Physics 121.
March 18, 2008.

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- Course Information
- Topics to be discussed today:
  - Variables used to describe rotational motion
  - The equations of motion for rotational motion

Course Announcements.

- Homework set # 6 is now available on the WEB and will be due on Saturday morning, March 22, at 8.30 am.

- All the material to be covered on Exam # 2 has now been discussed. Today we will start on material that will be covered on Exam # 3.

- Exam # 2 will take place on Tuesday March 25 at 8 am in Hubbell. It will cover the material discussed in Chapters 7, 8, and 9.

Physics 121.
Homework Set # 6.

Frank Wolfs — **Physics 121, Spring 2008**
Homework Set 06 — **Due date: 03/22/2008 at 08:30am EDT**

This assignment will be counted toward your final grade. You can attempt each problem 50 times; once you exceed this number of attempts, your solutions will not be recorded anymore. You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer. Note: to use scientific notion, use a notion like xxE+yy. It is important that you use a capital E; answers with a lower case e will be evaluated differently

Equations of motion with constant α.

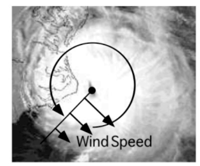
1. (20 pts) library/type19/prob05.pg
A pulsar is a rapidly rotating neutron star that emits radio pulses with precise synchronization, there being one such pulse for each rotation of the star. The period T of rotation is found by measuring the time between pulses. At present, the pulsar in the central region of the Crab nebula has a period of rotation of T = 0.03000000 s, and this is observed to be increasing at the rate of 0.00000146 s/yr. What is the angular velocity of the star? ____________
What is the angular acceleration of the pulsar? ____________
If its angular acceleration is constant, in how many years will the pulsar stop rotating? ____________
The pulsar originated in a super-nova explosion in the year A.D. 1054. What was the period of rotation of the pulsar when it was born? ____________

Equations of motion with constant α.

2. (20 pts) library/type19/prob02.pg
A wheel, starting from rest, has a constant angular acceleration of 0.3 rad/s^2. In a 1.8-s interval, it turns through an angle of 129 rad. How long has the wheel been in motion at the start of this 1.8-s interval? ____________

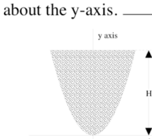
Rotational kinetic energy. Note: this is not a rigid body!

3. (20 pts) library/type20/prob01.pg
In a simple model of the wind speed associated with hurricane Emily, we assume there is calm eye 16.0 km in radius. The winds, which extend to a height of 7000 m, begin with a speed of 259.0 km/hr at the eye wall and decrease linearly with radial distance down to 0 km/hr at a distance of 101.0 km from the center. Assume the average density of the air from sea level to an altitude of 7000 m is 0.825 kg/m^3. Calculate the total kinetic energy of the winds. Note: To appreciate the hurricane's KE, compare your answer to the Hiroshima atomic bomb which had an energy equivalent to about 15,000 tons of TNT, representing an energy of about 6.00e+13 J. ____________

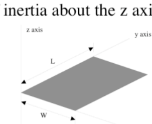


Calculating the moment of inertia

4. (20 pts) library/type20/prob02.pg
A uniform plate of height H = 0.79 m is cut in the form of a parabolic section. The lower boundary of the plate is defined by: y = 0.90 x^2. The plate has a mass of 3.57 kg. Find the moment of inertia of the plate about the y-axis. ____________



5. (20 pts) library/type20/prob04.pg
The surface density of a thin rectangle varies as:
$$\sigma(x,y) = (18kg/m^2) + (2kg/m^4)(x^2+y^2)$$
The rectangle has a length L = 0.75 m and a width W = 1.25 m.
What is moment of inertia about the z axis? ____________

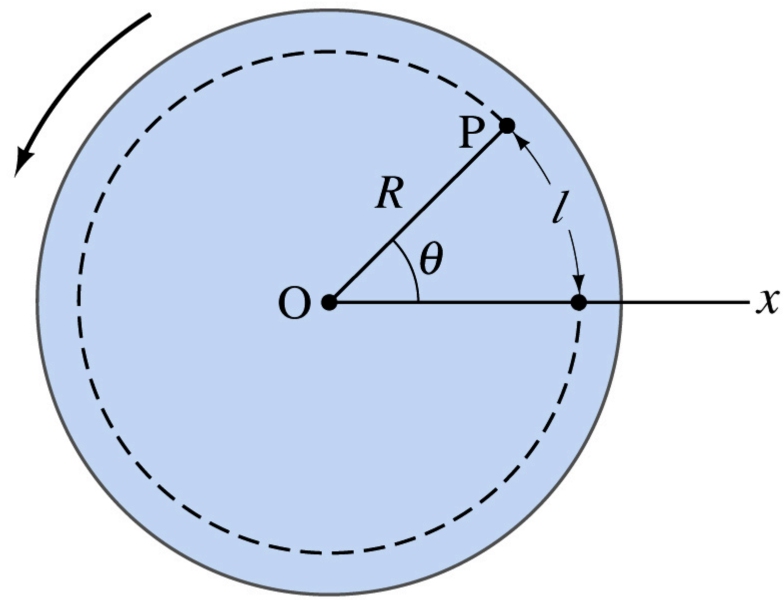


acceleration

Rotational motion: variables.

- In our discussion of rotational motion we will first focus on the rotation of **rigid** objects around a **fixed** axis.

