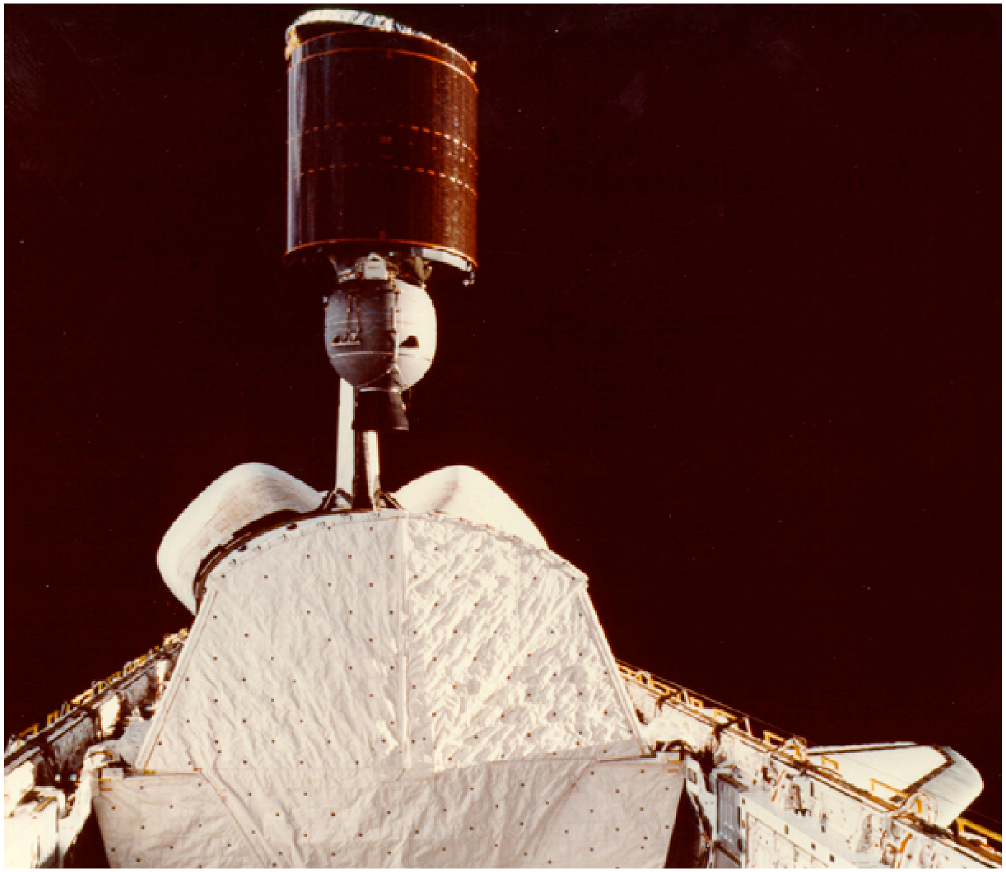
Physics 121.
March 20, 2008.



Physics 121.
March 20, 2008.

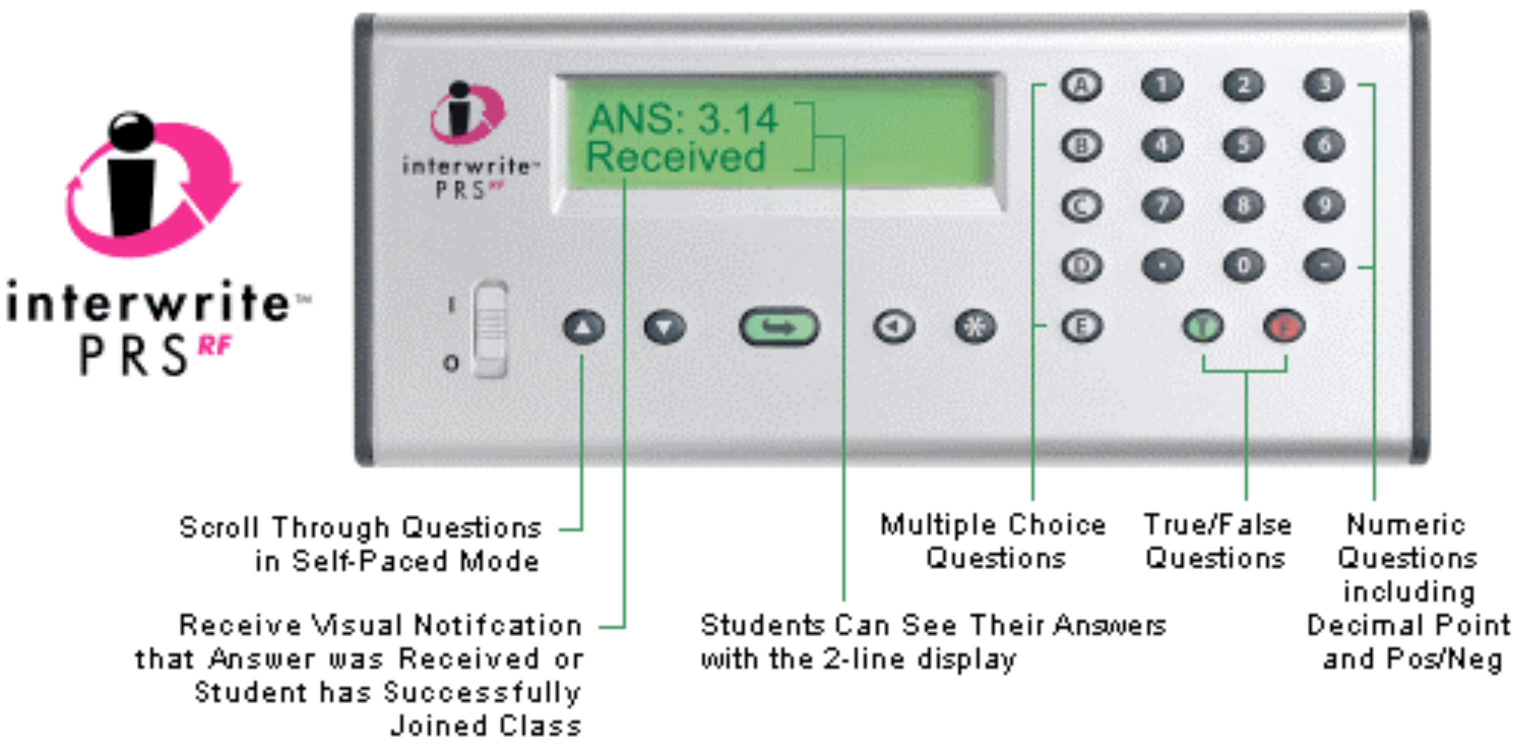
- Course Information
- Quiz
- Topics to be discussed today:
  - Rotational Variables (Review)
  - Torque
  - Rolling Motion
  - Review for Exam 2

Physics 121.
March 20, 2008.

- Homework set # 6 is now available on the WEB and will be due on Saturday morning, March 22, at 8.30 am.
- There will be no homework due on March 29.
- Exam # 2 will take place on Tuesday March 25 at 8 am in Hubbell. It will cover the material discussed in Chapters 7, 8, and 9.
- There will be no workshops or office hours on Tuesday - Friday next week.
- Extra office hours will be scheduled for Sunday 3/23 and Monday 3/24.

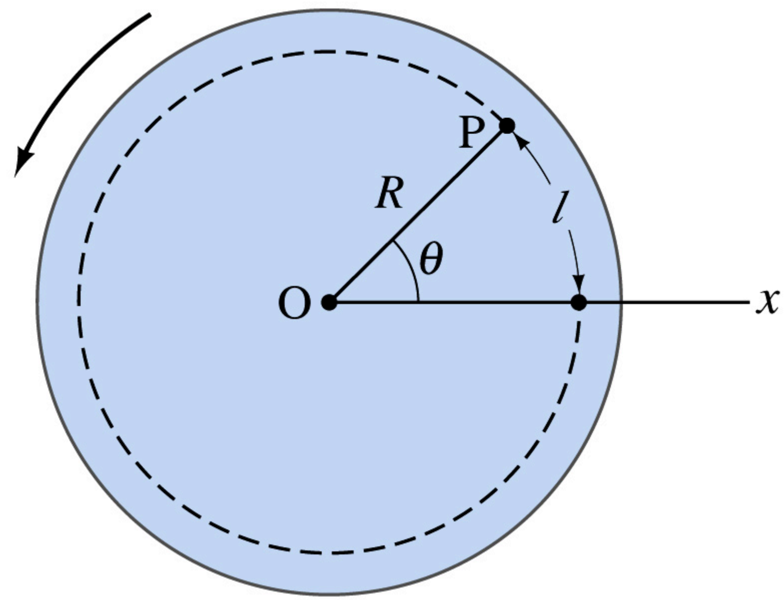
Physics 121.
Quiz lecture 16.

