Physics 121.
Thursday, February 28, 2008.

Frank L. H. Wolfs
Department of Physics and Astronomy, University of Rochester

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- Course Information
- Quiz?
- Topics to be discussed today:
  - The center of mass
  - Conservation of linear momentum
  - Systems of variable mass

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- Homework set # 5 is now available on the WEB and will be due next week on Saturday morning, March 8, at 8.30 am.
- This homework set contains WeBWorK problems and a video analysis.
- The most effective way to work on the assignment is to tackle 1 or 2 problems a day.
- If you run into problems, please contact me and I will try to help you solve your problems (Physics 121 problems that is).

Homework Set # 5.
WeBWorK problems.

Finding $U(x)$

The escape velocity

Conservation of energy (including “friction” losses)

Inelastic collisions and conservation of energy

Frank Wolfs — **Physics 121, Spring 2008**

Homework Set 05 — **Due date: 03/08/2008 at 08:30am EST**

This assignment will be counted toward your final grade. You can attempt each problem 15 times; once you exceed this number of attempts, your solutions will not be recorded anymore. You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer. Note: to use scientific notion, use a notion like xxE+yy. It is important that you use a capital E; answers with a lower case e will be evaluated differently

1. (10 pts) library/type12/prob05.pg

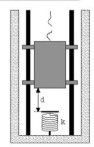
A particle moves along the x axis under the influence of a variable force $F(x) = 7.2\,x^2 + 5.1\,x$ where the force is measured in Newtons and the distance in meters. What is the potential energy associated with this force at $x = 3.0\,m$? Assume that $U(x) = 0\,J$ at $x = 0\,m$. ____________

2. (10 pts) library/type13/prob25.pg

At what speed should a space probe be fired from the earth if it is required to still be traveling at a speed of $3.62\,km/s$, even after coasting to an exceedingly great distance from the planet (a distance that is essentially infinite)? ____________

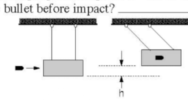
3. (10 pts) library/type13/prob19.pg

The cable of an elevator of mass $M = 3620\,kg$ snaps when the elevator is a rest at one of the floors of a skyscraper. At this point the elevator is a distance $d = 98.8\,m$ above a cushioning spring whose spring constant is $k = 16900\,N/m$. A safety device clamps the elevator against the guide rails so that a constant frictional force of $f = 6231\,N$ opposes the motion of the elevator. Find the maximum distance by which the cushioning spring will be compressed. ____________



4. (10 pts) library/type16/prob07.pg

A ballistic pendulum, as shown in the figure, was a device used in the past century to measure the speed of bullets. The pendulum consists of a large block of wood suspended from long wires. Initially, the pendulum is at rest. The bullet strikes the block horizontally and remains stuck in it. The impact of the bullet puts the block in motion, causing it to swing upward to a height h. If the bullet has a mass of $6.6\,g$, and the block of mass $7.9\,kg$ swings up to a height of $h = 2.000\,cm$, what was the speed of the bullet before impact? ____________



5. (10 pts) library/type16/prob04.pg

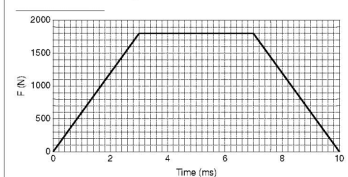
A 8450-kg rocket is set for vertical firing from the earth's surface. If the exhaust speed is $1450\,m/s$, how much gas must be ejected each second in order for the thrust to be equal to the weight of the rocket? ____________

How much gas must be ejected each second to give the rocket an initial upward acceleration of $21\,m/s^2$? ____________

6. (10 pts) library/type17/prob04.pg

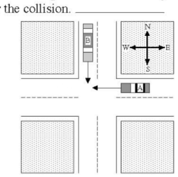
The Figure shows an approximate representation of the contact force versus time during the collision of a 37-g tennis ball with a wall. The initial speed of the ball is $170.3\,m/s$ perpendicular to the wall. What is the speed of the tennis ball after the collision?

