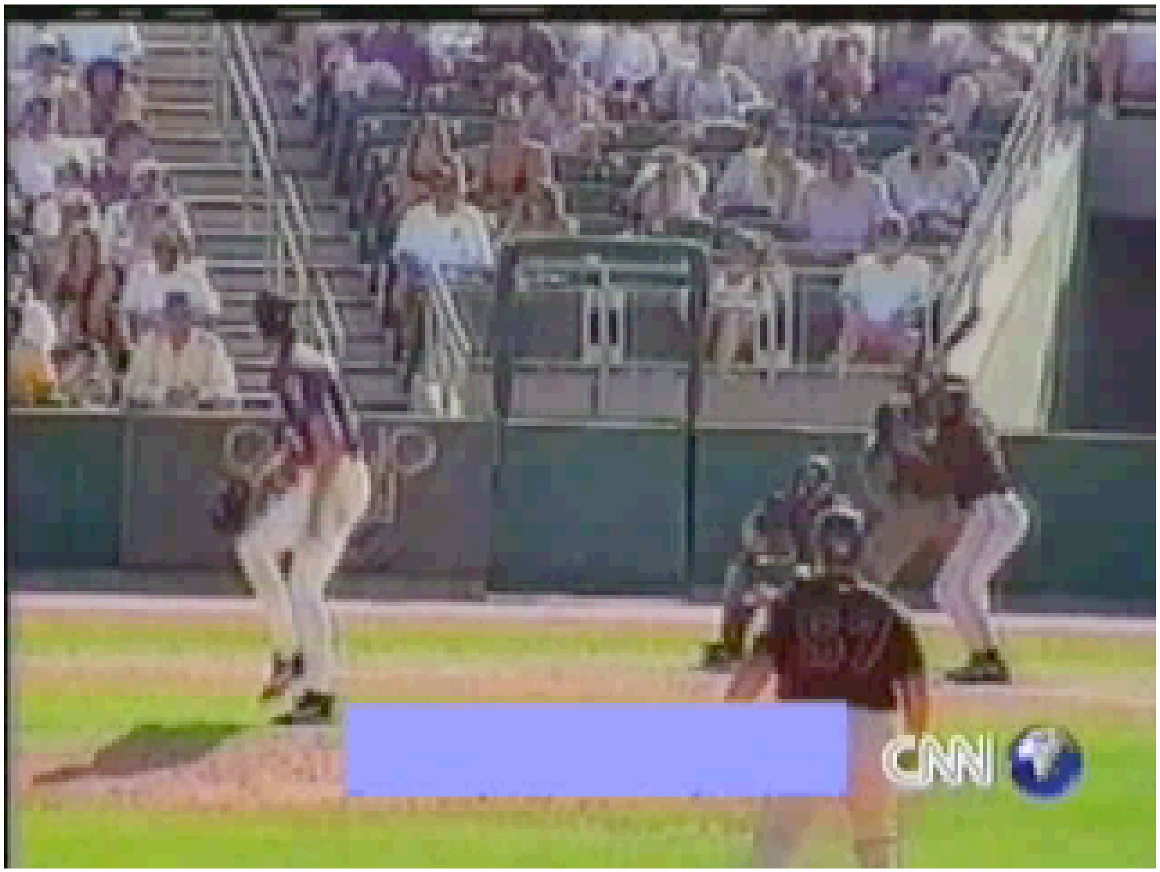
Physics 121.
Thursday, March 6, 2008.



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Physics 121.
Thursday, March 6, 2008.

