Physics 121.
March 18, 2008.

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- Course Information
- Topics to be discussed today:
  - Variables used to describe rotational motion
  - The equations of motion for rotational motion

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Course Announcements.

- Homework set # 6 is now available on the WEB and will be due on Saturday morning, March 22, at 8.30 am.
- All the material to be covered on Exam # 2 has now been discussed. Today we will start on material that will be covered on Exam # 3.
- Exam # 2 will take place on Tuesday March 25 at 8 am in Hubbell. It will cover the material discussed in Chapters 7, 8, and 9.

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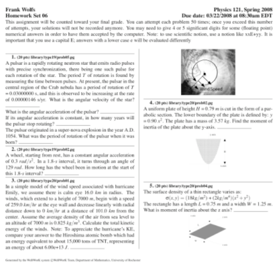
Physics 121.
Homework Set # 6.

Calculating the moment of inertia

Equations of motion with constant α.

Equations of motion with constant α.

Rotational kinetic energy. Note: this is not a rigid body!



Frank Wolfs — **Physics 121, Spring 2008**
Homework Set 06 — **Due date: 03/22/2008 at 08:30am EDT**

This assignment will be counted toward your final grade. You can attempt each problem 50 times; once you exceed this number of attempts, your solutions will not be recorded anymore. You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer. Note: to use scientific notion, use a notion like xxE+yy. It is important that you use a capital E; answers with a lower case e will be evaluated differently

1. (20 pts) library/type19/prob05.pg
A pulsar is a rapidly rotating neutron star that emits radio pulses with precise synchronization, there being one such pulse for each rotation of the star. The period T of rotation is found by measuring the time between pulses. At present, the pulsar in the central region of the Crab nebula has a period of rotation of T = 0.03000000 s, and this is observed to be increasing at the rate of 0.00000146 s/yr. What is the angular velocity of the star? ________
What is the angular acceleration of the pulsar? ________
If its angular acceleration is constant, in how many years will the pulsar stop rotating? ________
The pulsar originated in a super-nova explosion in the year A.D. 1054. What was the period of rotation of the pulsar when it was born? ________

2. (20 pts) library/type19/prob02.pg
A wheel, starting from rest, has a constant angular acceleration of 0.3 rad/s^2. In a 1.8-s interval, it turns through an angle of 129 rad. How long has the wheel been in motion at the start of this 1.8-s interval? ________

3. (20 pts) library/type20/prob01.pg
In a simple model of the wind speed associated with hurricane Emily, we assume there is calm eye 16.0 km in radius. The winds, which extend to a height of 7000 m, begin with a speed of 259.0 km/hr at the eye wall and decrease linearly with radial distance down to 0 km/hr at a distance of 101.0 km from the center. Assume the average density of the air from sea level to an altitude of 7000 m is 0.825 kg/m^3. Calculate the total kinetic energy of the winds. Note: To appreciate the hurricane's KE, compare your answer to the Hiroshima atomic bomb which had an energy equivalent to about 15,000 tons of TNT, representing an energy of about 6.00e+13 J. ________

4. (20 pts) library/type20/prob02.pg
A uniform plate of height H = 0.79 m is cut in the form of a parabolic section. The lower boundary of the plate is defined by: $y = 0.90\,x^2$. The plate has a mass of 3.57 kg. Find the moment of inertia of the plate about the y-axis. ________

5. (20 pts) library/type20/prob04.pg
The surface density of a thin rectangle varies as:
$$\sigma(x,y) = (18kg/m^2) + (2kg/m^4)(x^2+y^2)$$
The rectangle has a length L = 0.75 m and a width W = 1.25 m. What is moment of inertia about the z axis? ________

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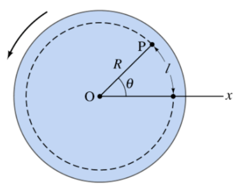
acceleration

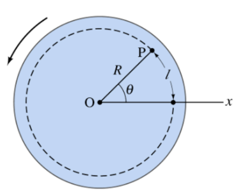
Rotational motion: variables.

- In our discussion of rotational motion we will first focus on the rotation of **rigid** objects around a **fixed** axis.
- The variables that are used to describe this type of motion are similar to those we use to describe linear motion:
  - Angular position
  - Angular velocity
  - Angular acceleration



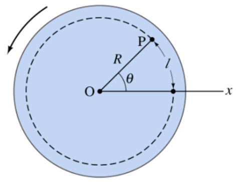
Rotational motion: variables.

- The angular position, measured in radians, is the angle of rotation of the object with respect to a reference position.
- The angular position of point P at this point in time is equal to θ. In order to uniquely define this position, we have assume that an the angular position is measured with respect to the x axis.

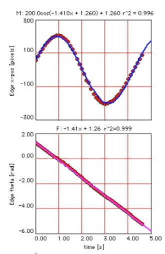


Rotational motion: variables.

