Physics 121.
Lecture 13.

Frank L. H. Wolfs
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Physics 121.
Lecture 13.

- Course Information
- Topics to be discussed today:
  - Conservation of linear momentum (a brief review)
  - One- and two-dimensional collisions (elastic and inelastic)

Physics 121.
Course Information.

- Homework set # 5 is now available on the WEB and will be due next week on Saturday morning, March 21, at 8.30 am.

- To download the collision videos:
  - OSX: use control-click while pointing to the movie links to download the linked file.
  - Windows: use right-click while pointing to the movie links to download the linked file.

- The most effective way to work on the assignment is to tackle 1 - 2 problems a day.

Midterm Exam # 2.
March 24, 2026.

- Midterm Exam # 2 is scheduled for Tuesday March 24 between 8 am and 9.20 pm.
- The material covered on Exam # 2 will Chapters 6, 7, 8, and 9.
- The format will be the same as the format of Exam # 1.
- I will review the material covered on the exam during my lecture on March 19.

The center of mass (a review).

- The position of the center of mass of a system of particles along the x-axis is defined as

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1}{M} \sum_i m_i x_i$$

and similar expressions for the y and z positions.

- The motion of the center of mass is determined by the externals forces acting on the system:

$$M\vec{a}_{cm} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

Linear momentum (a review).

- The product of the mass and velocity of an object is called the **linear momentum $\vec{p}$** of that object.
- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M \vec{v}_{cm}$$

Linear momentum (a review).

- The change in the linear momentum of the system can now be calculated:

$$\frac{d\vec{P}_{cm}}{dt} = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \sum_i m_i\vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

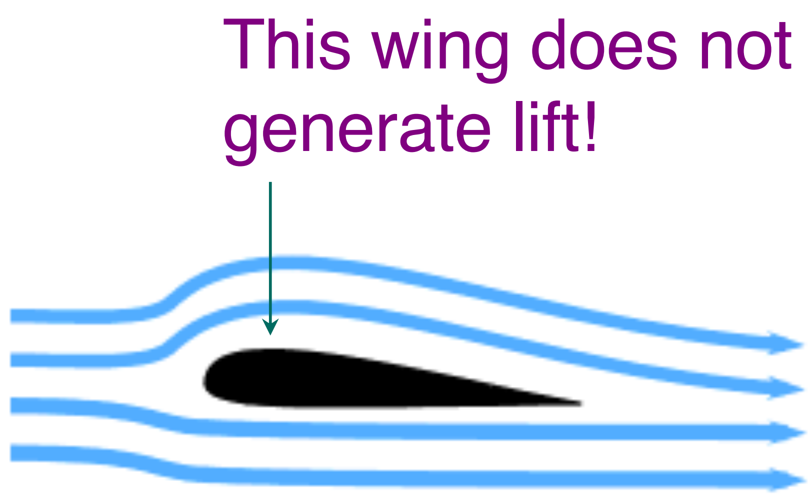
- This relations shows us that if there are no external forces, the total linear momentum of the system will be constant (independent of time).

Now we have all the tools to really understand lift!

Sorry Bernouilli!

Generating lift. Who needs Bernoulli?

- Most people who claim to understand the generation of lift, will rely on the Bernoulli effect to explain the principle of flight.
- If you are one of them, I am sorry to have to tell you that you do not understand the principle of flight.
- Although the Bernoulli effect has been used for many decades to explain the principle of flight, the Bernoulli effect can only account for a few percent of the lift generated by a wing, and it can not account for phenomena such as inverted flight.



http://www.aa.washington.edu/faculty/eberhardt/lift.htm

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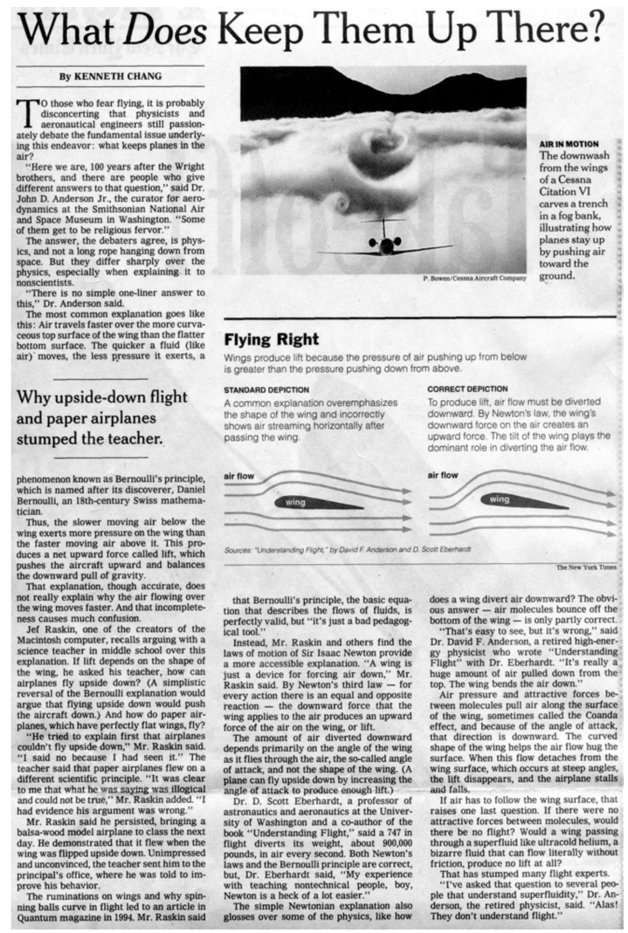
NY Times, January 2004.
100 Year Anniversary of Flight.