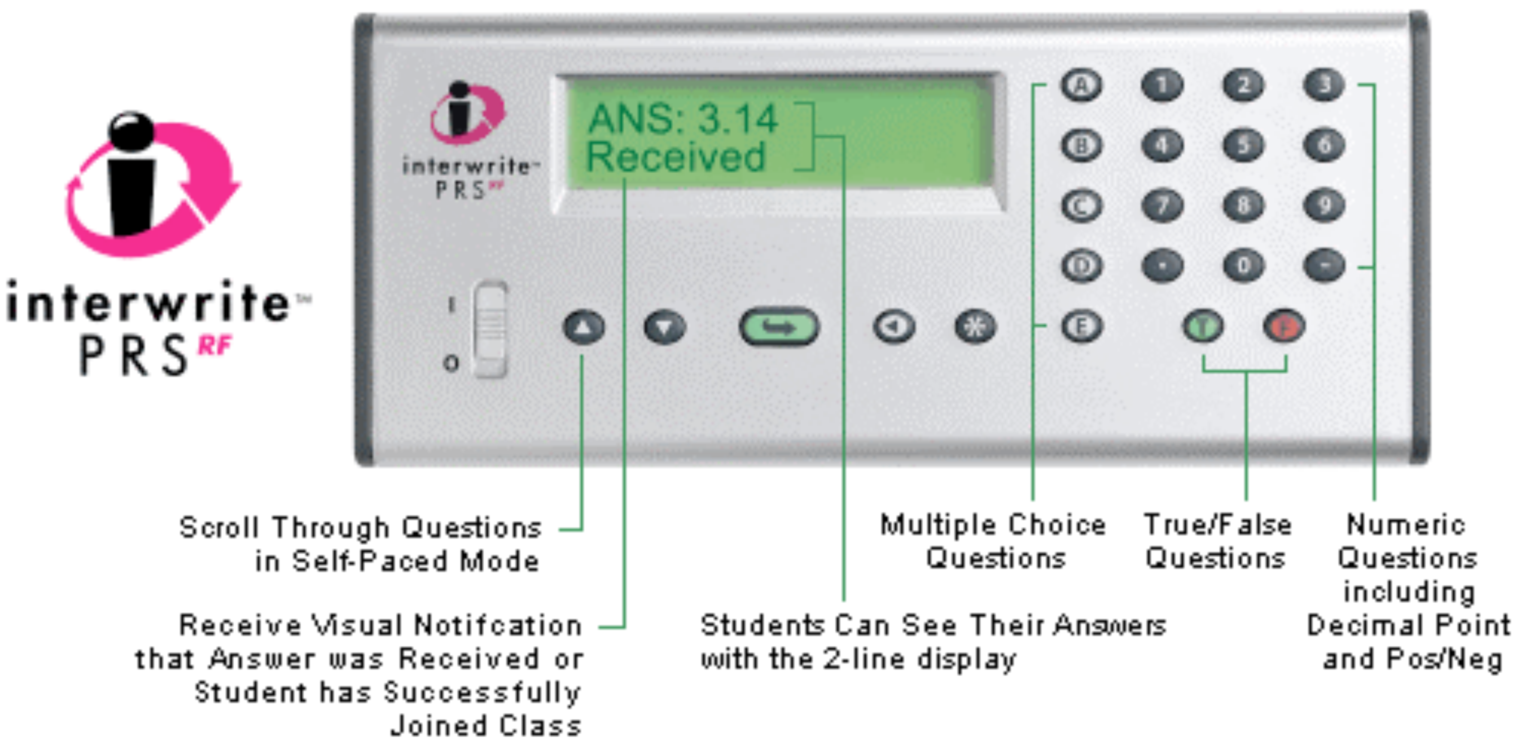
- Course Information
- Quiz
- Topics to be discussed today:
  - Conservation of linear momentum and one-dimensional collisions (a brief review)
  - Two-dimensional collisions (elastic and inelastic)

Course Information.

- Homework set # 5 is due on Saturday morning, March 8, at 8.30 am.
- Homework set # 6 will be available on Saturday morning, March 8, and will be due on Saturday March 22 at 8.30 am.
- I will have office hours today between 11.30 am and 1.30 pm in B&L 203A.
- Access to the Physics 121 webpages and the WeBWorK server will be intermittent during spring break due to hardware and software upgrades that will be carried out.

Physics 121.
Quiz lecture 14.

- The quiz today will have 4 questions!



Linear momentum (a quick review).

- The product of the mass and velocity of an object is called the **linear momentum p** of that object.
- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

- The change in the linear momentum of the system can now be calculated:

$$\frac{d\vec{P}_{cm}}{dt} = \frac{d}{dt}\left(M\vec{v}_{cm}\right) = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- This relations shows us that if there are no external forces, the total linear momentum of the system will be constant (independent of time).

Linear momentum (a quick review). Systems with variable mass.

- The first Rocket equation:

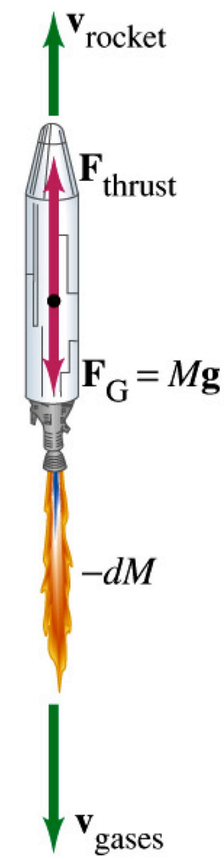
$$RU_0 = Ma_{\text{rocket}}$$

where
  - R = -dM/dt is the rate of fuel consumption.
  - U_0 = -u where u is the (positive) velocity of the exhaust gasses relative to the rocket.

This equation can be used to determine the rate of fuel consumption required for a specific acceleration.

- The **second rocket equation**:

$$v_f = v_i + u \ln\left(\frac{M_i}{M_f}\right)$$

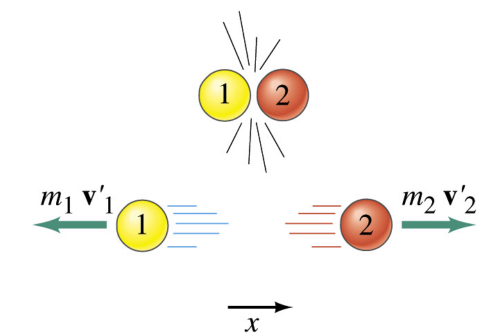


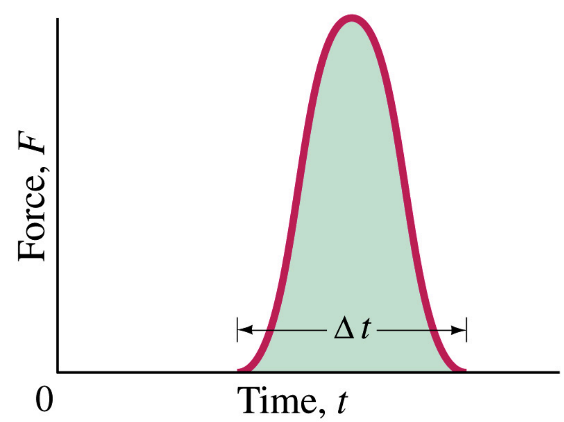
Collisions (a quick review). The collision force.