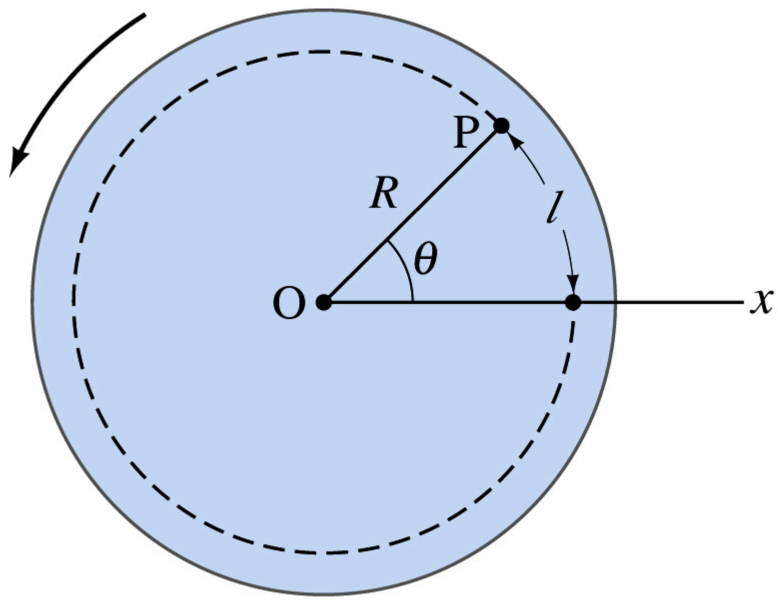
- The variables that are used to describe this type of motion are similar to those we use to describe linear motion:
    - Angular position
    - Angular velocity
    - Angular acceleration



Rotational motion: variables.

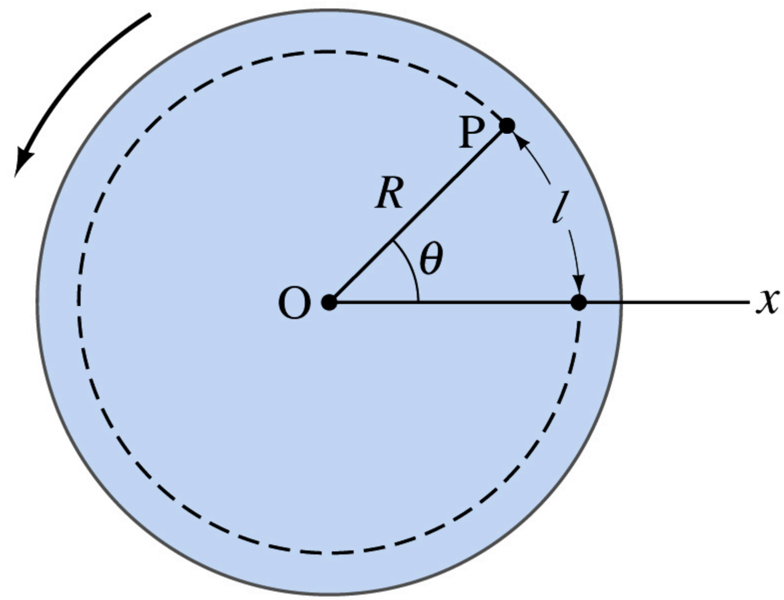
- The angular position, measured in radians, is the angle of rotation of the object with respect to a reference position.

- The angular position of point P at this point in time is equal to θ. In order to uniquely define this position, we have assume that an the angular position is measured with respect to the x axis.

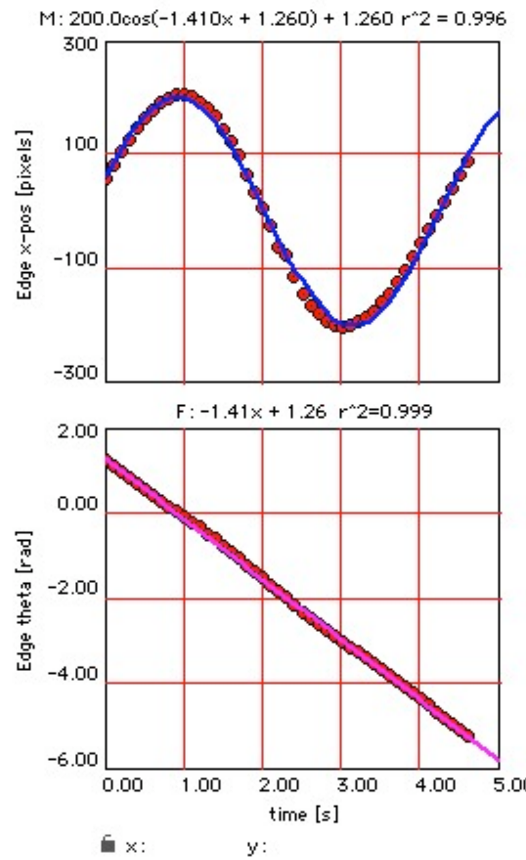


Rotational motion: variables.

- Note:
  - The angular position is always specified in radians!!!!
  - One radian is the angular displacement corresponding to a linear displacement $l = R$.
  - Make sure you keep track of the sign of the angular position!!!!!
  - An increase in the angular position corresponds to a counter-clockwise rotation; a decrease corresponds to a clockwise rotation.

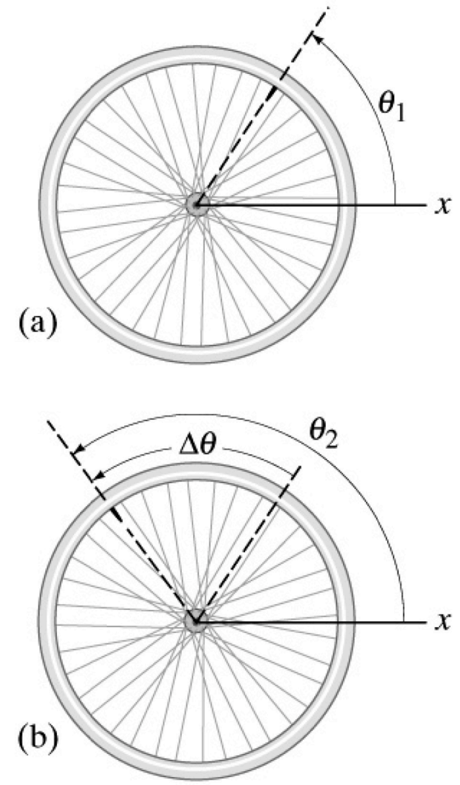


Complex motion in Cartesian coordinates is simple motion in rotational coordinates.