- What keeps them up there?
  - “it is disconcerting that physicists and aeronautical engineers still passionately debate the fundamental issues”
  - Jef Raskin, one of the creators of the Macintosh, was send to the principal’s office when he argued with his science teacher that the Bernoulli effect could not explain why paper airplane’s fly. According to the science teacher paper airplanes use a different scientific principle to fly.



What *Does* Keep Them Up There?

By KENNETH CHANG

TO those who fear flying, it is probably disconcerting that physicists and aeronautical engineers still passionately debate the fundamental issue underlying this endeavor: what keeps planes in the air?

"Here we are, 100 years after the Wright brothers, and there are people who give different answers to that question," said Dr. John D. Anderson Jr., the curator for aerodynamics at the Smithsonian National Air and Space Museum in Washington. "Some of them get to be religious fervor."

The answer, the debaters agree, is physics, and not a long rope hanging down from space. But they differ sharply over the physics, especially when explaining it to nonscientists.

"There is no simple one-liner answer to this," Dr. Anderson said.

The most common explanation goes like this: Air travels faster over the more curvaceous top surface of the wing than the flatter bottom surface. The quicker a fluid (like air) moves, the less pressure it exerts, a

Why upside-down flight and paper airplanes stumped the teacher.

phenomenon known as Bernoulli's principle, which is named after its discoverer, Daniel Bernoulli, an 18th-century Swiss mathematician.

Thus, the slower moving air below the wing exerts more pressure on the wing than the faster moving air above it. This produces a net upward force called lift, which pushes the aircraft upward and balances the downward pull of gravity.

That explanation, though accurate, does not really explain why the air flowing over the wing moves faster. And that incompleteness causes much confusion.

Jef Raskin, one of the creators of the Macintosh computer, recalls arguing with a science teacher in middle school over this explanation. If lift depends on the shape of the wing, he asked his teacher, how can airplanes fly upside down? (A simplistic reversal of the Bernoulli explanation would argue that flying upside down would push the aircraft down.) And how do paper airplanes, which have perfectly flat wings, fly?

"He tried to explain first that airplanes couldn't fly upside down," Mr. Raskin said. "I said no because I had seen it." The teacher said that paper airplanes flew on a different scientific principle. "It was clear to me that what he was saying was illogical and could not be true," Mr. Raskin added. "I had evidence his argument was wrong."

Mr. Raskin said he persisted, bringing a balsa-wood model airplane to class the next day. He demonstrated that it flew when the wing was flipped upside down. Unimpressed and unconvinced, the teacher sent him to the principal's office, where he was told to improve his behavior.

The ruminations on wings and why spinning balls curve in flight led to an article in Quantum magazine in 1994. Mr. Raskin said

P. Bowen/Cessna Aircraft Company

AIR IN MOTION The downwash from the wings of a Cessna Citation VI carves a trench in a fog bank, illustrating how planes stay up by pushing air toward the ground.

Flying Right

Wings produce lift because the pressure of air pushing up from below is greater than the pressure pushing down from above.

STANDARD DEPICTION
A common explanation overemphasizes the shape of the wing and incorrectly shows air streaming horizontally after passing the wing.

CORRECT DEPICTION
To produce lift, air flow must be diverted downward. By Newton's law, the wing's downward force on the air creates an upward force. The tilt of the wing plays the dominant role in diverting the air flow.

Sources: "Understanding Flight," by David F. Anderson and D. Scott Eberhardt

The New York Times

that Bernoulli's principle, the basic equation that describes the flows of fluids, is perfectly valid, but "it's just a bad pedagogical tool."

Instead, Mr. Raskin and others find the laws of motion of Sir Isaac Newton provide a more accessible explanation. "A wing is just a device for forcing air down," Mr. Raskin said. By Newton's third law — for every action there is an equal and opposite reaction — the downward force that the wing applies to the air produces an upward force of the air on the wing, or lift.

The amount of air diverted downward depends primarily on the angle of the wing as it flies through the air, the so-called angle of attack, and not the shape of the wing. (A plane can fly upside down by increasing the angle of attack to produce enough lift.)

Dr. D. Scott Eberhardt, a professor of astronautics and aeronautics at the University of Washington and a co-author of the book "Understanding Flight," said a 747 in flight diverts its weight, about 900,000 pounds, in air every second. Both Newton's laws and the Bernoulli principle are correct, but, Dr. Eberhardt said, "My experience with teaching nontechnical people, boy, Newton is a heck of a lot easier."

The simple Newtonian explanation also glosses over some of the physics, like how does a wing divert air downward? The obvious answer — air molecules bounce off the bottom of the wing — is only partly correct.

"That's easy to see, but it's wrong," said Dr. David F. Anderson, a retired high-energy physicist who wrote "Understanding Flight" with Dr. Eberhardt. "It's really a huge amount of air pulled down from the top. The wing bends the air down."

Air pressure and attractive forces between molecules pull air along the surface of the wing, sometimes called the Coanda effect, and because of the angle of attack, that direction is downward. The curved shape of the wing helps the air flow hug the surface. When this flow detaches from the wing surface, which occurs at steep angles, the lift disappears, and the airplane stalls and falls.

If air has to follow the wing surface, that raises one last question. If there were no attractive forces between molecules, would there be no flight? Would a wing passing through a superfluid like ultracold helium, a bizarre fluid that can flow literally without friction, produce no lift at all?