- The quiz today will have 3 questions!



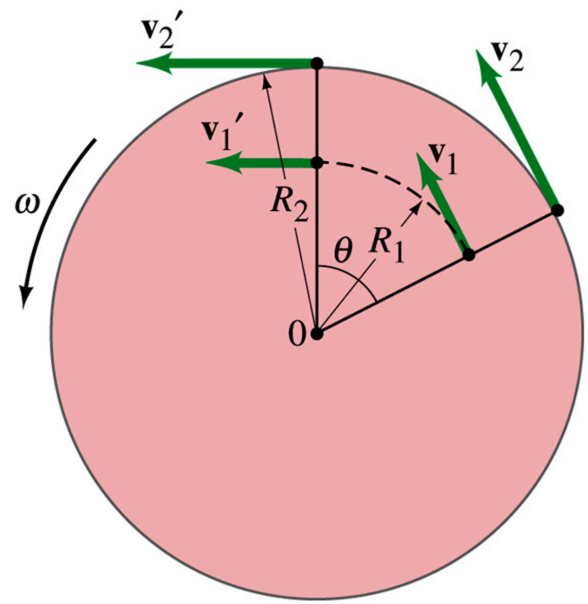
Rotational variables.
A quick review.

- The variables that are used to describe rotational motion are:
  - Angular position θ
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



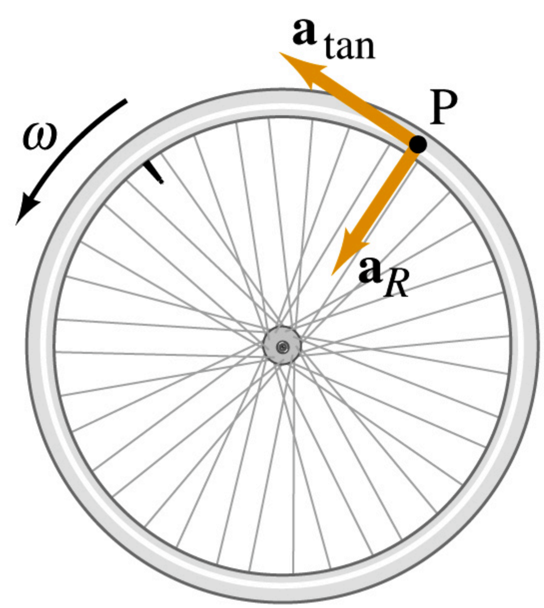
Rotational variables.
A quick review.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:
    - Each part of the rigid object has the same angular velocity.
    - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
    - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



Rotational variables.
A quick review.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P. The two components of the acceleration of point P are:
  - The radial component: this component is always present since point P carried out circular motion around the axis of rotation.
  - The tangential component: this component is present only when the angular acceleration is not equal to 0 rad/s^2.



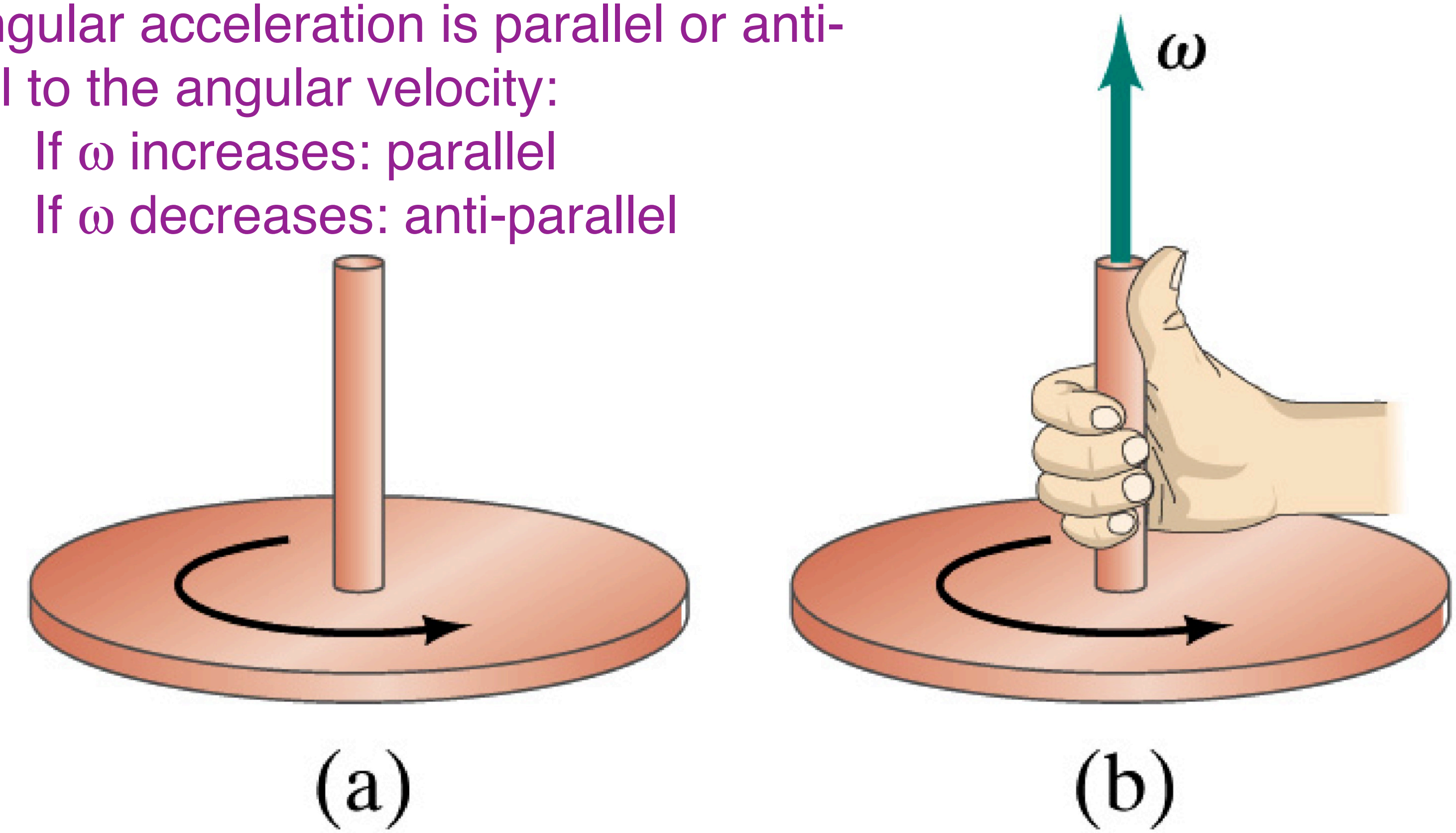
Rotational variables.
A quick review.

Angular velocity and acceleration are vectors! They have a magnitude and a direction. The direction of ω is found using the right-hand rule.