Rotational motion: variables.

- If we look at an object carrying out a rotation around a fixed axis, we will see that the angular position becomes a function of time.
- To describe the rotational motion, we introduce the concepts of angular velocity and angular acceleration.
- **Remember**: for linear motion we found it useful to introduce the concepts of linear velocity and linear acceleration.



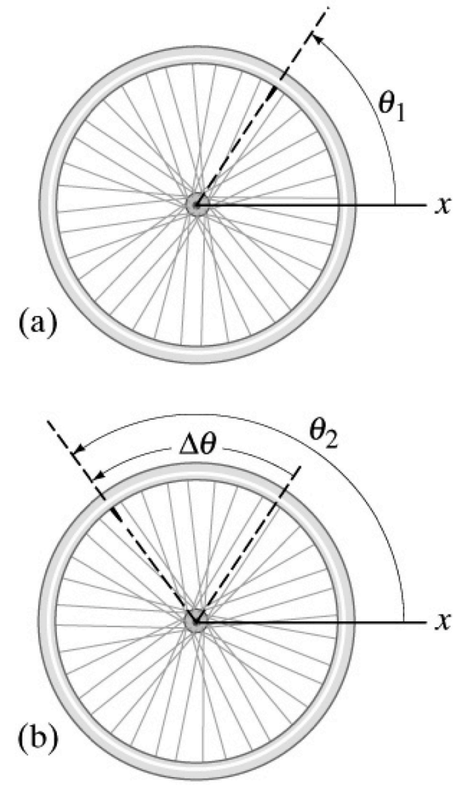
Angular acceleration (rad/s²)

Rotational motion: variables.

- For both velocity and acceleration we can talk about instantaneous and average.

- Angular velocity:
  - Definition: $\omega = \mathrm{d}\theta/\mathrm{d}t$
  - Symbol: ω
  - Units: rad/s

- Angular acceleration:
  - Definition: $\alpha = \mathrm{d}\omega/\mathrm{d}t = \mathrm{d}^2\theta/\mathrm{d}t^2$
  - Symbol: α
  - Units: rad/s^2



Angular acceleration (rad/s^2)

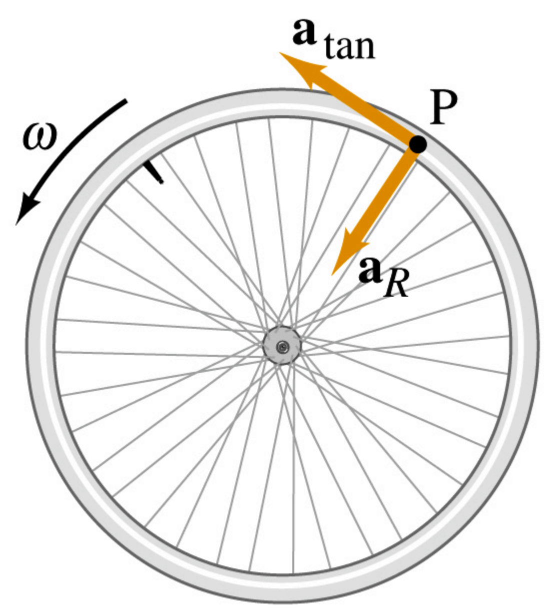
Rotational motion: constant acceleration.

- If the object experiences a constant angular acceleration, then we can describe its rotational motion with the following equations of motion:

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

- Note how similar these equations are to the equation of motion for linear motion!!!!!!



Rotational motion: constant acceleration. Example problem.

- A wheel starting from rest, rotates with a constant angular acceleration of 2.0 rad/s^2. During a certain 3.0 s interval it turns through 90 rad. (a) How long had the wheel been turning before the start of the 3.0 s interval ? (b). What was the angular velocity of the wheel at the start of the 3.0 s interval ?

- Define time $t = 0$ s as the time that the wheel is at rest. The angular velocity and the angle of rotation at a later time t are given by

$$\omega(t) = \alpha t \qquad\qquad \theta(t) = \frac{1}{2}\alpha t^2$$

Rotational motion: constant acceleration. Example problem.

- The change in the angular position $\Delta\theta$ during a time period Δt can now be calculated:

$$\Delta\theta(t)=\theta(t+\Delta t)-\theta(t)=\frac{1}{2}\alpha(t+\Delta t)^2-\frac{1}{2}\alpha t^2=\frac{1}{2}\alpha(\Delta t)^2+\alpha t\Delta t$$

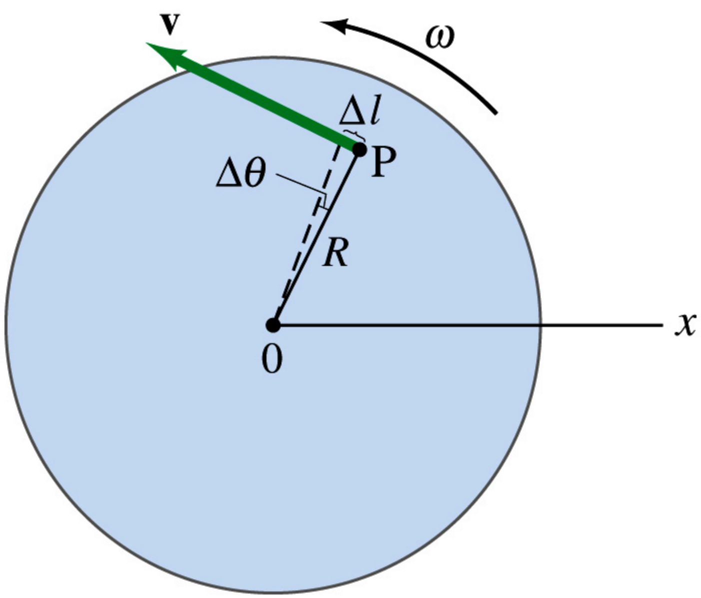
- Since the problem specifies $\Delta\theta$ and Δt we can now calculate the time t and the angular velocity at that time:

$$t=\frac{\Delta\theta-\frac{1}{2}\alpha(\Delta t)^2}{\alpha\Delta t} \qquad \omega=\alpha t=\frac{\Delta\theta-\frac{1}{2}\alpha(\Delta t)^2}{\Delta t}=\frac{\Delta\theta}{\Delta t}-\frac{1}{2}\alpha(\Delta t)$$

Rotational motion: variables.