That has stumped many flight experts.

"I've asked that question to several people that understand superfluidity," Dr. Anderson, the retired physicist, said. "Alas! They don't understand flight."

Generating lift.
Who needs Bernoulli?

- In order for a wing to generate lift, (producing an upward force) it must force the air down.
- The vertical component of the momentum associated with the downwash, must balanced by the vertical component associated with the lift generated by the wing.
- Thus, unless the air is forced downwards, the wing will not generate lift.

This wing does not generate lift!

This wing does generate lift!

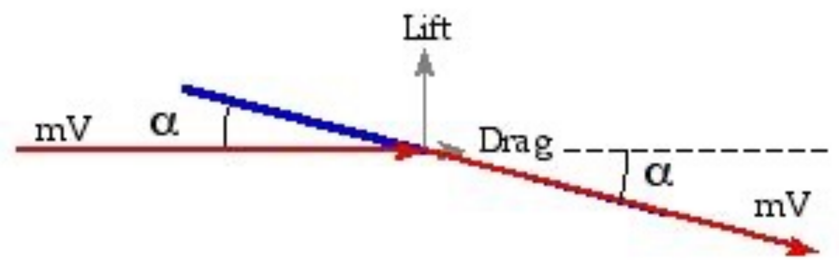
http://www.aa.washington.edu/faculty/eberhardt/lift.htm

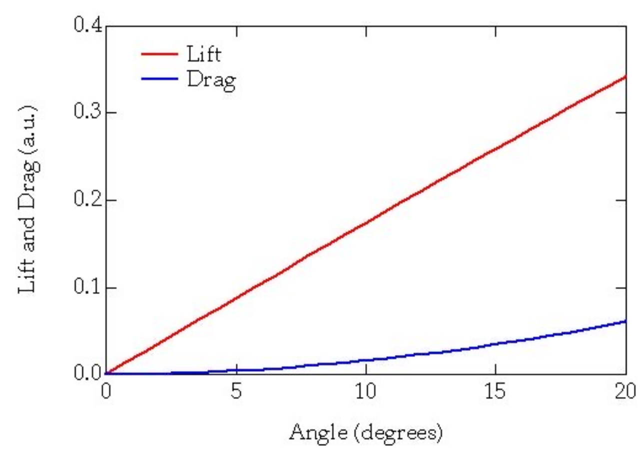
Generating lift. Who needs Bernoulli?

- The lift of a wing is generated by the downwards deflection of air.
- Consider an elastic collision between an air molecule of mass m and velocity V. The change in the vertical component of the momentum of the wing as a result of this single collision is equal to:

$$p_{lift} = mV\sin(\alpha)$$

- The lift generated will increase if V increases, if the air density increases, and/or if the angle of attack increases.





Generating lift.
An example.

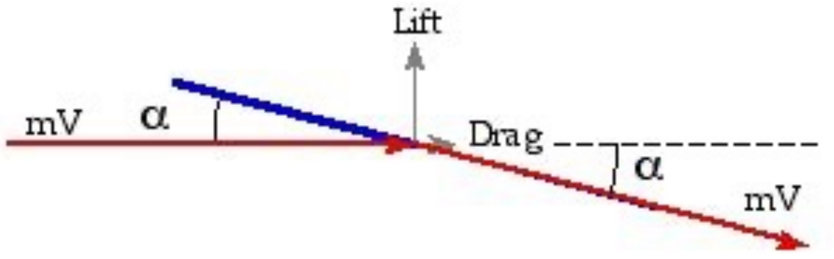
- Consider a Piper Warrior. This plane has a maximum weight of 2450 lb, and the required lift on takeoff is about 11,000 N.
- On takeoff, the airspeed is 60 knots = 31 m/s. Assume that on takeoff the angle of attack is 5°.
- To generate this lift requires that

$$Lift = \frac{dp_{lift}}{dt} = \frac{dm}{dt} V \sin(\alpha)$$

- This requires that

$$\frac{dm}{dt} = \frac{Lift}{V \sin(\alpha)} = 4100 \, \frac{\text{kg}}{\text{s}}$$

www.newpiper.com



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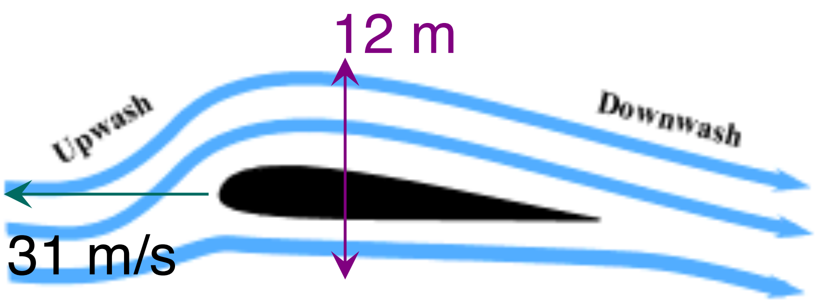
Generating lift.
Certainly not a surface phenomenon.

- The Warrior has a wing span of 35.0 ft (10.7 m) and a wing surface area of 170 ft^2 (15.8 m^2).
- The density of air at sea level is about 1 kg/m^3.
- In order to generate an air deflection rate of 4100 kg/s, the height of the layer of air involved must be about 12 m.
- Clearly, the generation of lift is not a surface phenomenon.

The Rochester Flying Club



http://www.aa.washington.edu/faculty/eberhardt/lift.htm

Generating lift.
What does it take to lift a Boeing 737?