- During a collision, a strong force is exerted on the colliding objects for a short period of time.
- The collision force is usually much stronger then any external force.
- The result of the collision force is a change in the linear momentum of the colliding objects.
- The change in the momentum of one of the objects is equal to

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$$

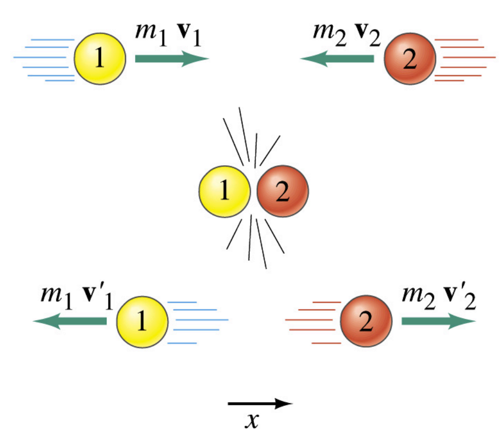
- This change is **collision impulse**.

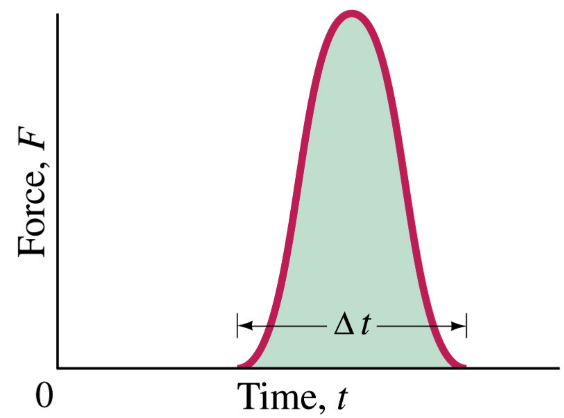




Elastic and inelastic collisions (a quick review).

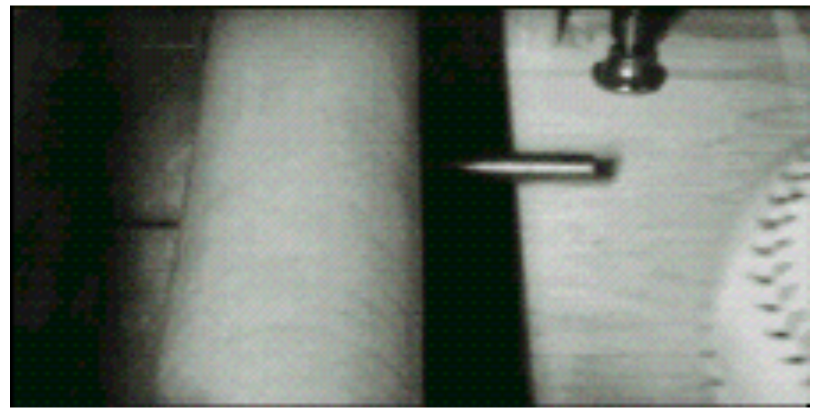
- If we consider both colliding object, then the collision force becomes an internal force and the total linear momentum of the system must be conserved if there are no external forces acting on the system.
- Collisions are usually divided into two groups:
  - Elastic collisions: **kinetic energy is conserved**.
  - Inelastic collisions: **kinetic energy is NOT conserved**.

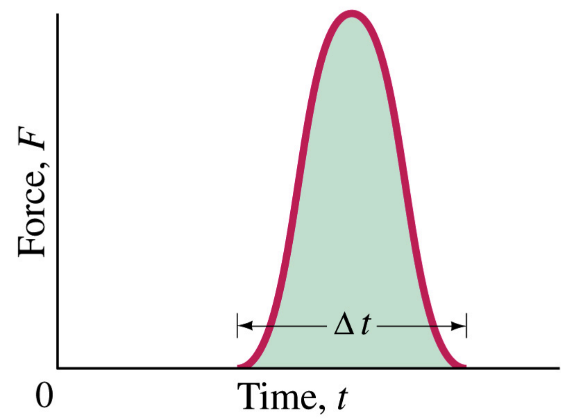




Collisions (a quick review).

- Kinetic energy does not need to be conserved during the time period that the collision force is acting on the system. The kinetic energy may even become 0 for a short period of time.
- During the time period that the collision force is non-zero, some or all of the initial kinetic energy may be converted into potential energy (for example, the potential energy associated with deformation).





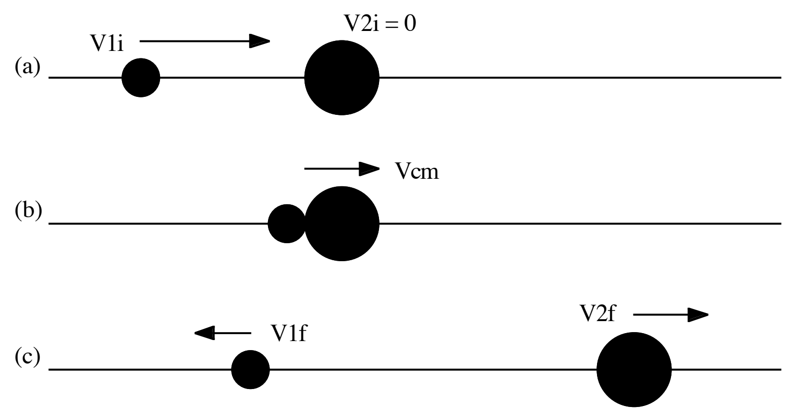
Elastic collisions in one dimension (a quick review).