- Note:
  - The angular position is always specified in radians!!!!
  - One radian is the angular displacement corresponding to a linear displacement $l = R$.
  - Make sure you keep track of the sign of the angular position!!!!!
  - An increase in the angular position corresponds to a counter-clockwise rotation; a decrease corresponds to a clockwise rotation.

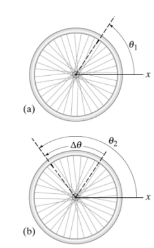


Complex motion in Cartesian coordinates is simple motion in rotational coordinates.



Rotational motion: variables.

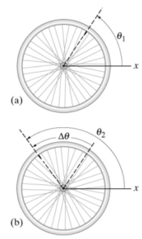
- If we look at an object carrying out a rotation around a fixed axis, we will see that the angular position becomes a function of time.
- To describe the rotational motion, we introduce the concepts of angular velocity and angular acceleration.
- **Remember**: for linear motion we found it useful to introduce the concepts of linear velocity and linear acceleration.



Angular acceleration (rad/s²)

Rotational motion: variables.

- For both velocity and acceleration we can talk about instantaneous and average.
- Angular velocity:
  - Definition: $\omega = d\theta/dt$
  - Symbol: ω
  - Units: rad/s
- Angular acceleration:
  - Definition: $\alpha = d\omega/dt = d^2\theta/dt^2$
  - Symbol: α
  - Units: rad/s^2



Angular acceleration (rad/s^2)

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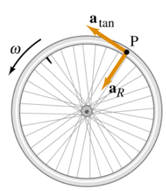
Rotational motion: constant acceleration.

- If the object experiences a constant angular acceleration, then we can describe its rotational motion with the following equations of motion:

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

- Note how similar these equations are to the equation of motion for linear motion!!!!!!



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Rotational motion: constant acceleration. Example problem.

- A wheel starting from rest, rotates with a constant angular acceleration of 2.0 rad/s^2. During a certain 3.0 s interval it turns through 90 rad. (a) How long had the wheel been turning before the start of the 3.0 s interval ? (b). What was the angular velocity of the wheel at the start of the 3.0 s interval ?
- Define time $t = 0$ s as the time that the wheel is at rest. The angular velocity and the angle of rotation at a later time t are given by

$$\omega(t) = \alpha t \qquad \theta(t) = \frac{1}{2}\alpha t^2$$

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Rotational motion: constant acceleration. Example problem.

- The change in the angular position $\Delta\theta$ during a time period Δt can now be calculated:

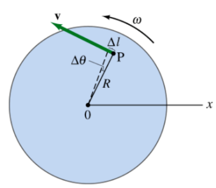
$$\Delta\theta(t) = \theta(t+\Delta t) - \theta(t) = \frac{1}{2}\alpha(t+\Delta t)^2 - \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha(\Delta t)^2 + \alpha t \Delta t$$

- Since the problem specifies $\Delta\theta$ and Δt we can now calculate the time t and the angular velocity at that time:

$$t = \frac{\Delta\theta - \frac{1}{2}\alpha(\Delta t)^2}{\alpha \Delta t} \qquad \omega = \alpha t = \frac{\Delta\theta - \frac{1}{2}\alpha(\Delta t)^2}{\Delta t} = \frac{\Delta\theta}{\Delta t} - \frac{1}{2}\alpha(\Delta t)$$

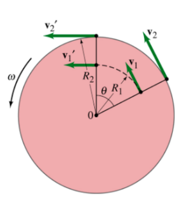
Rotational motion: variables.

- The linear velocity of a part of the rigid body is related to the angular velocity of the object.
- Consider point P:
  - It this point makes one complete revolution, it travels a distance $2\pi R$.
  - When the angular position changes by $d\theta$, point P moves a distance $dl = 2\pi R(d\theta/2\pi) = R d\theta$.
  - The linear velocity of point P is equal to $v = dl/dt = R d\theta/dt = R\omega$.



Rotational motion: variables.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:
  - Each part of the rigid object has the same angular velocity.
  - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
  - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



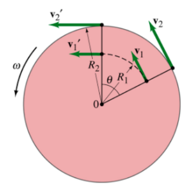
Relation between rotational and linear variables.

- Although in rotational motion we prefer to use rotational variables, we can also express the motion in terms of linear variables:

$$s = r\theta$$

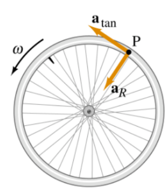
$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$



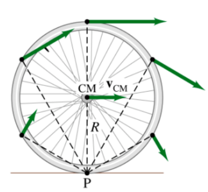
Rotational motion: acceleration.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P. The two components of the acceleration of point P are:
  - The radial component: this component is always present since point P carried out circular motion around the axis of rotation.
  - The tangential component: this component is present only when the angular acceleration is not equal to 0 rad/s^2.



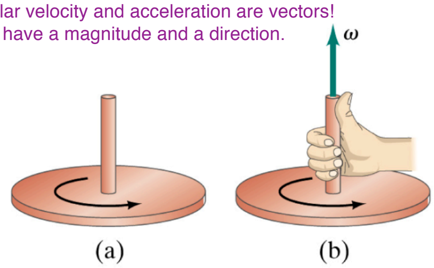
Rolling motion.