- At 6.02 am on 3/22/03, CO 282 took of from San Antonio (TX). For this particular flight, the weight of the plane was 106,000 pounds and the rotation speed was 131 knots (numbers provided by Mark d'Arpino, the first officer on this flight).
- For this flight:
  - *dm/dt* = 80,000 kg/s (80,000 m^3/s)
  - Note: (*dm/dt*)/*M* = 1.7 for this 737 compared to 3.7 for the Warrior.
  - The wingspan of this 737 is 94.9 ft, which indicates that a 41 m thick layer of air is involved in the generation of lift.

Photographer: B. Leibowitz

Photographer: M. Abbott

Pictures from airliners.net

Generating lift.

- The effect of the downwash of the wing of this citation, flying low over a fog layer, is clearly visible.

- More details about the generation of lift can be found in "*Understanding Flight*" by Anderson and Eberhardt. Note: Anderson is a retired high-energy physicists.

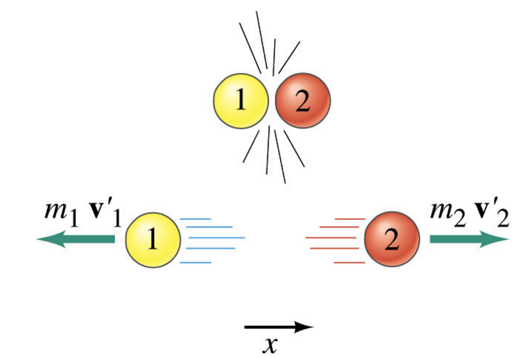
Photograph by Paul Bowen, courtesy of Cessna Aircraft, Co.

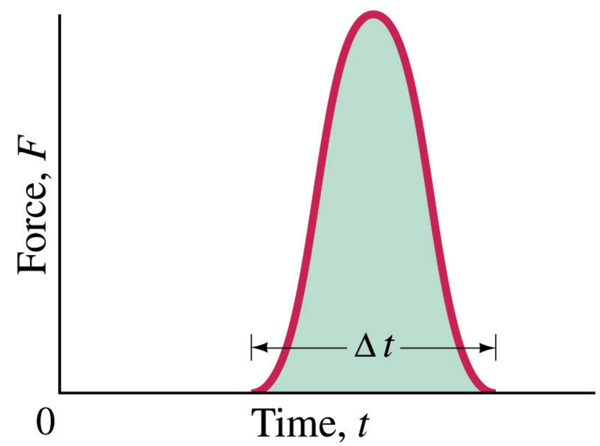
Back to more mundane topics: collisions.

Collisions.
The collision force.

- During a collision, a strong force is exerted on the colliding objects for a short period of time.
- The collision force is usually much stronger than any external force.
- The result of the collision force is a change in the linear momentum of the colliding objects.
- The change in the momentum of one of the objects is equal to

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$





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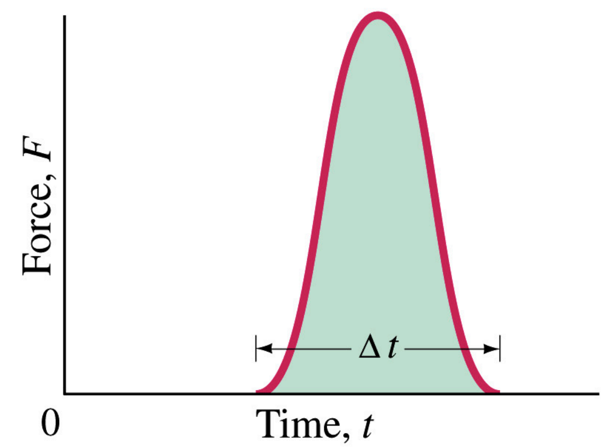
Collisions.
The collision impulse.

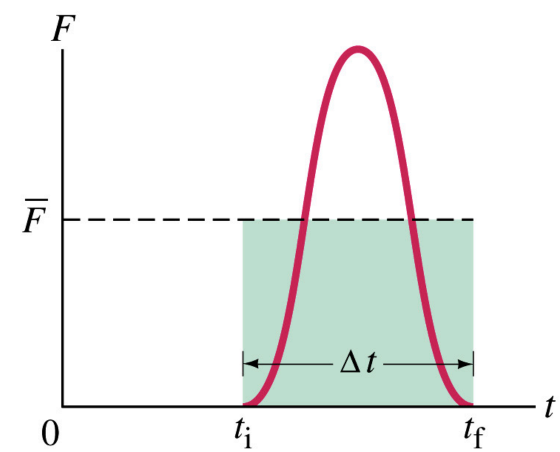
- If we measure the change in the linear momentum of an object, we will obtain information about the integral of the force acting on it:

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

- The integral of the force is called the collision impulse $\vec{J}$:

$$\vec{J} = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$





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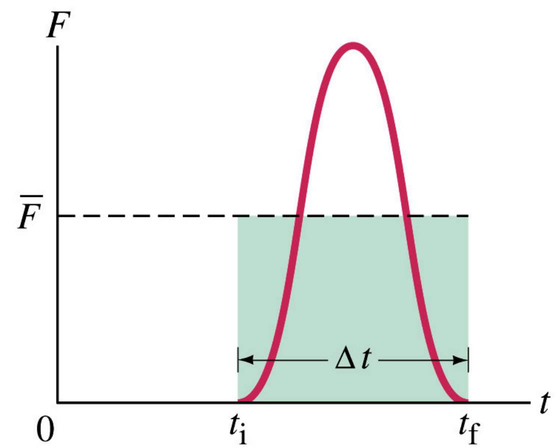
Collisions.
The collision impulse.