- The solution for the final velocity of mass m_1 is:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

- The solution for the final velocity of mass m_2 is:

$$v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) = \frac{2m_1}{m_1 + m_2} v_{1i}$$

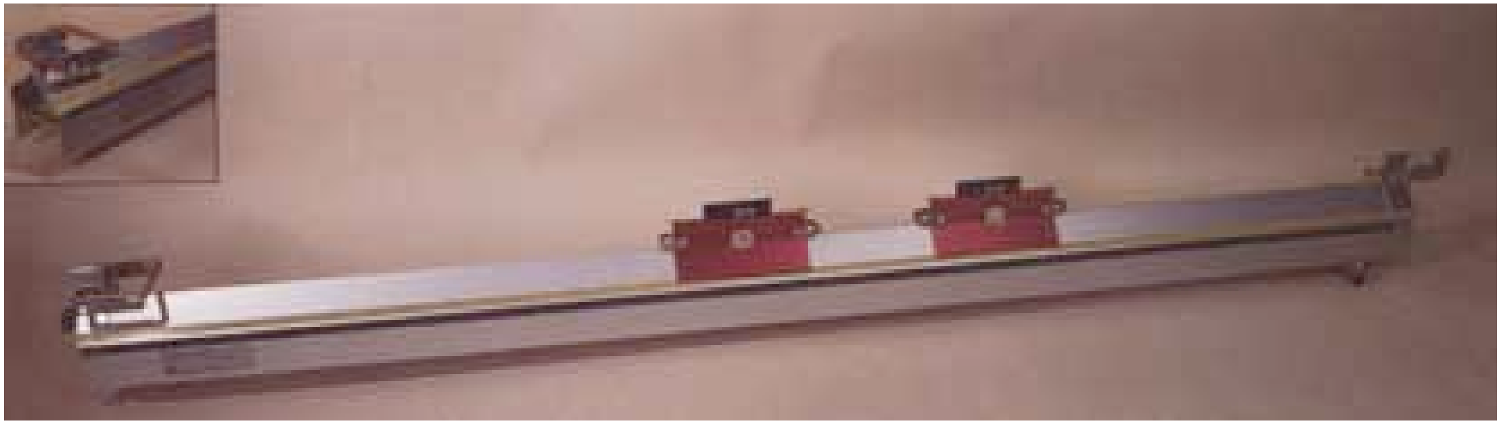


Elastic collisions in one dimension (a quick review).

- Let us focus on specific examples where one cart (cart 2) is at rest:
  - $m_1 = m_2$: $v_{1f} = 0$ m/s, $v_{2f} = v_{1i}$
  - $m_1 = 2m_2$: $v_{1f} = (1/3)v_{1i}$, $v_{2f} = (4/3)\, v_{1i}$
  - $2m_1 = m_2$: $v_{1f} = -(1/3)v_{1i}$, $v_{2f} = (2/3)\, v_{1i}$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

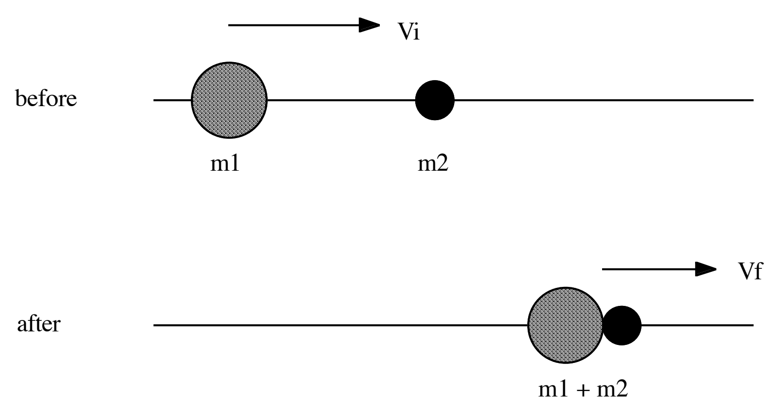
$$v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) = \frac{2m_1}{m_1 + m_2} v_{1i}$$



TEL-Atomic, http://www.telatomic.com/at.html

Inelastic collisions in one dimension (a quick review).