The angular acceleration is parallel or anti-parallel to the angular velocity:

If ω increases: parallel

If ω decreases: anti-parallel



Rotational Kinetic Energy. A quick review.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i \left(\omega r_i\right)^2 = \frac{1}{2}\left\{\sum_i m_i r_i^2\right\}\omega^2 = \frac{1}{2} I\omega^2$$

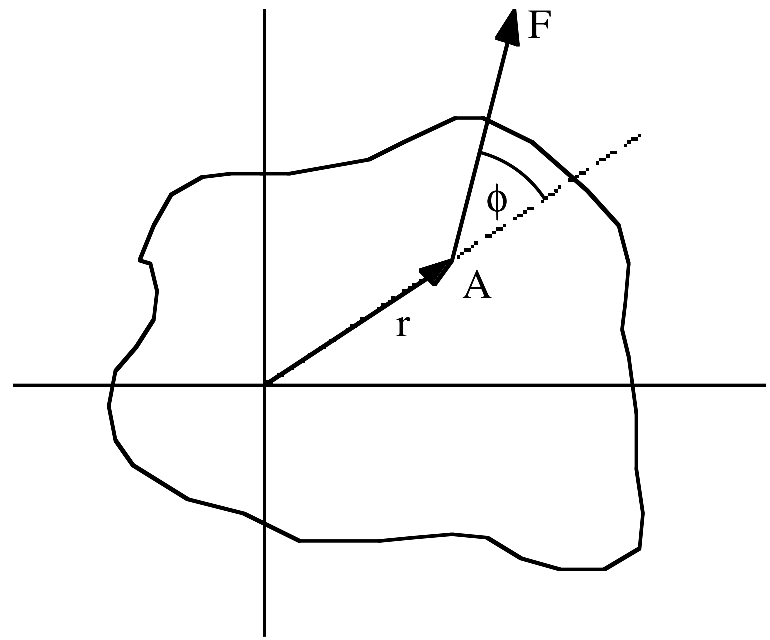
- The kinetic energy is proportional to square of the rotational velocity ω. Note: the equation is similar to the translational kinetic energy (1/2 mv^2) except that instead of being proportional to the the mass m of the object, the rotational kinetic energy is proportional to the **moment of inertia I** of the object:

$$I = \sum_i m_i r_i^2 \quad \text{or} \quad I = \int r^2 dm$$

Note: units of *I*: kg m^2

Torque.

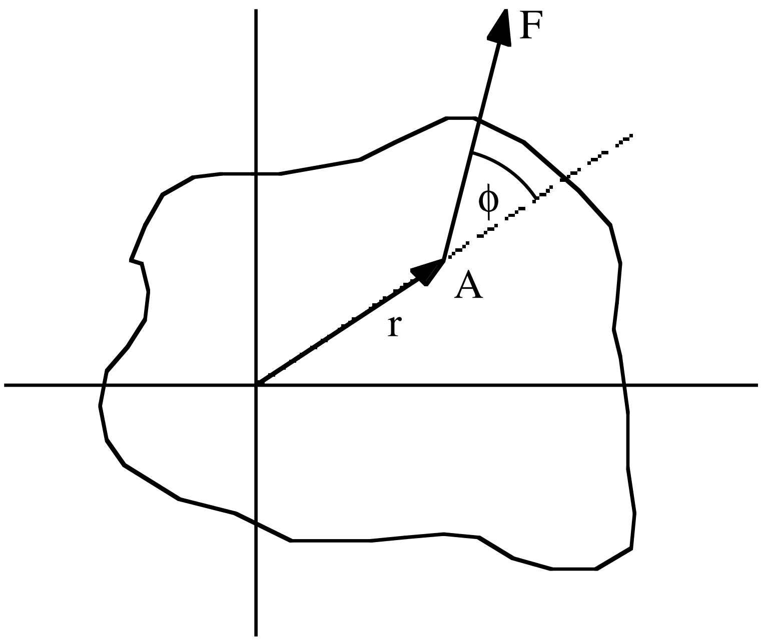
- Consider a force F applied to an object that can only rotate.
- The force F can be decomposed into two two components:
  - A radial component directed along the direction of the position vector r. The magnitude of this component is $F\cos\phi$. This component will not produce any motion.
  - A tangential component, perpendicular to the direction of the position vector r. The magnitude of this component is $F\sin\phi$. This component will result in rotational motion.



Torque.

- If a mass m is located at the position on which the force is acting (and we assume any other masses can be neglected), it will experience a linear acceleration equal to $F\sin\phi/m$.
- The corresponding angular acceleration is equal to $F\sin\phi/(mr)$.
- Since in rotational motion the moment of inertia plays an important role, we will rewrite the angular acceleration in terms of the moment of inertia:

$$\alpha = rF\sin\phi \,/\, (mr^2) = rF\sin\phi \,/\, I$$



Torque.

- Consider rewriting the previous equation in the following way:

$$rF\sin\phi = I\alpha$$

- The left-hand-side of this equation is called the torque τ of the force F:

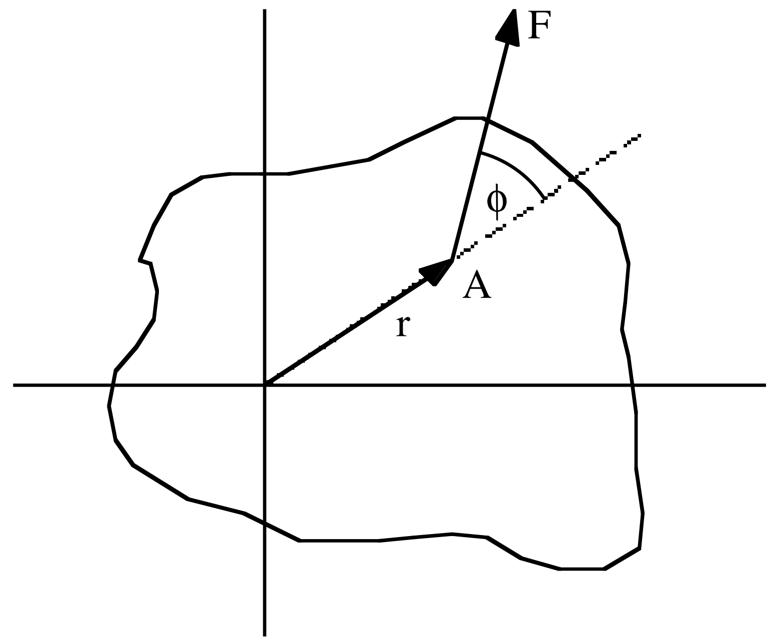
$$\tau = I\alpha$$

- This equation looks similar to Newton's second law for linear motion:

$$F = ma$$

- Note:

<u>linear</u>	<u>rotational</u>
mass m	moment I
force F	torque τ

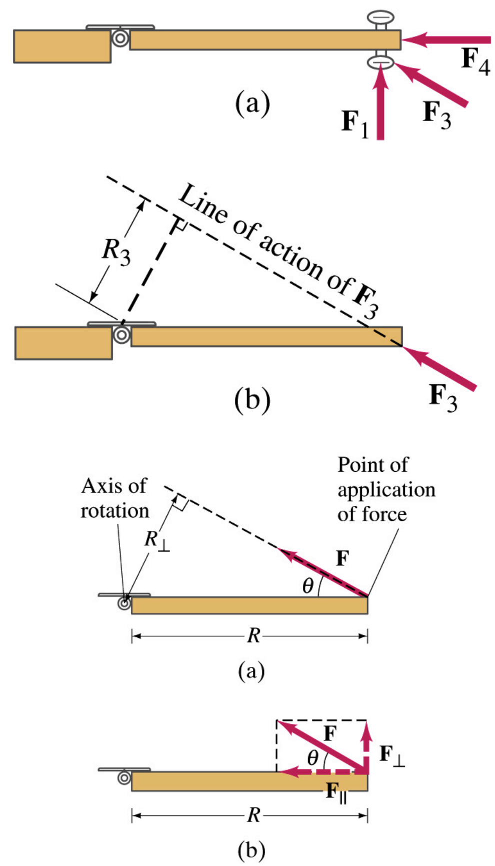


Torque.

