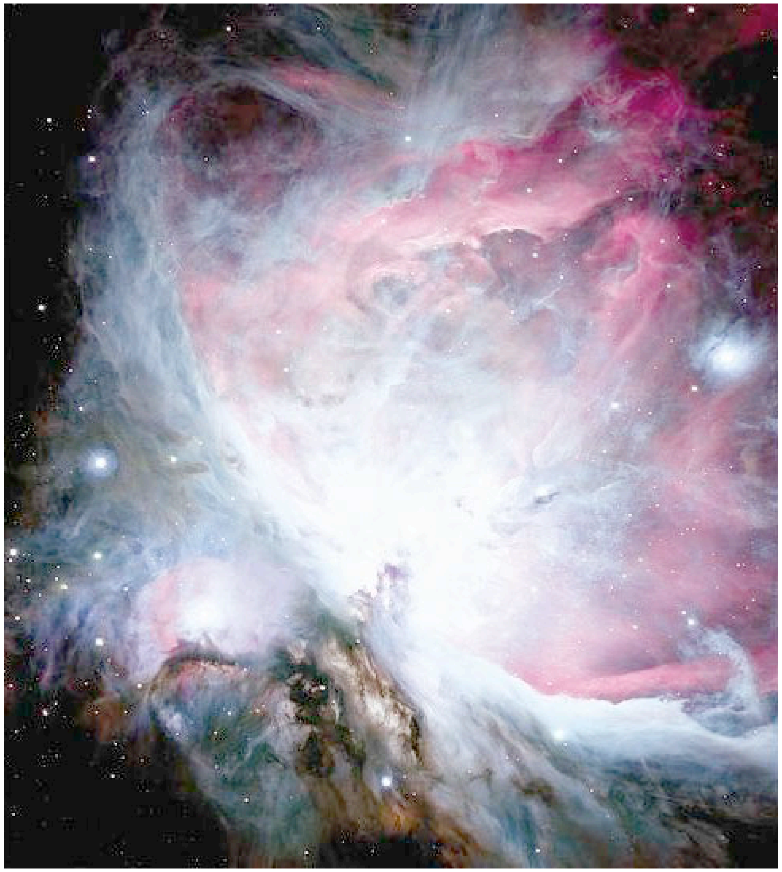


# Conservation of angular momentum.

- Ignoring the mass of the bicycle wheel, the external torque will be close to zero if we use the center of the disk as our reference point.
- Since the external torque is zero, angular momentum thus should be conserved.
- I can change the orientation of the wheel by applying internal forces. In which direction will I need to spin to conserve angular momentum?



# Done for today!
# More about rotational motion on Thursday.



**The Orion Nebula from CFHT**
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