- Consider you are involved in a collision: you first move with 55 mph and after the collision you are at rest.
- The change in momentum is thus fixed and the collision impulse is also fixed:

$$\vec{J} = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

- What happens to you depends on the magnitude of the force! An increase in time *dt* results in a reduction of the force.

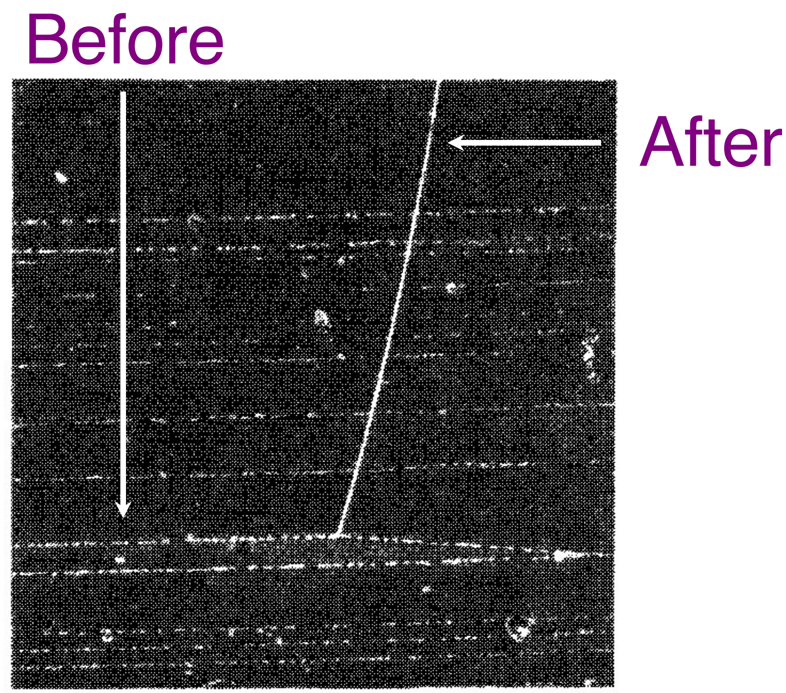


Collisions.
The collision impulse.

- Increasing the time required to come to a stop reduced the average force.
- This reduction in the average force can mean the difference between life and death.
- The human body can tolerate accelerations up to 10 - 15 times the gravitational acceleration.
- An acceleration of 10*g* brings an object traveling at 55 mph to rest over a distance of 3 m (9 feet).

Collisions.
The collision impulse.

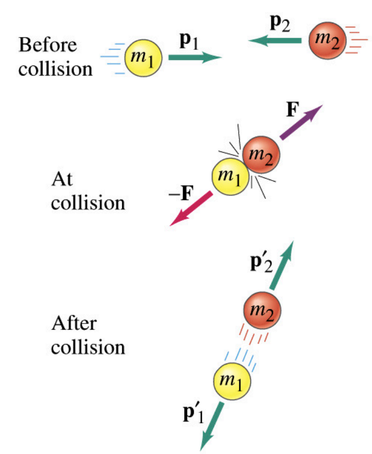
- Interactions between sub-atomic particles are usually studied by comparing their momenta before and after an interaction.
- The change in their momenta provides us with information about the collision impulse.
- Determining the force from the collision impulse required a knowledge of the time dependence of the interaction.

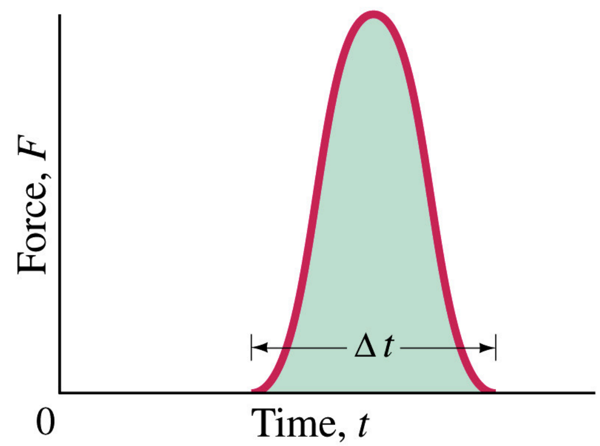


Collision between protons.

Elastic and inelastic collisions.

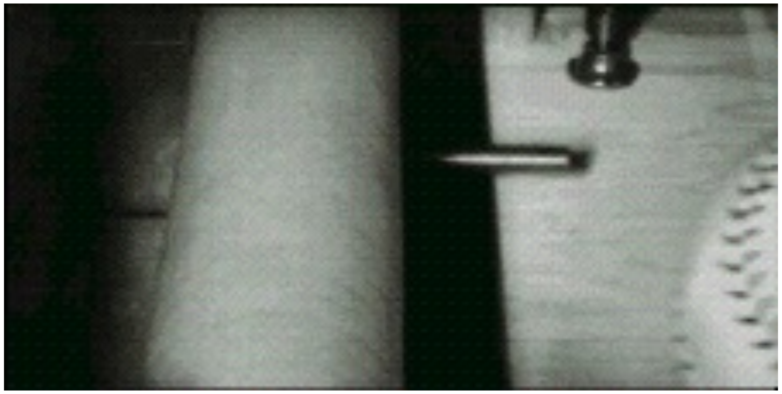
- If we consider both colliding object, then the collision force becomes an internal force, and the total linear momentum of the system must be conserved if there are no external forces acting on the system.
- Collisions are usually divided into two groups:
  - **Elastic collisions**: kinetic energy is conserved.
  - **Inelastic collisions**: kinetic energy is NOT conserved.