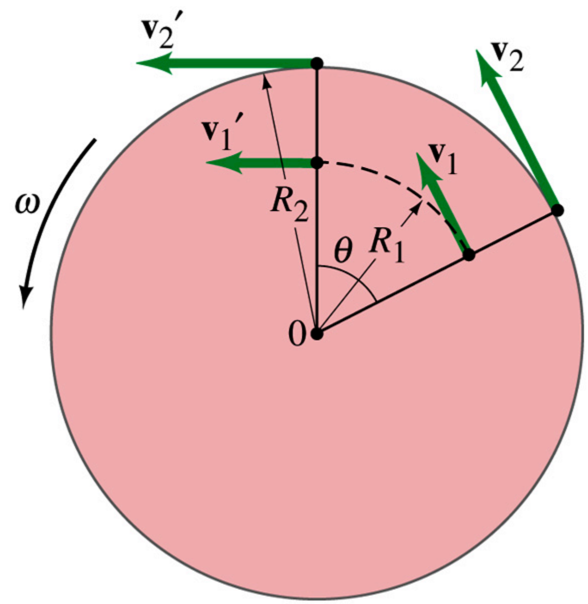
- The linear velocity of a part of the rigid body is related to the angular velocity of the object.

- Consider point P:
  - It this point makes one complete revolution, it travels a distance $2\pi R$.
  - When the angular position changes by $d\theta$, point P moves a distance $dl = 2\pi R(d\theta/2\pi) = R d\theta$.
  - The linear velocity of point P is equal to $v = dl/dt = R d\theta/dt = R\omega$.



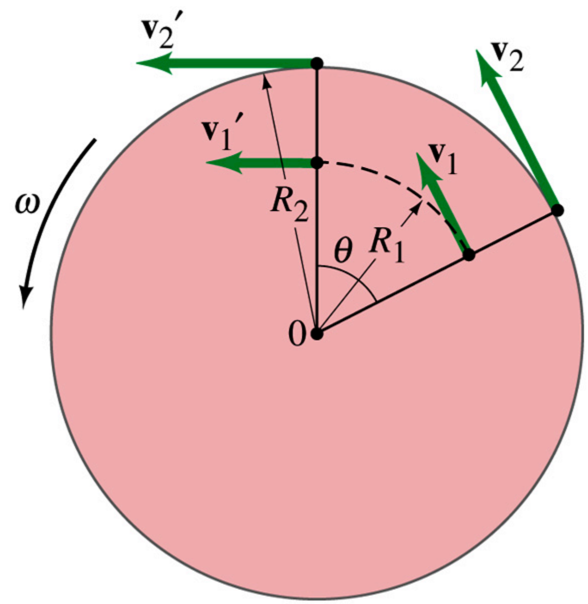
Rotational motion: variables.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:
  - Each part of the rigid object has the same angular velocity.
  - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
  - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



Relation between rotational and linear variables.

- Although in rotational motion we prefer to use rotational variables, we can also express the motion in terms of linear variables:



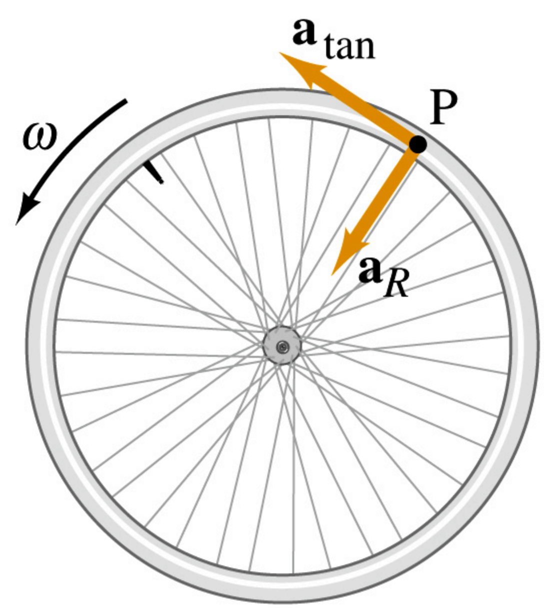
$$s = r\theta$$

$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

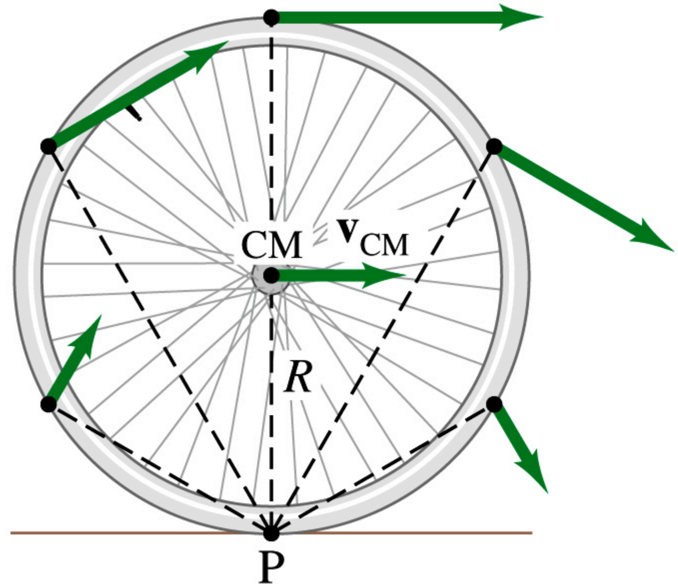
Rotational motion: acceleration.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P. The two components of the acceleration of point P are:
  - The radial component: this component is always present since point P carried out circular motion around the axis of rotation.
  - The tangential component: this component is present only when the angular acceleration is not equal to 0 rad/s^2.