- In inelastic collisions, kinetic energy is not conserved.
- A special type of inelastic collisions are the completely inelastic collisions, where the two objects stick together after the collision.
- Conservation of linear momentum in a completely inelastic collision requires that

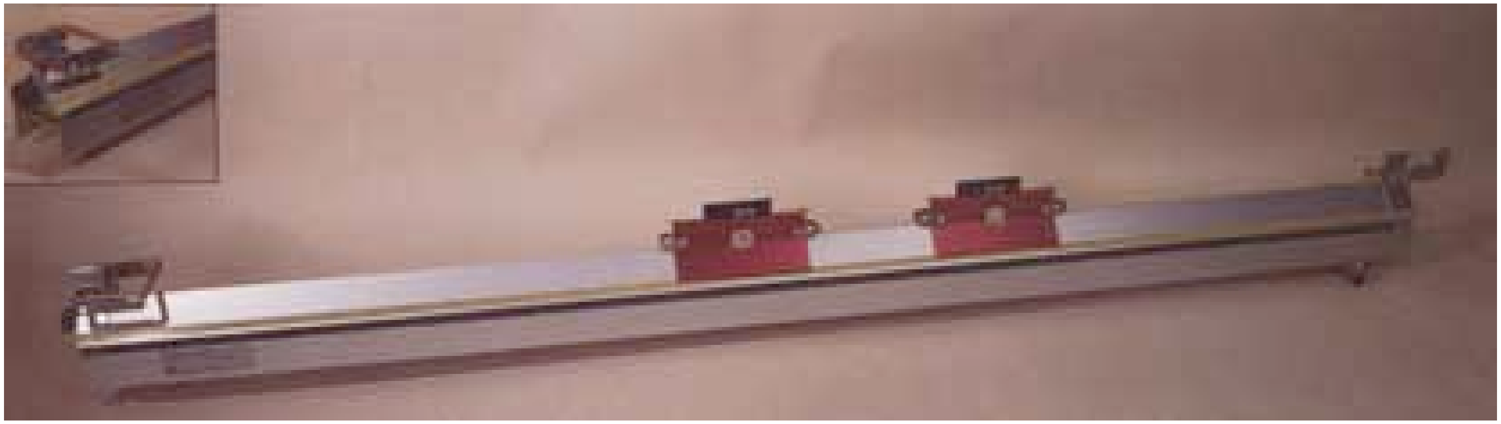


$$m_1 v_\mathrm{i} = (m_1 + m_2) v_\mathrm{f} \longrightarrow v_f = \frac{m_1}{m_1 + m_2} v_i$$

Inelastic collisions in one dimension (a quick review).

- Let us focus on one specific example of a procedure to measure the velocity of a bullet:
  - We shoot a 0.3 g bullet into a cart
  - The final velocity of the cart is measured and conservation of linear momentum can be used to determine the velocity of the bullet:

$$V_i = (m_1 + m_2)v_f/m_1$$



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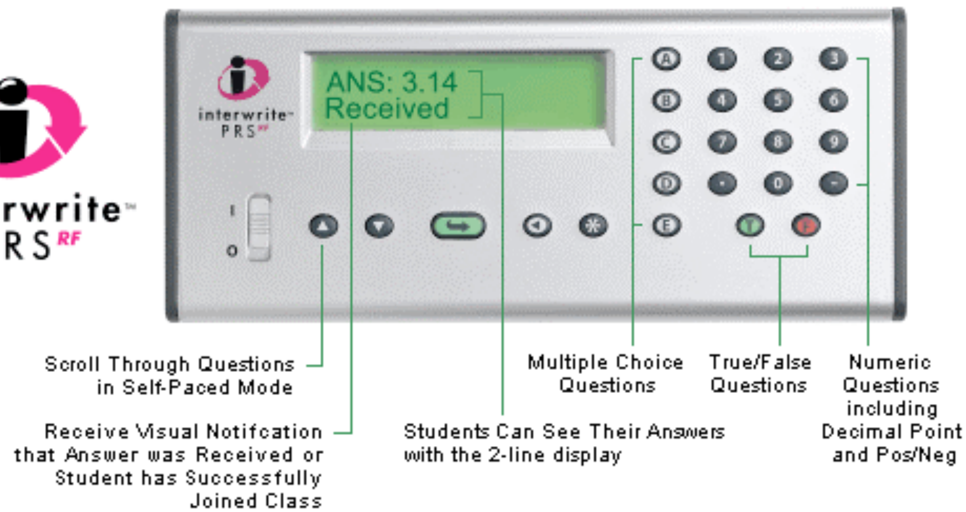
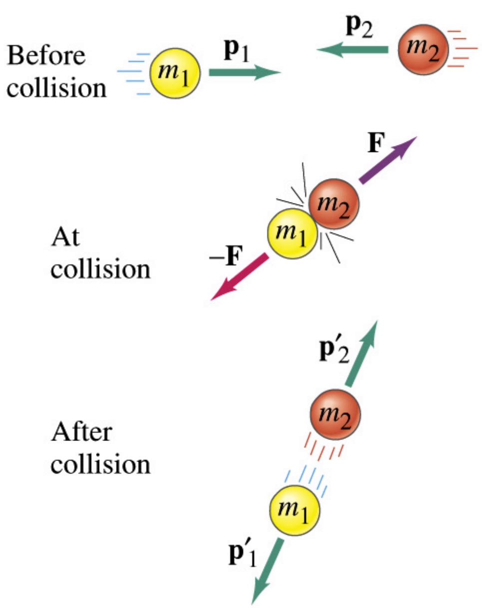
Collisions in 1D.
A review.