____



7. (10 pts) library/type18/prob02.pg

Two vehicles A and B are traveling west and south, respectively, toward the same intersection where they collide and lock together. Before the collision A (total weight 1740 N) is moving with a speed of 50 m/s and B (total weight 1960 N) has a speed of 40 m/s. Find the magnitude of the velocity of the interlocked vehicles after the collision. ____________



What is the direction of motion of the interlocked vehicles after the collision? Give your answer in terms of the angle that the velocity vector makes with respect to East (East = 0°, North = 90°, West = 180°, and South = 270°). ____________

1

Rocket motion

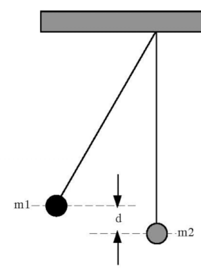
Collision force

2D collision

Homework Set # 5.
WeBWorK problems.

Elastic collisions and conservation of energy

8. **(10 pts) library/type18/prob05.pg**
Two 21-*cm* long pendulums (each made of a massless string and a ball) are initially situated as shown in the figure. The masses of the left and right balls are $m_1 = 135$ gram and $m_2 = 310$ gram , respectively. The first pendulum is released from a height $d =$ 8.4 cm and strikes the second.

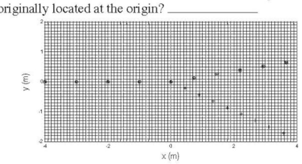


Assuming that the collision is completely inelastic and neglecting the mass of the strings and any frictional effects, how high does the center of mass rises after the collision? ____________

Generated by the WeBWorK system ©WeBWorK Team, Department of Mathematics, University of Rochester

2D collision

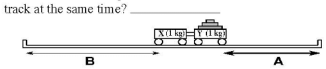
9. **(10 pts) library/type18/prob09.pg**
The Figure shows the result of a collision of a 3.75 *kg* ball with an object located at the origin. The position of the objects are shown at 4.6 *s* intervals. What is the mass of the object that was originally located at the origin? ____________



What is the change in the kinetic energy as a result of the collision? ____________

Explosion in 1D

10. **(10 pts) library/type18/prob30.pg**
The drawing below shows two laboratory carts (each has a mass of 1.0 *kg*) X and Y in contact with a compressed exploder spring between them. The mass on cart Y is 2.13 *kg*, distance A is 6 *cm*, distance B is 9 *cm*. What mass must be placed on cart X, such that after the explosion both carts will hit the ends of the track at the same time? ____________