- In general the torque associated with a force F is equal to

$$\vec{\tau} = rF\sin\phi = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

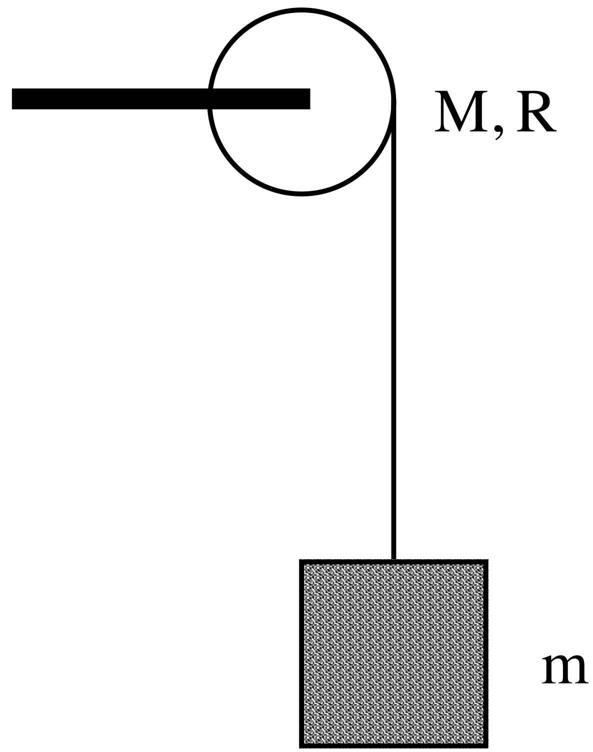
- The arm of the force (also called the moment arm) is defined as $r\sin\phi$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle ϕ is 90°.



Rotational motion. Sample problem.

- Consider a uniform disk with mass *M* and radius *R*. The disk is mounted on a fixed axle. A block with mass *m* hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension of the cord.

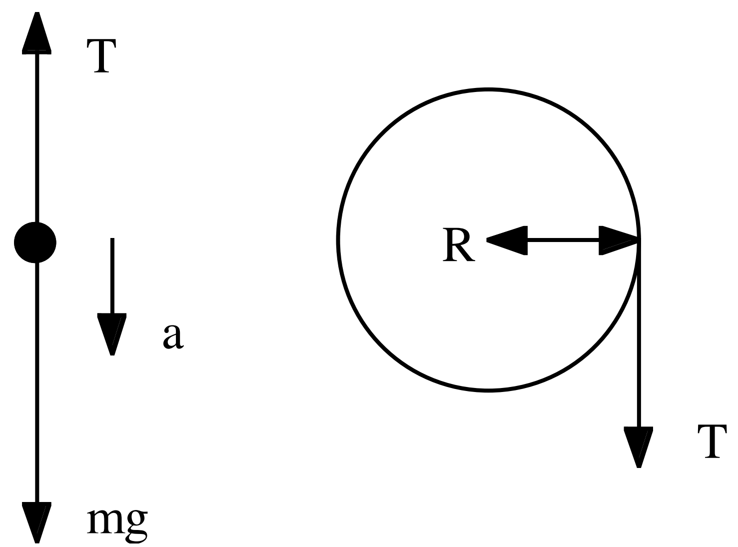
- Expectations:
  - Linear acceleration should approach *g* when *M* approaches 0 kg.



Rotational motion. Sample problem.

- Start with considering the forces and torques involved.
- Define the sign convention to be used.
- The block will move down and we choose the positive and we choose the positive y axis in the direction of the linear acceleration.
- The net force on mass m is equal to

$$ma = mg - T$$



Rotational motion. Sample problem.

- The net torque on the pulley is equal to

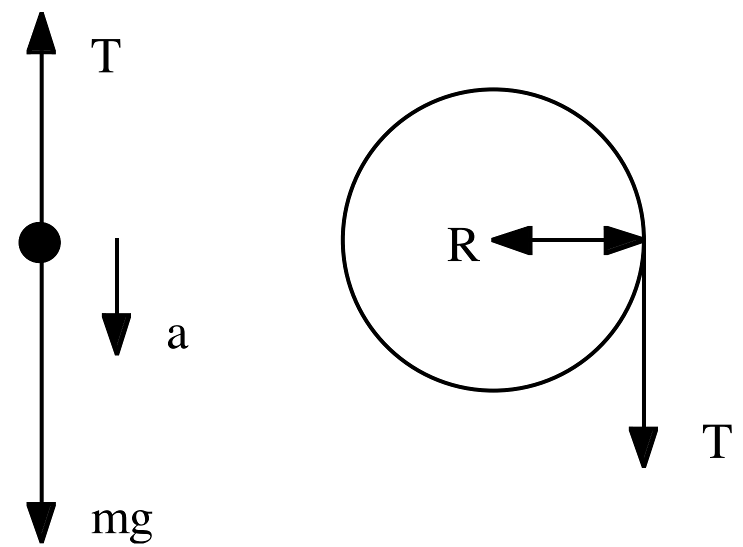
$$\tau = RT$$

- The resulting angular acceleration is equal to

$$\alpha = \frac{\tau}{I} = \frac{RT}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

- Assuming the cord is not slipping we can determine the linear acceleration:

$$a = \frac{2T}{M}$$

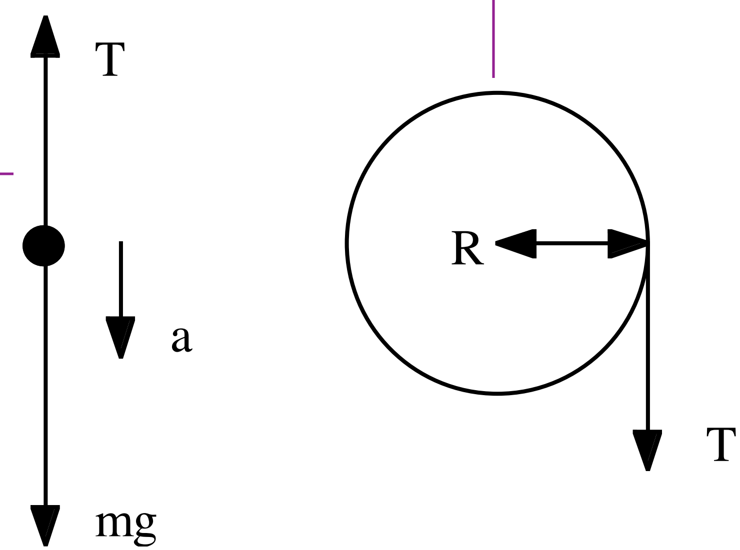


Rotational motion.
Sample problem.

- We now have two expressions for a:

$$a = \frac{2T}{M}$$

$$a = g - \frac{T}{M}$$



- Solving these equations we find:

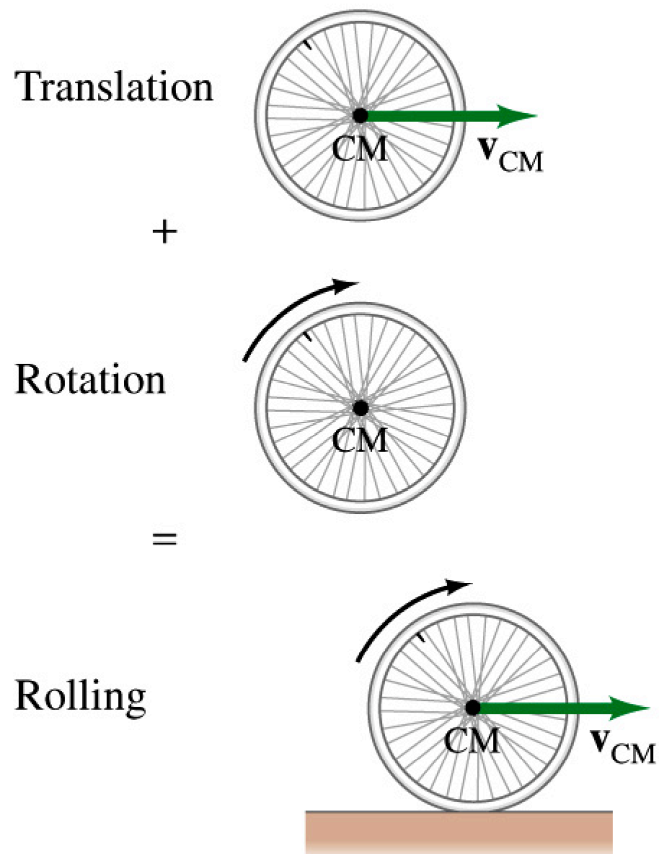
$$T = \frac{M}{M + 2m} mg$$

$$a = \frac{2m}{M + 2m} g$$

Note: $a = g$ when $M = 0$ kg!!!