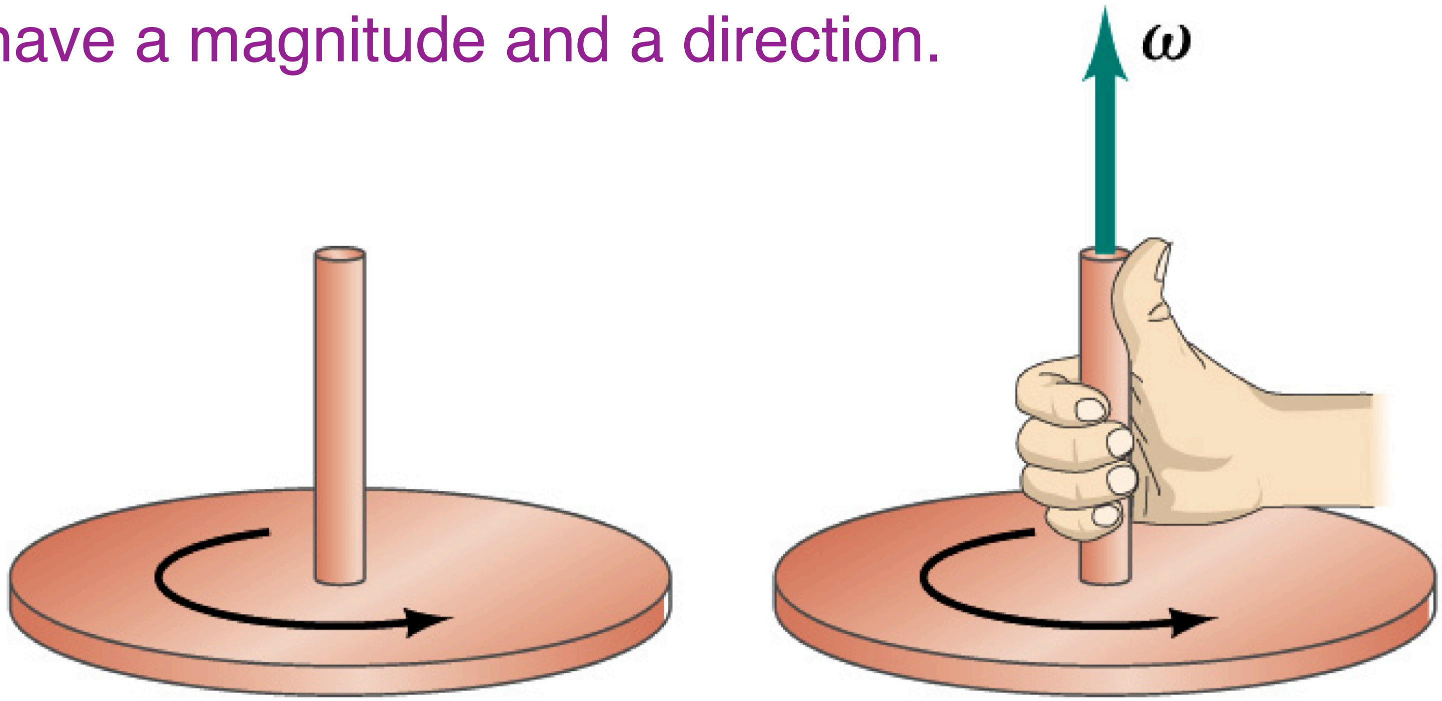
Rolling motion.

- To describe rolling motion we need to use both translational and rotational motion.
- The rolling motion can be described in terms of pure rotational motion with respect to the contact point P which is always at rest.
- The rotation axis around P is called the instantaneous axis and will move when the wheel rolls.



Direction of the angular velocity. Use your right hand!

Angular velocity and acceleration are vectors!
They have a magnitude and a direction.