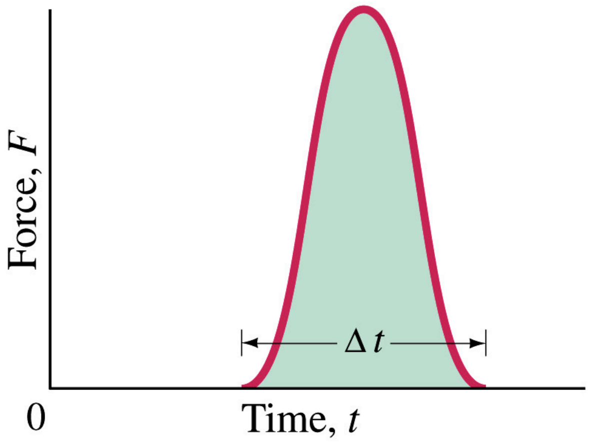




Elastic and inelastic collisions.

- Kinetic energy does not need to be conserved during the time period that the collision force is acting on the system. The kinetic energy may become 0 J for a short period of time.
- During the time period that the collision force is non-zero, some or all of the initial kinetic energy may be converted into potential energy (for example, the potential energy associated with deformation).





4 Minute 14 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 14 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

Elastic collisions in one dimension.

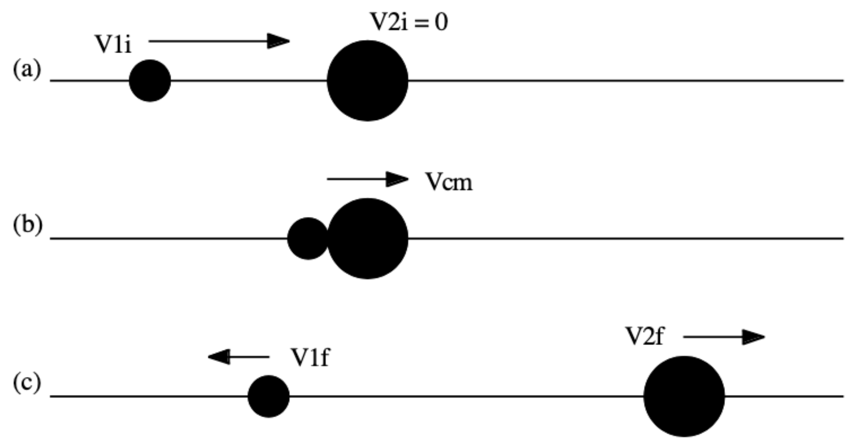
- Consider the elastic collision show in the Figure.
- Conservation of linear momentum requires that

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

- Conservation of kinetic energy requires that

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Two equations with two unknown!



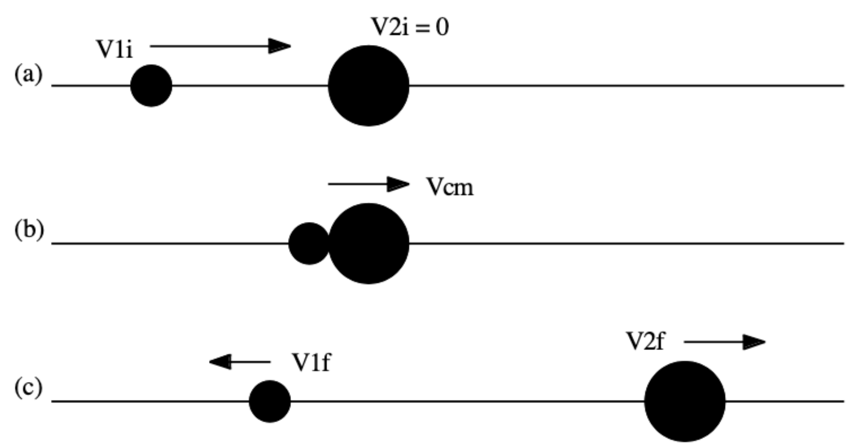
Elastic collisions in one dimension.

- The solution for the final velocity of mass m_1 is:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

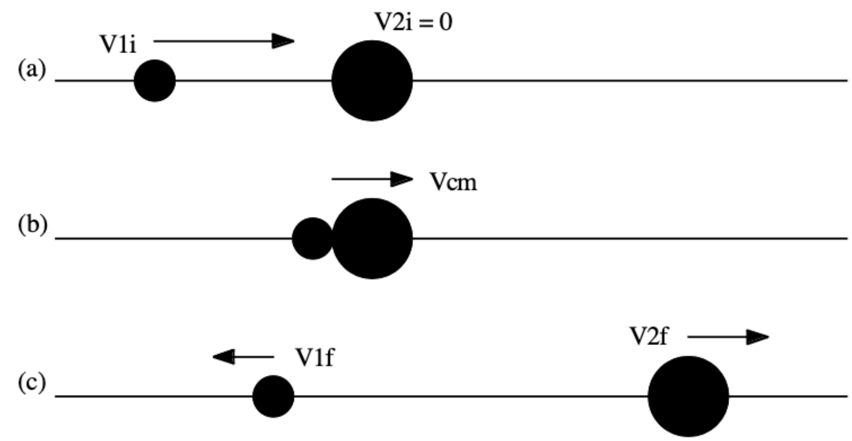
- The solution for the final velocity of mass m_2 is:

$$v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) = \frac{2m_1}{m_1 + m_2} v_{1i}$$



Elastic collisions in one dimension. Special cases.