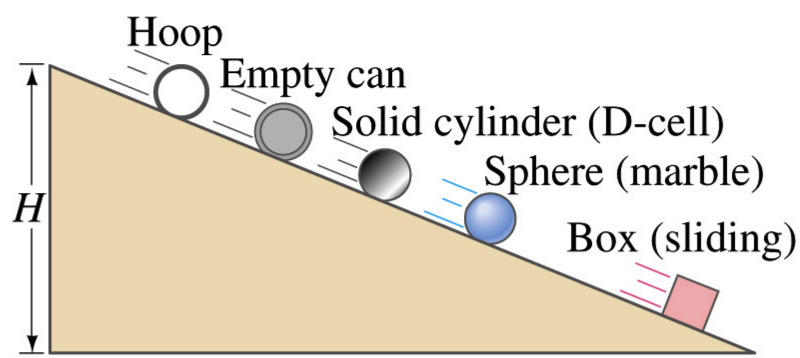
Rolling motion.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
  - Translational kinetic energy = (1/2) Mv_{cm}^2.
  - Rotational kinetic energy = (1/2) $I_{cm}\omega^2$.
- Assuming the wheel does not slip: $\omega = v / R$.



Rolling motion.

- Consider two objects of the same mass but different moments of inertia, released from rest from the top of an inclined plane:
  - Both objects have the same initial mechanical energy (assuming their CM is located at the same height).
  - At the bottom of the inclined plane they will have both rotational and translational kinetic energy.
  - Which object will reach the bottom first?



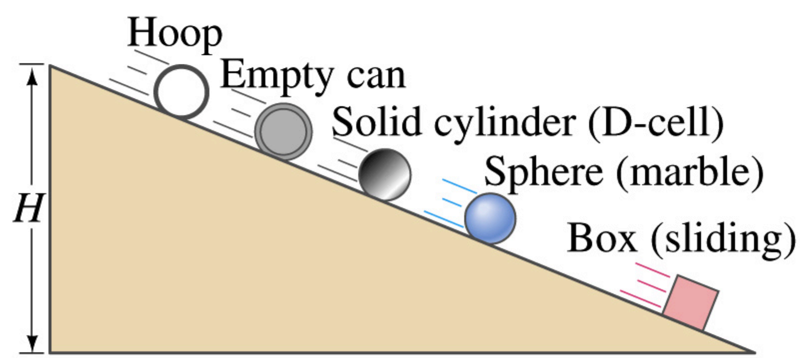
Rolling motion.

- Initial mechanical energy = mgH.
- Final mechanical energy = $(1/2)\, mv_{cm}^2 + (1/2)\, I_{cm}\, \omega^2$.
- Assuming no slipping, we can rewrite the final mechanical energy as $(1/2)\{m+I_{cm} / R^2\}v_{cm}^2$.
- Conservation of energy implies:

$$(1/2)\{m+I_{cm} / R^2\}v_{cm}^2 = mgH$$

or

$$(1/2)\{1+I_{cm} / mR^2\}v_{cm}^2 = gH$$



The smaller I_{cm}, the larger v_{cm} at the bottom of the incline.

Physics 121 Review
Midterm Exam # 2

Review Physics 121.
Midterm Exam # 2.

- Main topics covered:
  - Work and Energy (chapters 7 - 8):
    - Work and Work-Energy Theorem.
    - Kinetic and Potential Energy.
    - Conservative and Non-Conservative Forces.
    - Conservation of Energy.
  - Linear Momentum and Collisions (chapter 9):
    - Center-of-Mass and Motion of the Center-of-Mass.
    - Conservation of Linear Momentum.
    - Rocket Equations.
    - Elastic and Inelastic Collisions.
- The material covered on the exam is the material covered in Chapters 7, 8, and 9.

Chapter 7

Work and Energy

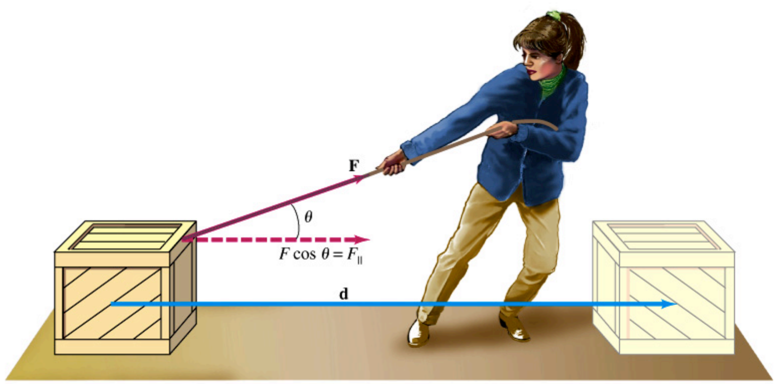
Chapter 7.
Work: Definition.

- When a force F is applied to an object, it may produce a displacement d.

- The work W done by the force F is defined as

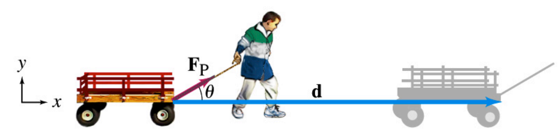
$$W = \vec{\mathbf{F}} \bullet \vec{\mathbf{d}} = Fd\cos\theta$$

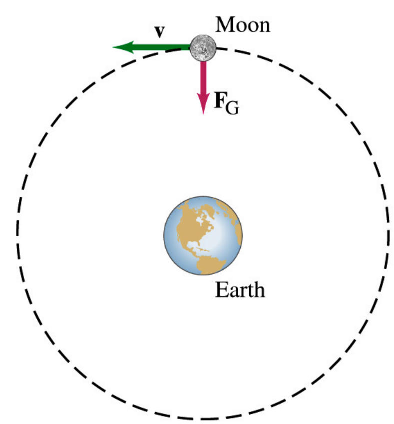
where ϕ is the angle between the force F and the displacement d.



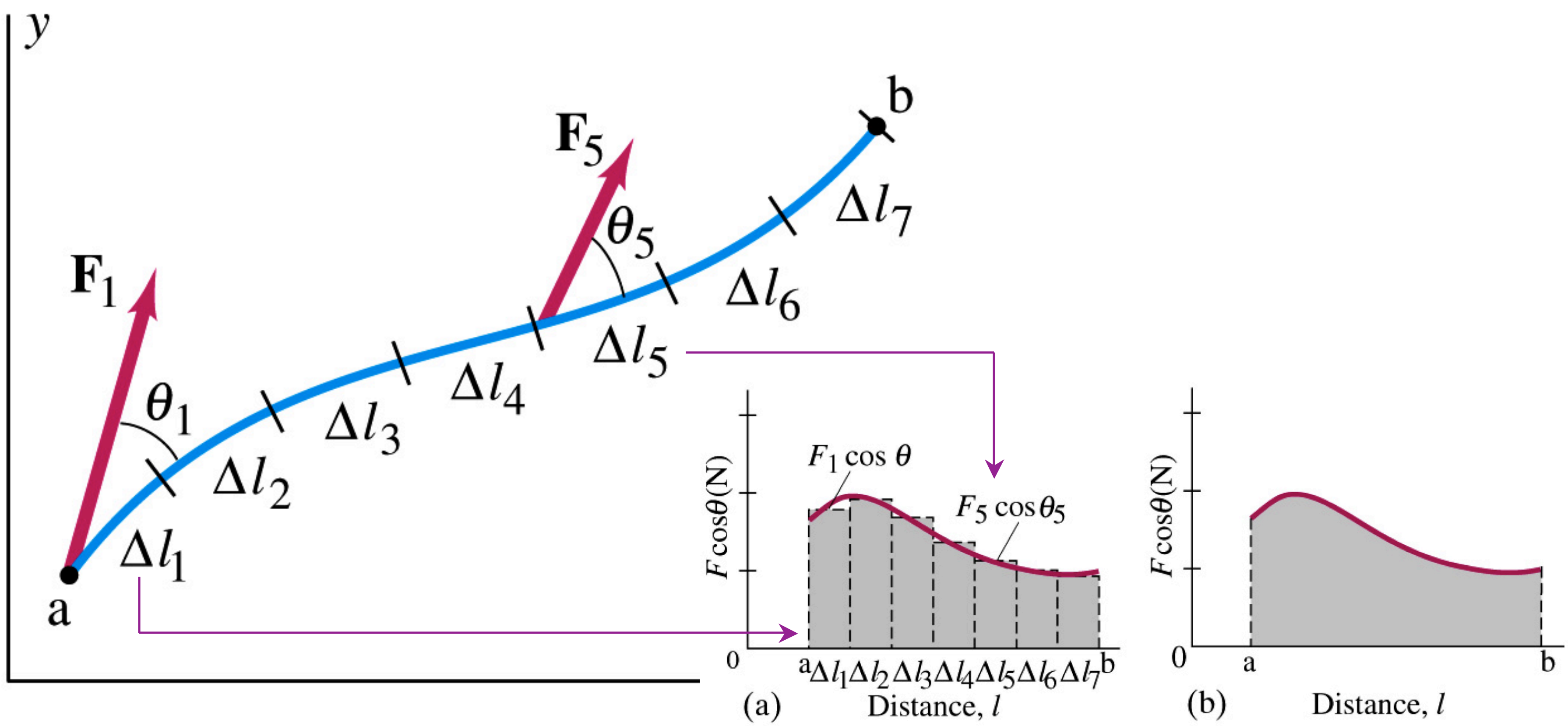
Chapter 7.
Work: Units.