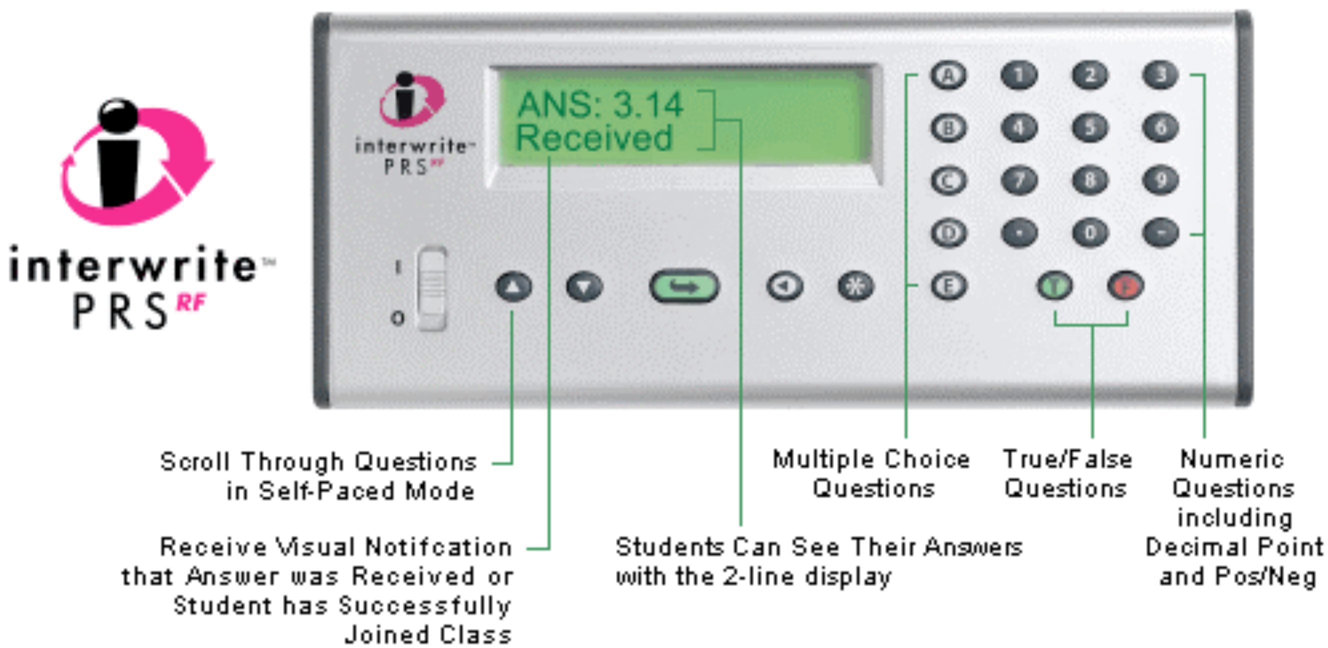


Physics 121
Thursday, February 28, 2008.

- We will grade Exam # 1 this weekend, and the grades will be distributed via email on Monday.
- The exam will be returned to you during workshops next week. Please carefully look at the exam and if you made any mistakes, try to understand why what you did was not correct. If you disagree with the grade you received, you need to come and talk to me. Your TAs can not change your exam grade.
- The next exam will take place on March 25. Please do not wait until the day before the exam to start studying!

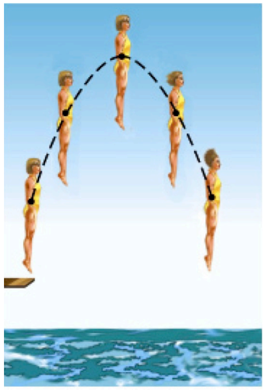
Physics 121.
Quiz lecture 12.

- The quiz today will have 3 questions and provide me with feedback on Exam # 1.
- Note: each answer you submit is correct!



The Center of Mass.

- Up to now we have ignored the shape of the objects were are studying.
- Objects that are not point-like appear to carry out more complicated motions than point-like objects (e.g. the object may be rotating during its motion).
- We will find that we can use whatever we have learned about motion of point-like objects if we consider the motion of the center-of-mass of the extended object.



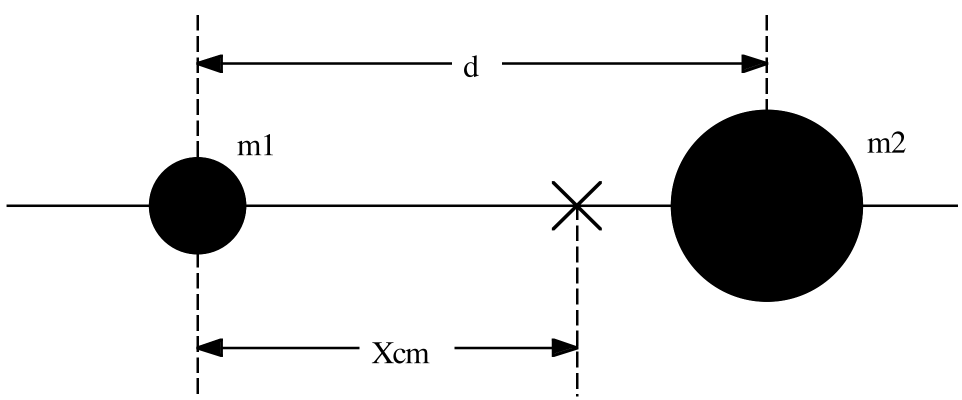
(a)

(b)

But what is the center of mass and where is it located?