- To describe rolling motion we need to use both translational and rotational motion.
- The rolling motion can be described in terms of pure rotational motion with respect to the contact point P which is always at rest.
- The rotation axis around P is called the instantaneous axis and will move when the wheel rolls.



Direction of the angular velocity. Use your right hand!

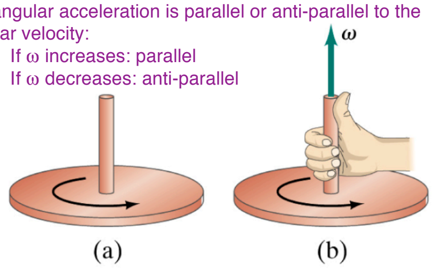
Angular velocity and acceleration are vectors!
They have a magnitude and a direction.



Direction of the angular acceleration.

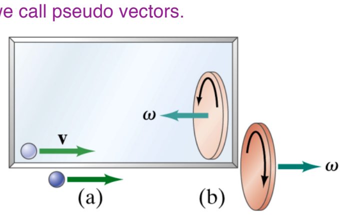
The angular acceleration is parallel or anti-parallel to the angular velocity:

- If ω increases: parallel
- If ω decreases: anti-parallel



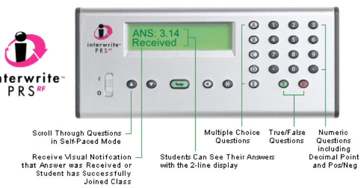
Pseudo vectors.

Angular velocity and accelerations are not real vectors! They do no behave as real vectors under reflection: they are what we call pseudo vectors.



Let's test your understanding of the basic concepts!

- Let's test our understanding of the basic aspects of rotational variables by working on the following concept problems:
  - Q15.1
  - Q15.2
  - Q15.3
  - Q15.4
  - Q15.5



Rotational kinetic energy.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2}\left\{\sum_i m_i r_i^2\right\}\omega^2$$

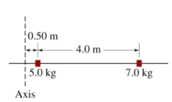
- The kinetic energy is proportional to the square of the rotational velocity ω. Note: the equation is similar to the translational kinetic energy (1/2 mv^2) except that instead of being proportional to the the mass m of the object , the rotational kinetic energy is proportional to the **moment of inertia *I*** of the object:

$$I = \sum_i m_i r_i^2 \quad \Rightarrow \quad K = \frac{1}{2} I\omega^2$$

Note: units of *I*: kg m^2

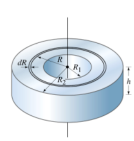
The moment of inertia. Calculating *I*.

- The moment of inertia of an objects depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:



$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

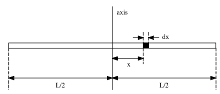


$$I = \int r^2 dm$$

Moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its center of mass?
- Consider a slice of the rod, with width dx, located a distance x from the rotation axis. The mass dm of this slice is equal to

$$dm = \frac{m}{L}dx$$

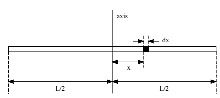


Moment of inertia. Sample problem.

- The moment of inertia dI of this slice is equal to

$$dI = x^2 dm = \frac{m}{L}x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:

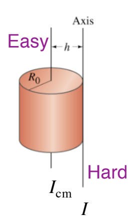


$$I = \int_{-L/2}^{L/2} dI = \int_{-L/2}^{L/2} \frac{m}{L}x^2 dx = \frac{m}{L}\left\{\frac{1}{3}\left(\frac{L}{2}\right)^3 - \frac{1}{3}\left(-\frac{L}{2}\right)^3\right\} = \frac{1}{12}mL^2$$

Moment of inertia. Parallel-axis theorem.

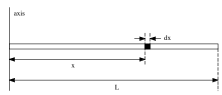
- Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:

$$I = I_{cm} + Mh^2$$



Moment of inertia. Sample problem.

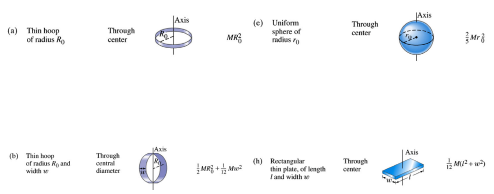
- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its left corner?



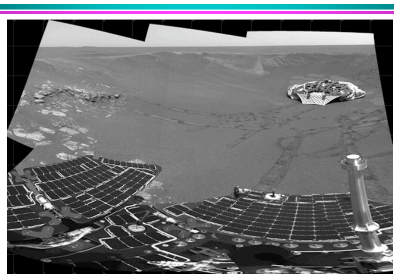
- We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:

$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$$

Moment of inertia. Do not memorize them! You will get Figure 10.20 on our exams!



Done for today! Much more about rotations on Thursday!



Opportunity rover indicates ancient Mars was wet.
Credit: Mars Exploration Rover Mission, JPL, NASA