- The unit of work is the Joule (abbreviated J).
- Per definition, 1 J = 1 Nm = 1 kg m^2/s^2.
- There are many important examples of forces that do not do any work. For example, the gravitational force between the earth and the moon does not do any work! Note: in this case, the speed of the moon does not change.





Chapter 7.
Work done by a varying force.



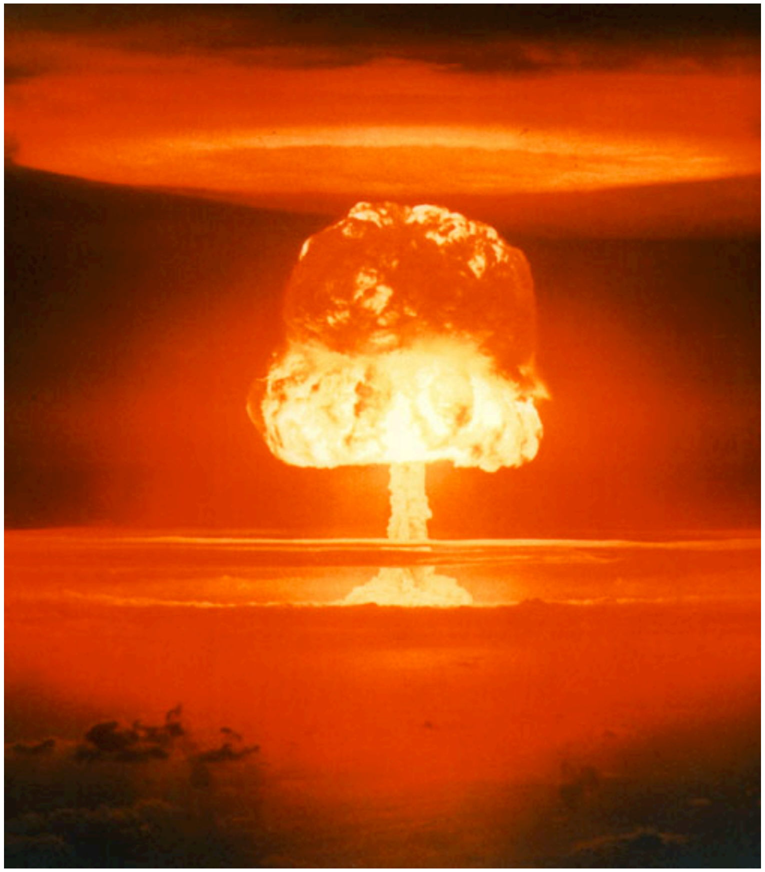
Chapter 7.
Power: Definition and units.

- In many cases, the work done by a tool is less important than the the rate with which the work can be done.

- Power of defined as work per unit time:

$$P = \frac{dW}{dt}$$

- The unit of power is the Watt, abbreviated by a W (1 W = 1 J/s = 1 kg m^2/s^3)



Chapter 7.
Work-Energy theorem.

- The Work-Energy theorem states:

The net work done on an object is equal to the change in its kinetic energy.

or $W = \Delta K$. The kinetic energy of an object is defined as $(1/2)Mv^2$.

Chapter 8

Conservation of Energy

Chapter 8.
Conservation of mechanical energy

- Consider what would happen if we define the mechanical energy of a system to be equal to the sum of the kinetic energy K and the potential energy U:

$$E = K + U$$

- If the total mechanical energy is constant, we must require that $\Delta E = 0$, or

$$\Delta K + \Delta U = 0$$

- We conclude that any change in the kinetic energy ΔK must be accompanied by an equal but opposite change in the potential energy ΔU.

Chapter 8.
Potential energy in one dimension.

- Per definition, the change in potential energy is related to the work done by the force:

$$\Delta U = -W = -\int_{x_0}^{x} F(x')\,dx'$$

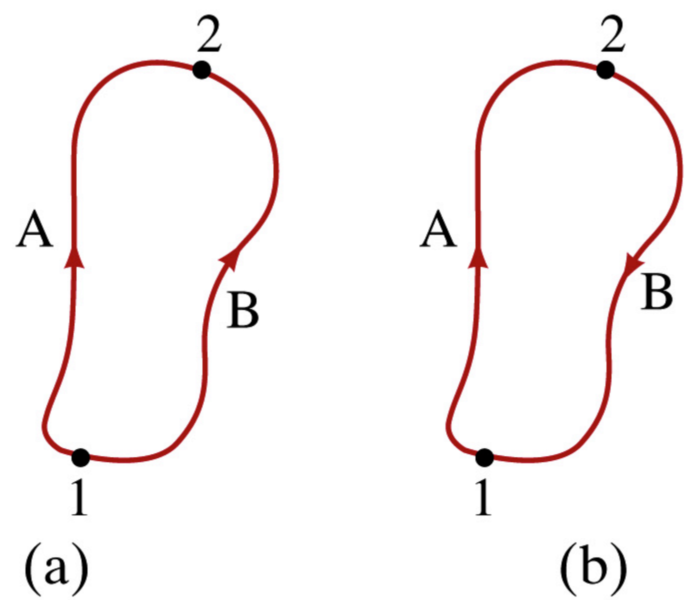
- The potential energy at x can thus be related to the potential energy at a point x_0:

$$U(x) = U(x_0) + \Delta U = U(x_0) - \int_{x_0}^{x} F(x')\,dx'$$

- **Note**: the units of potential energy are the units of energy (the Joule).

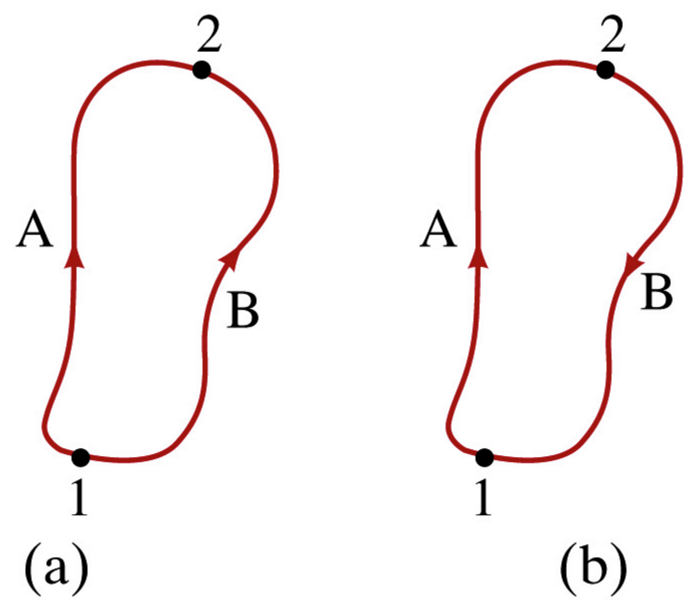
Chapter 8.
Potential energy and path dependence.

- The difference between the potential energy at (2) and at (1) depends on the work done by the force F along the path between (1) and (2).
- But there are many roads that lead from (1) to (2). The potential at (2) is uniquely defined only if the work done is path independent.
- This is not true for all forces. For example, the work done by the friction force is always negative, and the work is path dependent.