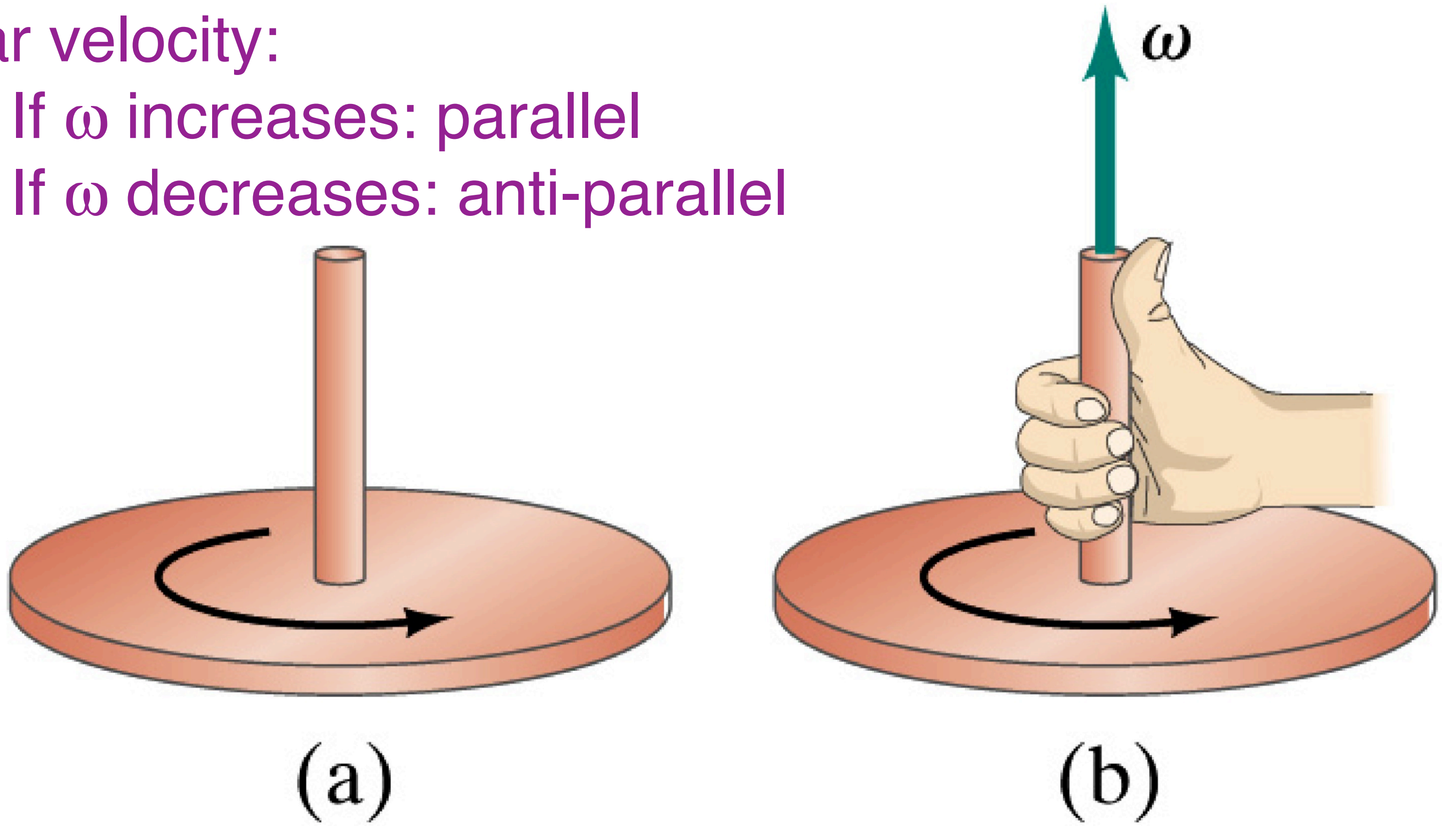
(a) (b)

Direction of the angular acceleration.

The angular acceleration is parallel or anti-parallel to the angular velocity:

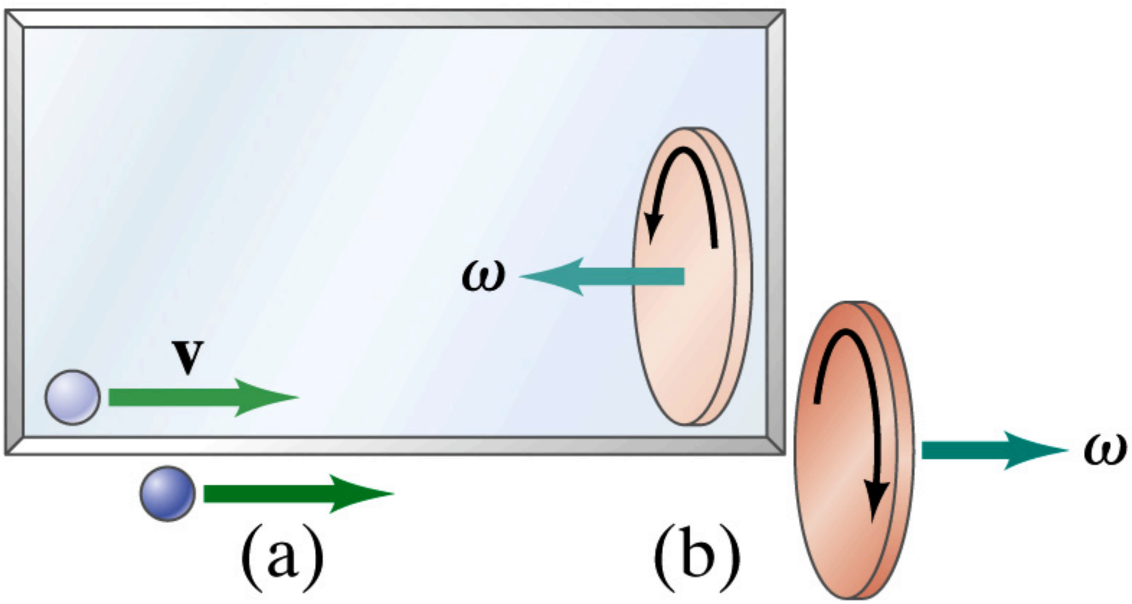
If ω increases: parallel

If ω decreases: anti-parallel



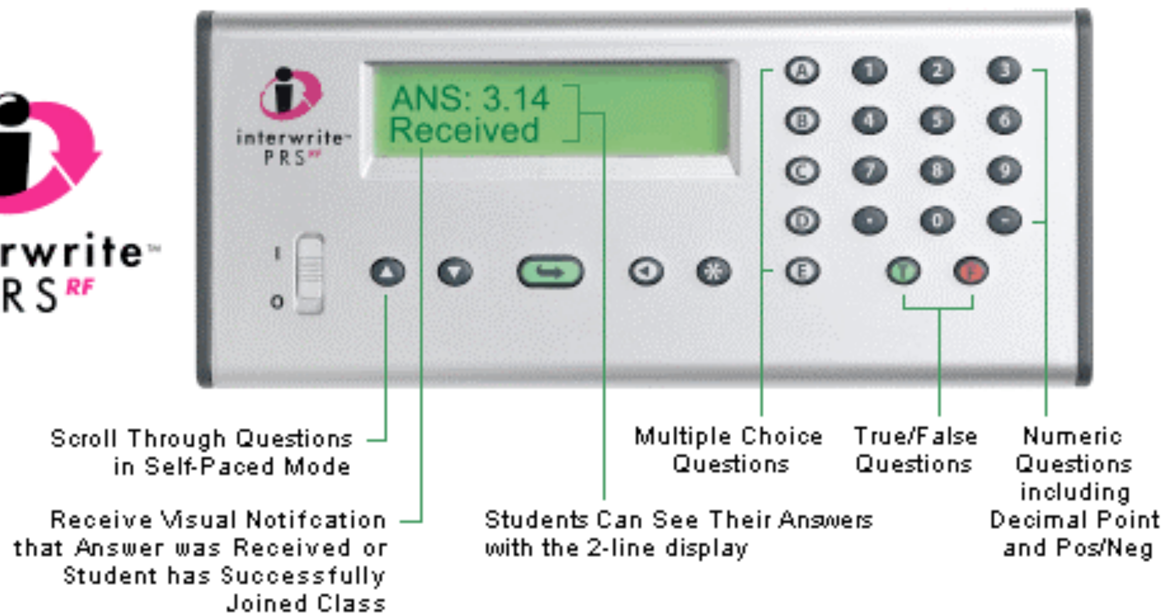
Pseudo vectors.