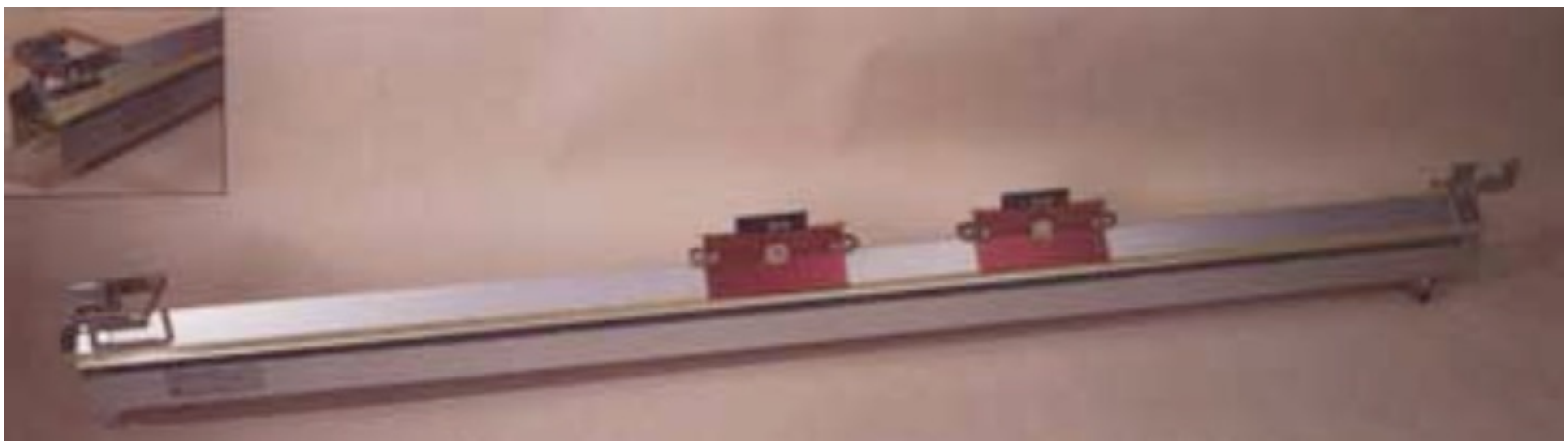
- $m_1 = m_2$:
  - $v_{1f} = 0$ m/s
  - $v_{2f} = v_{1i}$
- $m_2 >> m_1$:
  - $v_{1f} = - v_{1i}$
  - $v_{2f} = (2m_1/m_2)\, v_{1i}$
- $m_1 >> m_2$:
  - $v_{1f} = v_{1i}$
  - $v_{2f} = 2v_{1i}$



Note: the motion of the center of mass is not changed due to the collision.

Elastic collisions in one dimension.

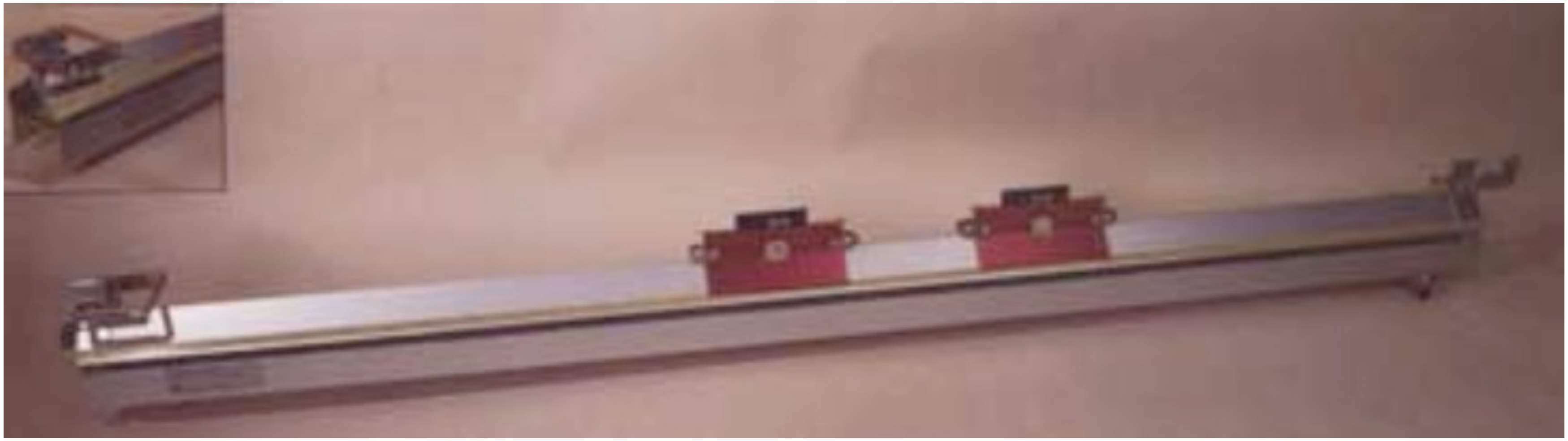
- We can use an air track to study elastic collisions.
- The velocity of the carts can be determined if we measure the length of time required to pass through the photo gates: velocity = length/time.