Chapter 8.
Conservative and non-conservative forces.

- If the work is independent of the path, the work around a closed path will be equal to 0 J.
- A force for which the work is independent of the path is called a **conservative force**. Examples: spring force, gravitational force.
- A force for which the work depends on the path is called a **non-conservative force**. Examples: friction force, drag force.



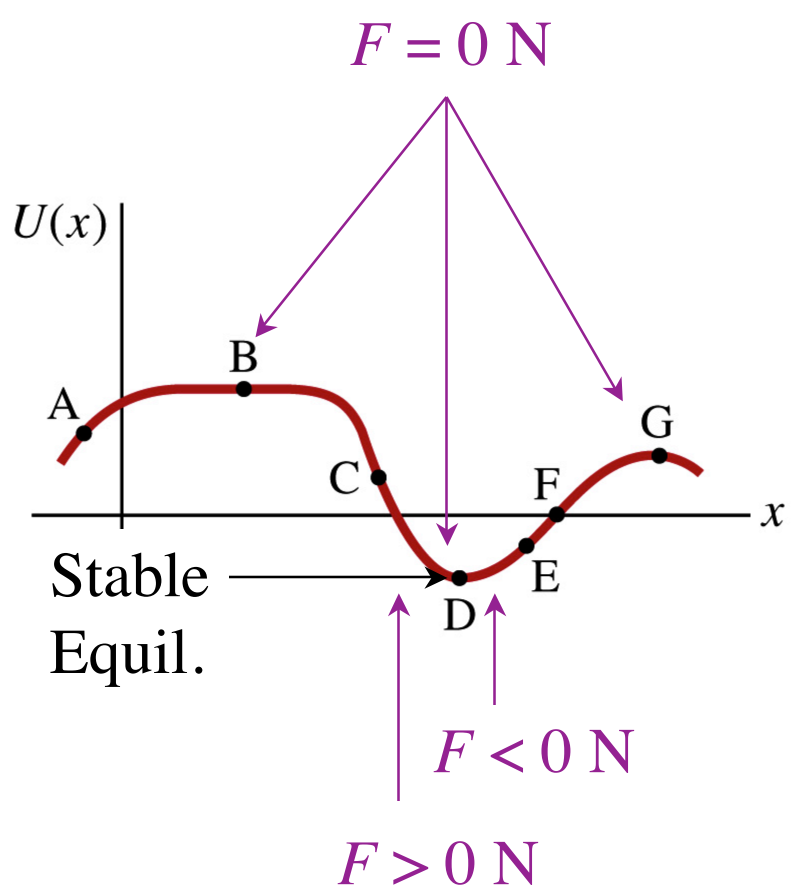
Chapter 8.
Potential energy in one dimension.

- The potential energy is directly related to the force acting on the object.
  - If we know the force, we can calculate the change dU:

$$\Delta U = -W = -\int_{x_0}^{x} F(x')dx'$$

  - If we know the change dU, we can calculate the force:

$$F(x) = -\frac{dU}{dx}$$



Chapter 8.
Conservation of energy and dissipative forces.

- When dissipative forces, such as friction forces, are present, mechanical energy is no longer conserved.

- For example, a friction force will reduce the speed of a moving object, thereby dissipating its kinetic energy.

- The amount of energy dissipated by these non-conservative forces can be calculated if we know the magnitude and direction of these forces along the path followed by the object we are studying:

$$\Delta K + \Delta U = W_{\mathrm{NC}}$$

where W_{NC} is the work done by the non-conservative forces.

Chapter 9

Linear momentum

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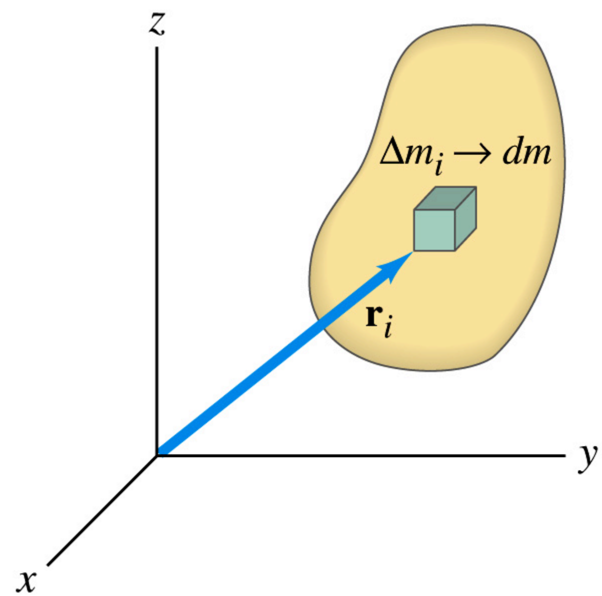
Chapter 9.
The center of mass.

- Up to now we have ignored the shape of the objects were are studying.
- we can use whatever we have learned about motion of point-like objects if we consider the motion of the center-of-mass of the extended object.
- The center-of-mass of an object is defined as

$$\vec{\mathbf{r}}_{cm} = \frac{1}{M} \int_{Volume} \vec{\mathbf{r}} dm$$



Chapter 9.
Motion of the center of mass.

- To examine the motion of the center of mass we start with its position and then determine its velocity and acceleration:

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

$$M\vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

$$M\vec{a}_{cm} = \sum_i m_i \vec{a}_i$$

- The expression for Ma_{cm} can be rewritten in terms of the forces on the individual components:

$$M\vec{a}_{cm} = \frac{d}{dt}\left(M\vec{v}_{cm}\right) = \frac{d\vec{P}_{cm}}{dt} = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- If the net external force is 0, the center of mass will move with constant velocity.

Chapter 9.
Linear momentum.

- The linear momentum of an object is the product of it mass and its velocity. For en extended object, the total linear momentum is equal to

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

- The change in the linear momentum of the system can now be calculated:

$$\frac{d\vec{P}_{cm}}{dt} = \frac{d}{dt}\left(M\vec{v}_{cm}\right) = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \sum_i m_i \vec{v}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- This relations shows us that if there are no external forces, the total linear momentum of the system will be constant (independent of time).

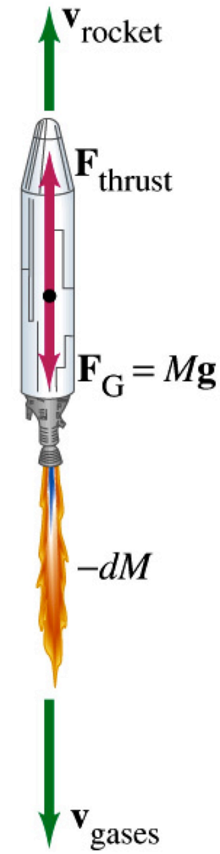
Chapter 9.
Systems with variable mass.

- The **first Rocket equation**:

$$RU_0 = Ma_{\text{rocket}}$$

where
  - R = -dM/dt is the rate of fuel consumption.
  - U_0 = -u where u is the (positive) velocity of the exhaust gasses relative to the rocket.

This equation can be used to determine the rate of fuel consumption required for a specific acceleration.



- The **second rocket equation**:

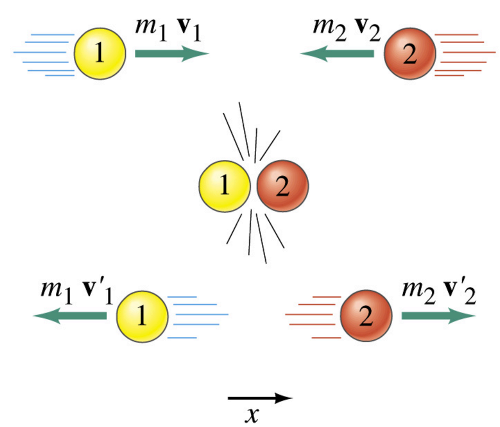
$$v_f = v_i + u \ln\left(\frac{M_i}{M_f}\right)$$

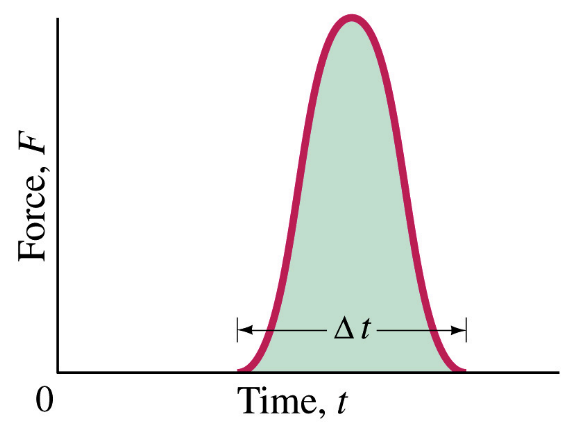
Chapter 9.
Collisions.