Angular velocity and accelerations are not real vectors! They do no behave as real vectors under reflection: they are what we call pseudo vectors.



Let’s test your understanding of the basic concepts!

- Let’s test our understanding of the basic aspects of rotational variables by working on the following concept problems:
  - Q15.1
  - Q15.2
  - Q15.3
  - Q15.4
  - Q15.5



Rotational kinetic energy.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i \left(\omega r_i\right)^2 = \frac{1}{2}\left\{\sum_i m_i r_i^2\right\}\omega^2$$

- The kinetic energy is proportional to the square of the rotational velocity ω. Note: the equation is similar to the translational kinetic energy (1/2 mv^2) except that instead of being proportional to the the mass m of the object , the rotational kinetic energy is proportional to the **moment of inertia *I*** of the object:

$$I = \sum_i m_i r_i^2 \quad \Rightarrow \quad K = \frac{1}{2} I \omega^2$$

Note: units of *I*: kg m^2

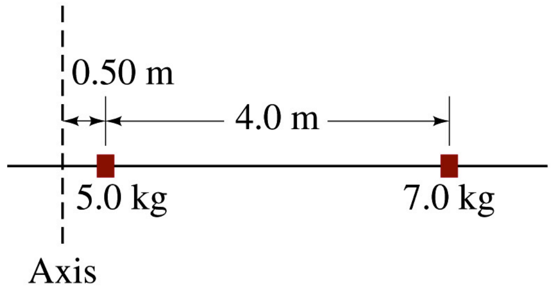
The moment of inertia. Calculating I.

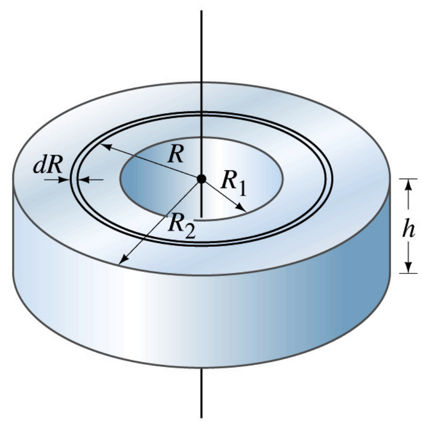
- The moment of inertia of an objects depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$



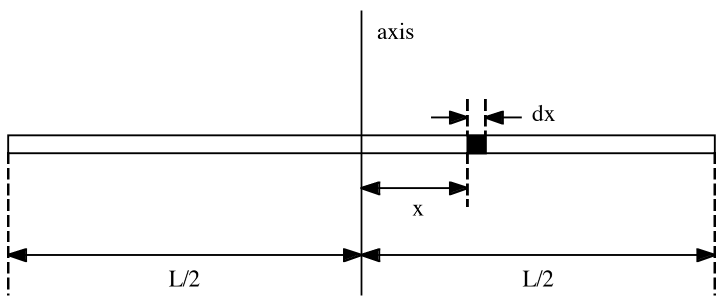


Moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its center of mass?

- Consider a slice of the rod, with width dx, located a distance x from the rotation axis. The mass dm of this slice is equal to

$$dm = \frac{m}{L} dx$$

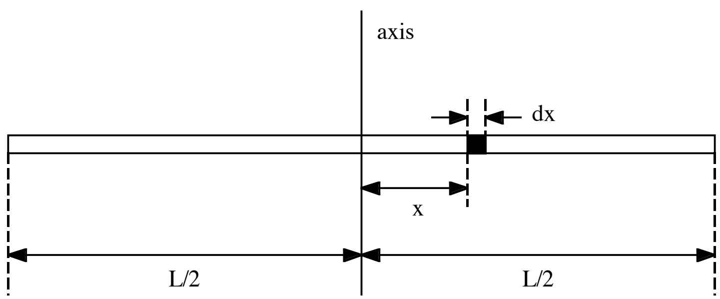


Moment of inertia. Sample problem.

- The moment of inertia dI of this slice is equal to

$$dI = x^2 dm = \frac{m}{L} x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:

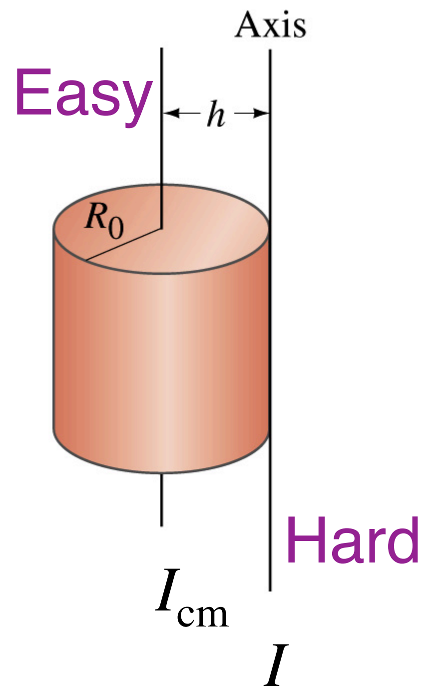


$$I = \int_{-L/2}^{L/2} dI = \int_{-L/2}^{L/2} \frac{m}{L} x^2 dx = \frac{m}{L}\left\{\frac{1}{3}\left(\frac{L}{2}\right)^3 - \frac{1}{3}\left(-\frac{L}{2}\right)^3\right\} = \frac{1}{12} mL^2$$

Moment of inertia. Parallel-axis theorem.

- Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:

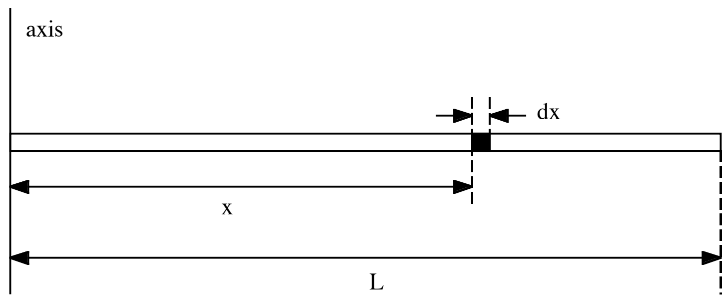
$$I = I_{cm} + Mh^2$$



Moment of inertia.
Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its left corner?

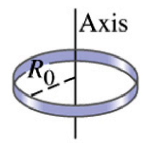
- We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:



$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$$

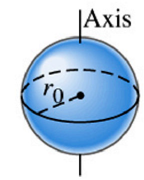
Moment of inertia. Do not memorize them! You will get Figure 10.20 on our exams!

(a) Thin hoop of radius R_0 — Through center



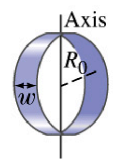
MR_0^2

(e) Uniform sphere of radius r_0 — Through center



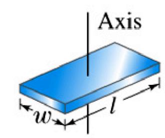
$\frac{2}{5}Mr_0^2$

(b) Thin hoop of radius R_0 and width w — Through central diameter



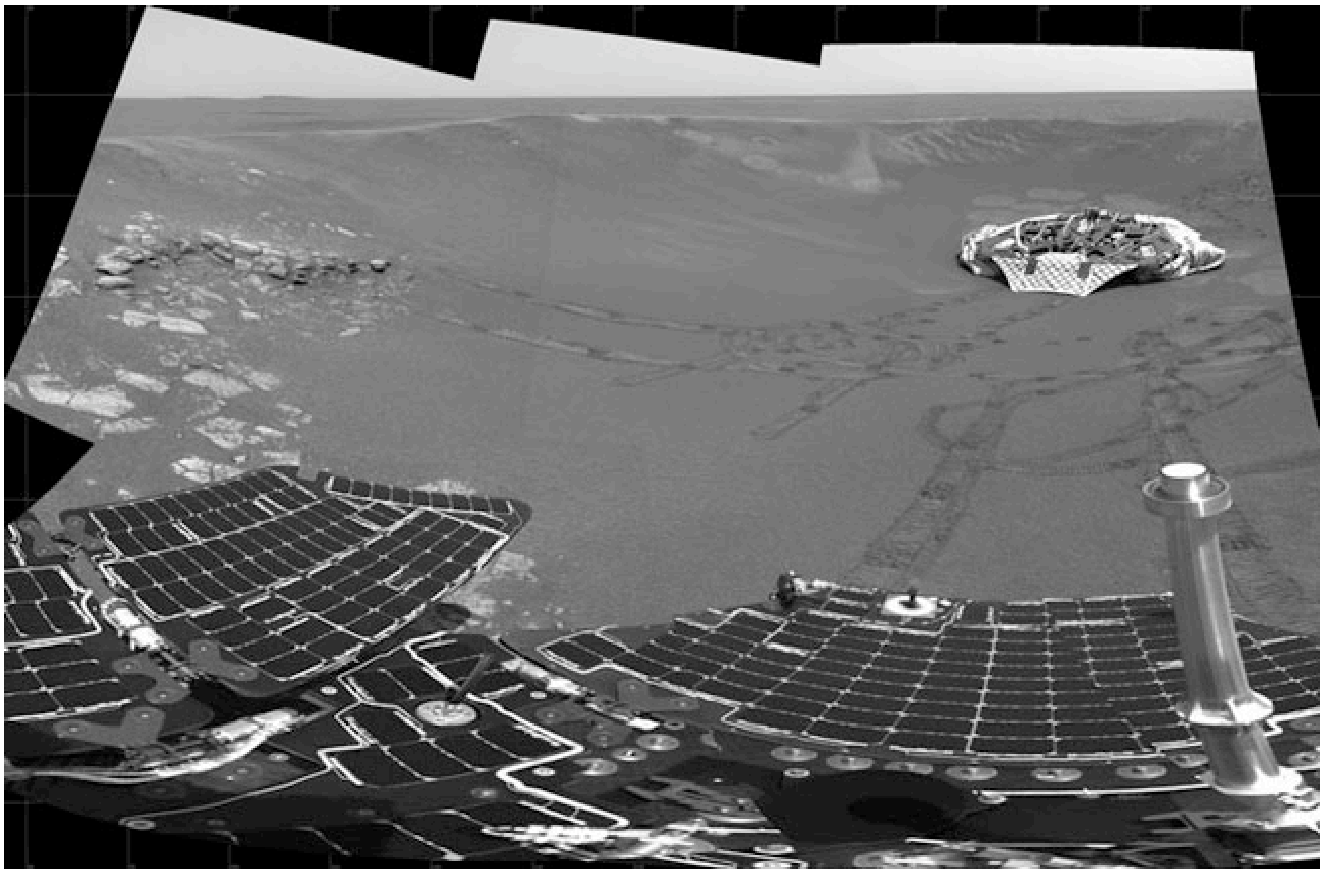
$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$

(h) Rectangular thin plate, of length l and width w — Through center



$\frac{1}{12}M(l^2 + w^2)$

Done for today!
Much more about rotations on Thursday!



Opportunity rover indicates ancient Mars was wet.
Credit: Mars Exploration Rover Mission, JPL, NASA