TEL-Atomic, http://www.telatomic.com/at.html

Elastic collisions in one dimension.

- Let us focus on specific examples where one cart (cart 2) is at rest:
  - $M_1 = M_2$: $v_{1f} = 0$ m/s, $v_{2f} = v_{1i}$
  - $M_1 = 2M_2$: $v_{1f} = (1/3)v_{1i}$, $v_{2f} = (4/3)\ v_{1i}$
  - $2M_1 = M_2$: $v_{1f} = -(1/3)v_{1i}$, $v_{2f} = (2/3)\ v_{1i}$



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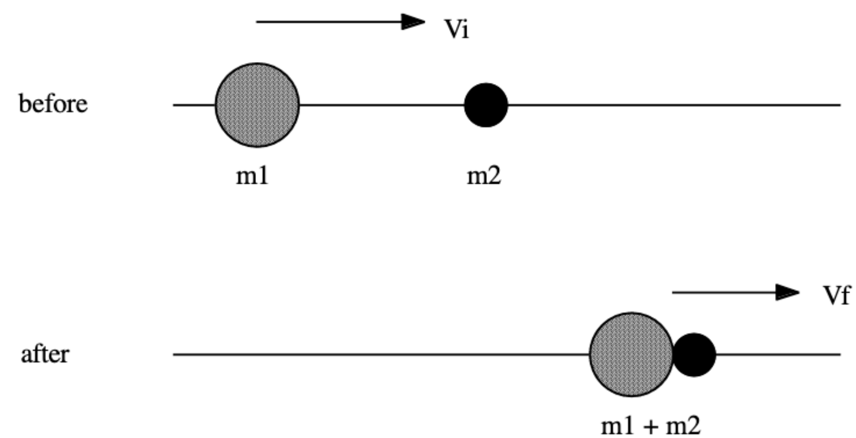
Inelastic collisions in one dimension.

- In inelastic collisions, kinetic energy is not conserved.
- A special type of inelastic collisions are the completely inelastic collisions, where the two objects stick together after the collision.
- Conservation of linear momentum in a completely inelastic collision requires that

$$m_1 v_i = (m_1 + m_2) v_f$$



Inelastic collisions in one dimension.

- The final velocity of the system is equal to

$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

- The final kinetic energy of the system is equal to

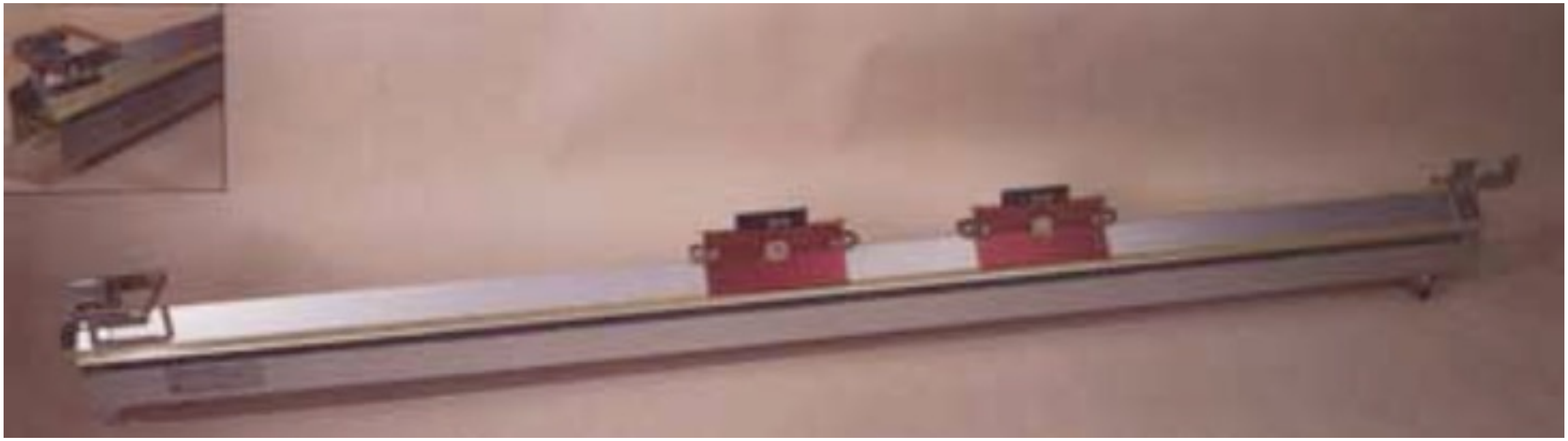
$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2} v_i\right)^2$$
$$= \frac{m_1}{m_1 + m_2} K_i$$

- Note: not all of the kinetic energy can be lost, even in a completely inelastic collision, since the motion of the center of mass must still be present. Only if our reference frame is chosen such that the center-of-mass velocity is zero, will the final kinetic energy in a completely inelastic collision be zero.

Inelastic collisions in one dimension.

- Let us focus on one specific example of a procedure to measure the velocity of a bullet:
  - We shoot a 0.3 g bullet into a cart.
  - The final velocity of the cart is measured, and conservation of linear momentum can be used to determine the velocity of the bullet:

$$v_i = \frac{(M_1 + M_2)}{M_1} v_f$$



TEL-Atomic, http://www.telatomic.com/at.html

Conceptual Questions.

- Let's practice what we learned (and read).

Done for today!
Have a great Wolfs break!

White Boat Rock on Mars
Credit: Mars Exploration Rover Mission, JPL, NASA