- Let's test our understanding of the concepts of linear momentum and collisions by working on the following concept problems:
  - Q14.1
  - Q14.2

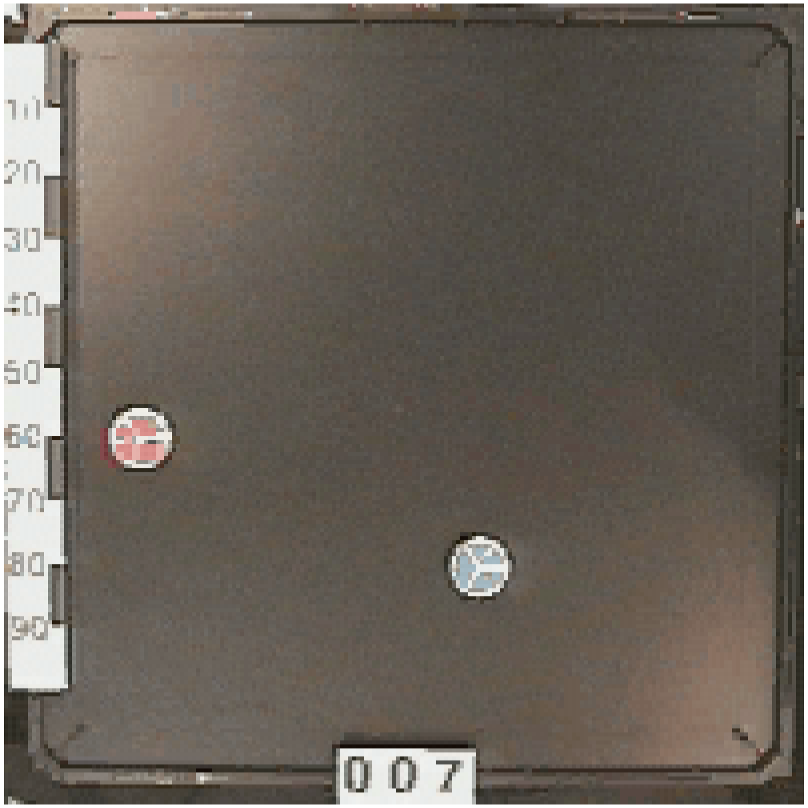


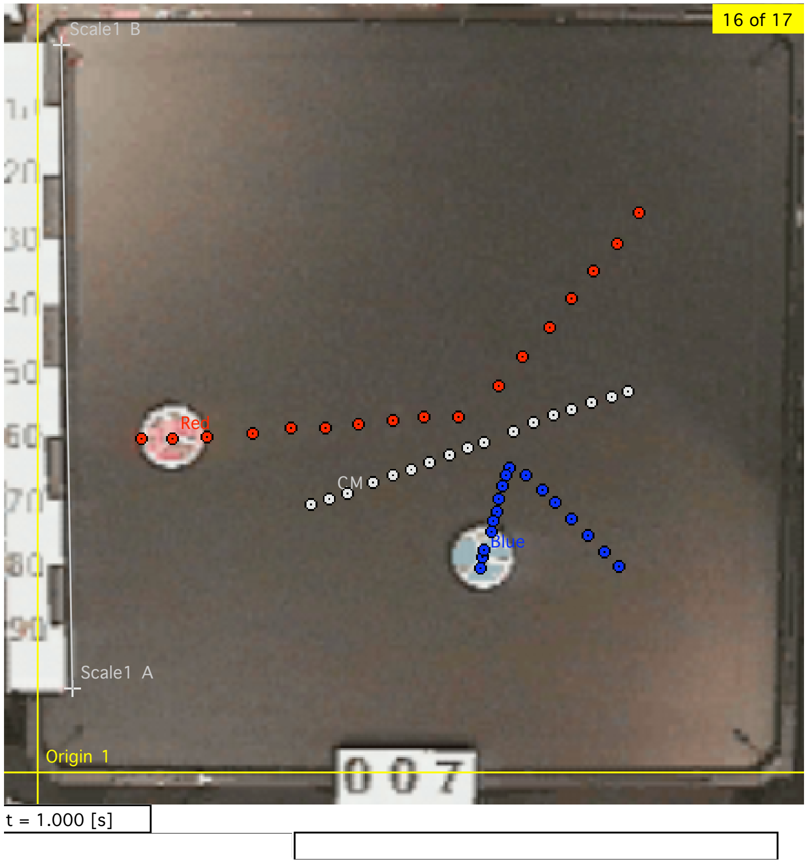
Collisions in two or three dimensions.

- Collisions in two or three dimensions are approached in the same way as collisions in one dimension.
- The x, y, and z components of the linear momentum must be conserved if there are no external forces acting on the system.
- The collisions can be elastic or inelastic.



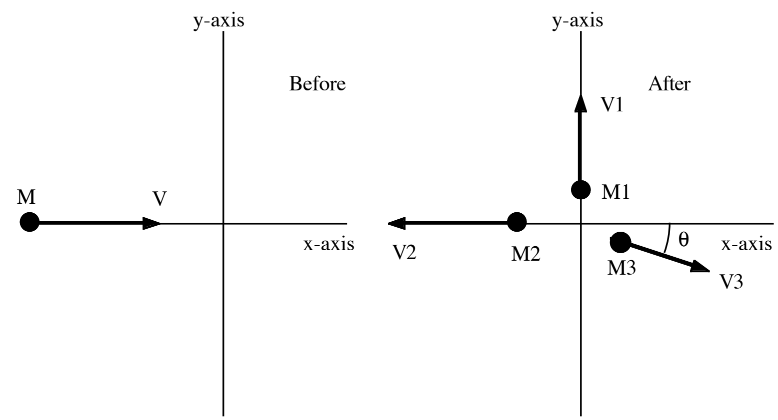
Collisions in two dimensions.
Elastic Collisions.