- During a collision, a strong force is exerted on the colliding objects for a short period of time.
- The collision force is usually much stronger then any external force.
- The result of the collision force is a change in the linear momentum of the colliding objects.
- The change in the momentum of one of the objects is equal to

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$

- The integral of the force is called the collision impulse J.

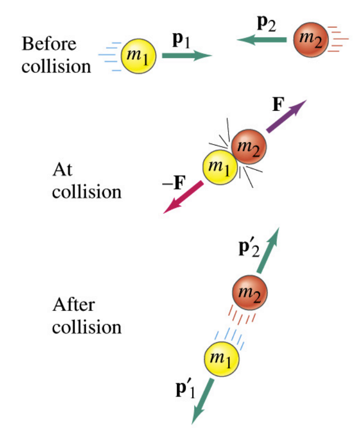


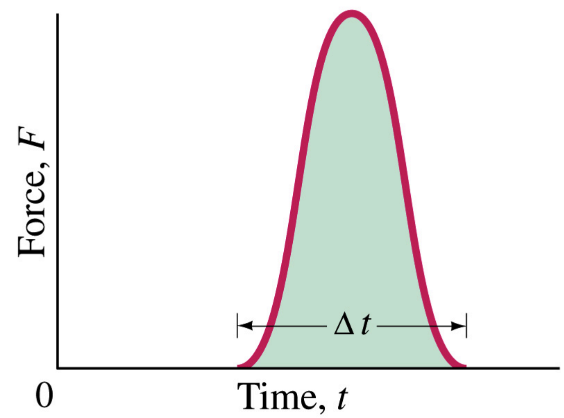


Chapter 9. Collisions.

- If we consider both colliding object, then the collision force becomes an internal force and the total linear momentum of the system must be conserved if there are no external forces acting on the system.

- Collisions are usually divided into two groups:
  - Elastic collisions: kinetic energy is conserved.
  - Inelastic collisions: kinetic energy is NOT conserved.





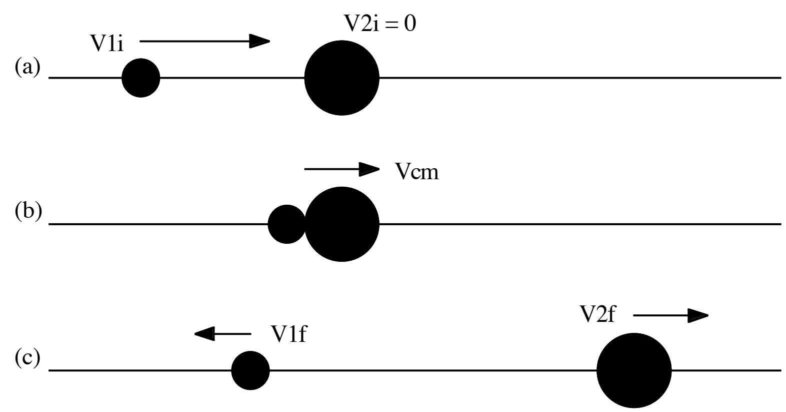
Chapter 9.
Elastic collisions in one dimension.

- The final state of an elastic collision in one dimension is completely defined by the initial conditions.
- We can always use a reference frame where one of the objects is at rest.
- The initial velocity of mass m_1 is v_{1i}. Mass m_2 is at rest.
- The final velocity of mass m_1 is:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

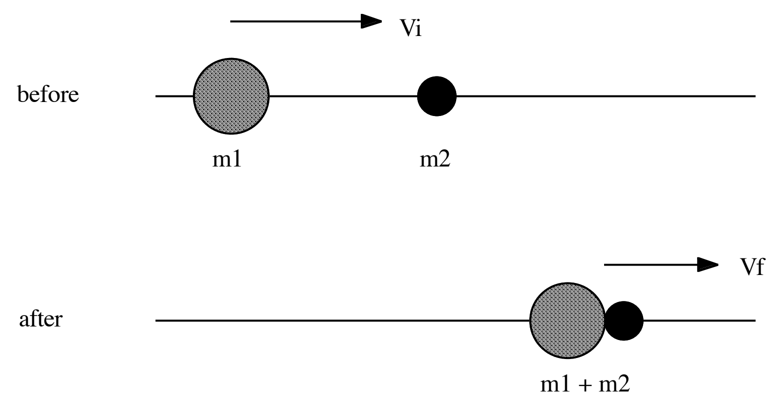
- The final velocity of mass m_2 is:

$$v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) = \frac{2m_1}{m_1 + m_2} v_{1i}$$



Chapter 9.
Inelastic collisions in one dimension.

- In inelastic collisions, kinetic energy is not conserved.
- A special type of inelastic collisions are the **completely inelastic collisions**, where the two objects stick together after the collision.
- Conservation of linear momentum in a completely inelastic collision requires that $m_1 v_i = (m_1 + m_2) v_f$.
- We can thus determine the final velocity and kinetic energy of the system:

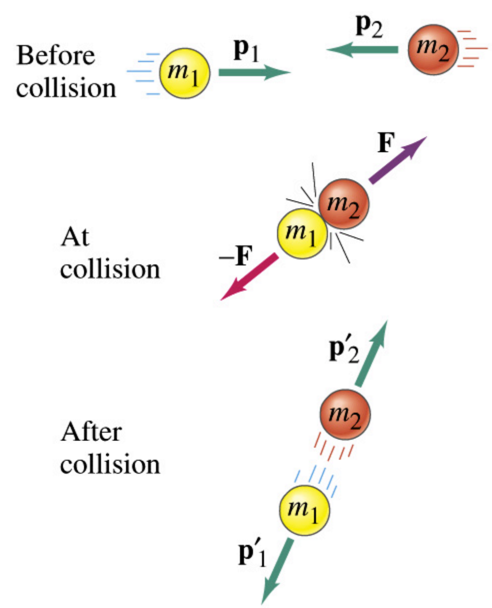


$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

$$K_f = \frac{1}{2}(m_1 + m_2) v_f^{\,2} = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2} v_i\right)^2 = \frac{m_1}{m_1 + m_2} K_i$$

Chapter 9.
Collisions in two or three dimensions.

- Collisions in two or three dimensions are approached in the same way as collisions in one dimension.
- The x, y, and z components of the linear momentum must be conserved if there are no external forces acting on the system.
- The collisions can be elastic or inelastic.
- Even for two-dimensional elastic collisions, the final state is not fully defined by the initial state.



Final remarks.

- The hardest part of each problem is recognizing the approach to take. Different approaches may lead to the same answer, but can differ greatly in difficulty.

- A suggestion:
  - Look at the end of chapter problems. There is only a limited number of types of question one can ask.
  - But .... Since the questions are grouped by section, you know already what approach to use based on the section to which the problems are assigned.
  - Some students benefit from copying the questions, cutting them out, writing the chapter/section numbers on the back, mixing them up, and then reading through them and determining what approach you would take if you would see that question on the exam (compare it with the focus of the section to which the problem was assigned).

Final remarks.

- You will only need your pen, a pencil, and an eraser. Being awake might also help!
- You will find a formula sheet attached to the exam. This sheet will be distributed via email before the exam and will also be available from the Physics 121 website,
- The exam will start at 8 am and end at 9.30 am. If you show up late you will just have less time to finish. Over the years I have heard every excuse possible for being late, but I have never heard one that I accepted.

Good luck preparing for the exam.

Done for today!
More about rotational motion next week.

N49's Cosmic Blast
Credit: Hubble Heritage Team (STScI / AURA), Y. Chu (UIUC) et al., NASA