Collisions in two or three dimensions. Example problem.

- A 20-kg body is moving in the direction of the positive x-axis with a speed of 200 m/s when, owing to an internal explosion, it breaks into three parts. One part, whose mass is 10 kg, moves away from the point of explosion with a speed of 100 m/s along the positive y-axis. A second fragment, with a mass of 4 kg, moves along the negative x-axis with a speed of 500 m/s.
  - What is the speed of the third (6 kg) fragment ?
  - How much energy was released in the explosion (ignore gravity)?



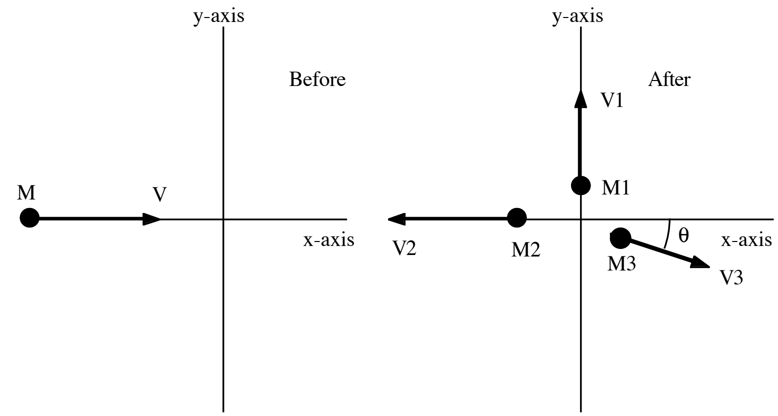
Collisions in two or three dimensions. Example problem.

- There are no external forces and linear momentum must thus be conserved.
- Conservation of linear momentum along the *x* axis requires

$$MV = m_3 v_3 \cos\theta_3 - m_2 v_2$$

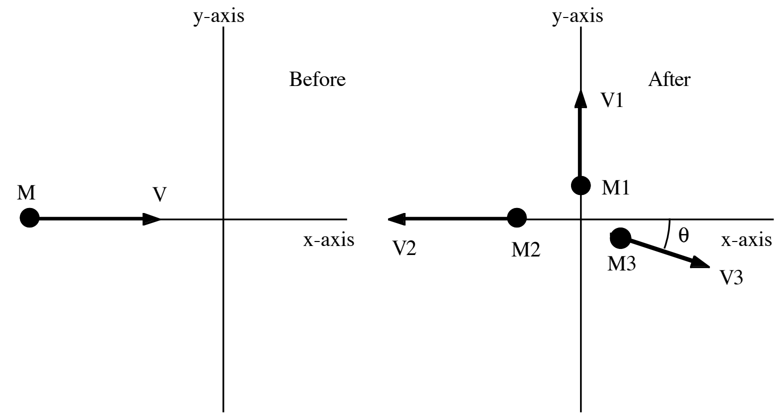
- Conservation of linear momentum along the *y* axis requires

$$0 = m_1 v_1 - m_3 v_3 \sin\theta_3$$



Collisions in two or three dimensions. Example problem.

- What do we know:
  - Speed and direction of mass *M*
  - Speed and direction of mass 1
  - Speed and direction of mass 2
- What do we need to know:
  - Speed and direction of mass 3
- Since we have two equations with two unknown, we can find the speed and direction of mass 3.
- Once we know the speed of mass 3 we can calculate the amount of energy released.



Collisions in two or three dimensions. Example problem.

- Our two equations can be rewritten as

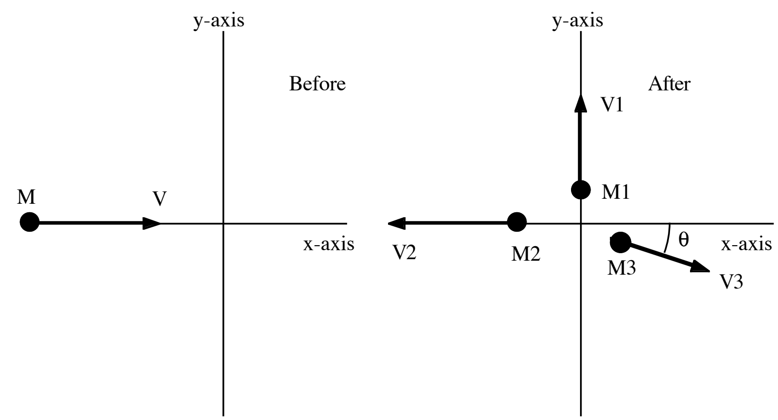
$$m_3 v_3 \cos\theta_3 = MV + m_2 v_2$$

$$m_3 v_3 \sin\theta_3 = m_1 v_1$$

- We can solve this equation by squaring each equation and adding them together:

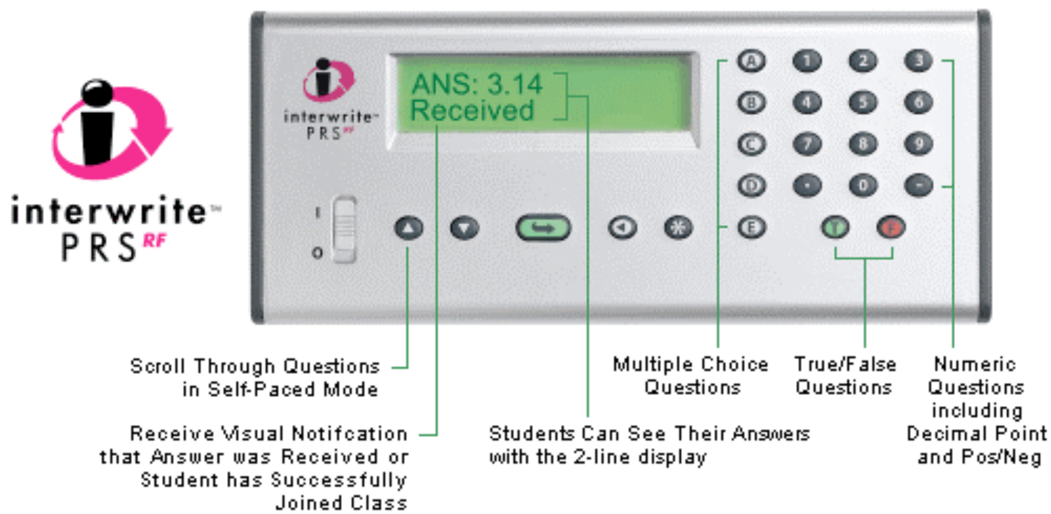
$$\left(m_3 v_3\right)^2 = \left(MV + m_2 v_2\right)^2 + \left(m_1 v_1\right)^2$$

- This equation tells us that v_3 = 1,014 m/s.
- The energy release is 3.23 MJ.

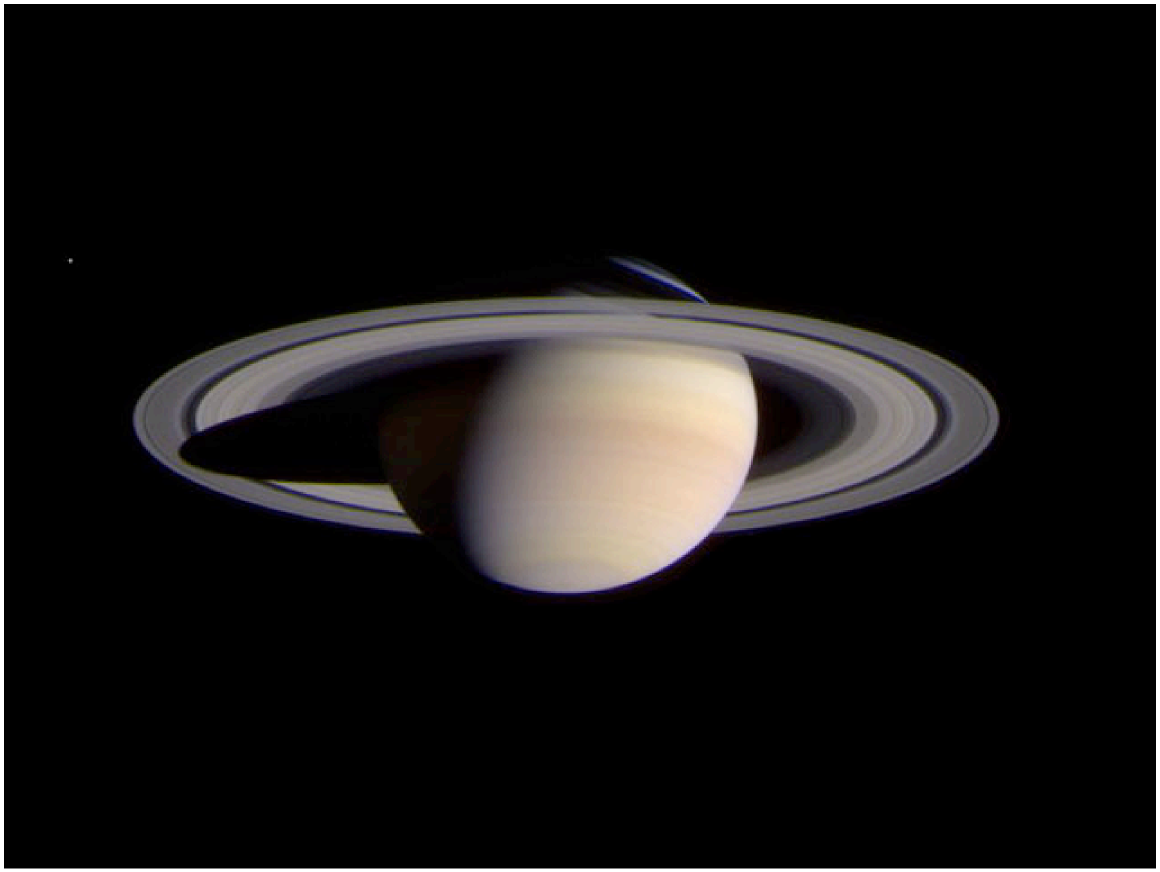


A few more questions.

- Let's test our understanding of the concepts of linear momentum and collisions by working on the following concept problems:
  - Q14.3
  - Q14.4
  - Q14.5



Done for today!
After break: rotational motion.



Cassini Closes in on Saturn Credit:
Cassini Imaging Team, SSI, JPL, ESA, NASA