- We will start with considering one-dimensional objects. For an object consisting out of two point masses, the center of mass is defined as

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1}{M} \sum_i m_i x_i$$

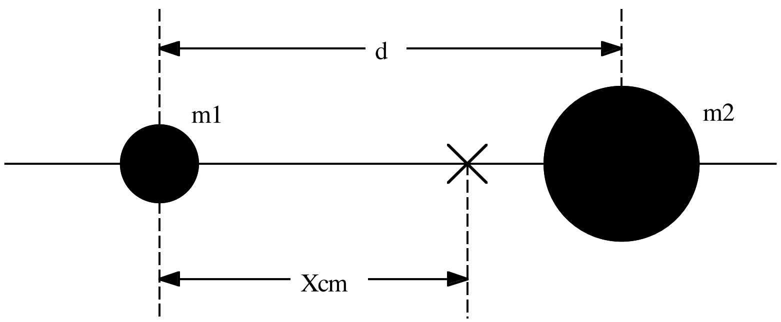


But what is the center of mass and where is it located?

- Let's look at this particular system.
- Since we are free to choose our coordinate system in a way convenient to us, we choose it such that the origin coincides with the location of mass m_1.
- The center of mass is located at

$$x_{cm} = \frac{m_2 d}{m_1 + m_2}$$

Note: the center of mass does not need to be located at a position where there is mass!

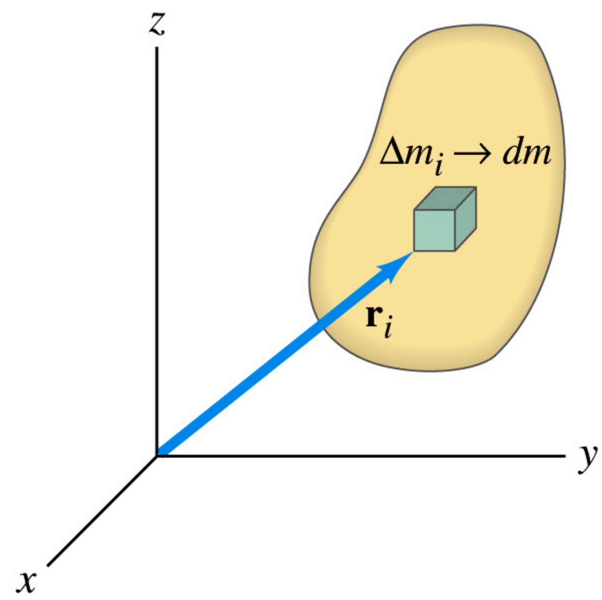


But what is the center of mass and where is it located?

- In two or three dimensions the calculation of the center of mass is very similar, except that we need to use vectors.

- If we are not dealing with discreet point masses we need to replace the sum with an integral.

$$\vec{\mathbf{r}}_{cm} = \frac{1}{M}\int_V \vec{\mathbf{r}}\,dm$$



Department of Physics and Astronomy, University of Rochester

But what is the center of mass and where is it located?

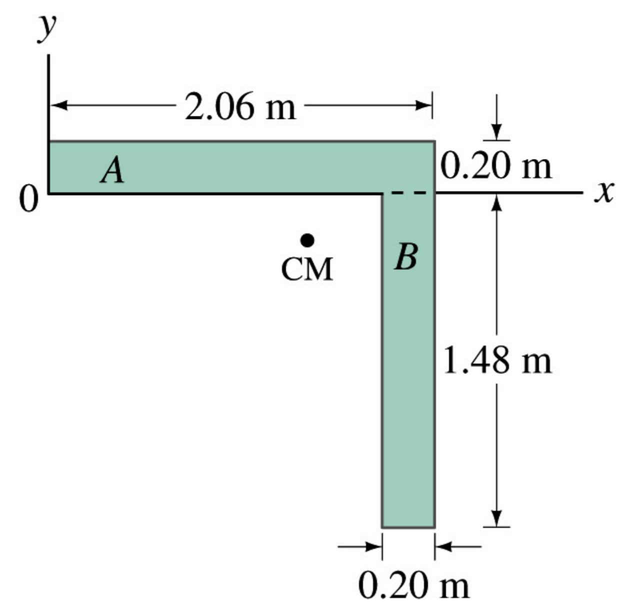
- We can also calculate the position of the center of mass of a two or three dimensional object by calculating its components separately:

$$x_{cm} = \frac{1}{M}\sum_i m_i x_i$$

$$y_{cm} = \frac{1}{M}\sum_i m_i y_i$$

$$z_{cm} = \frac{1}{M}\sum_i m_i z_i$$

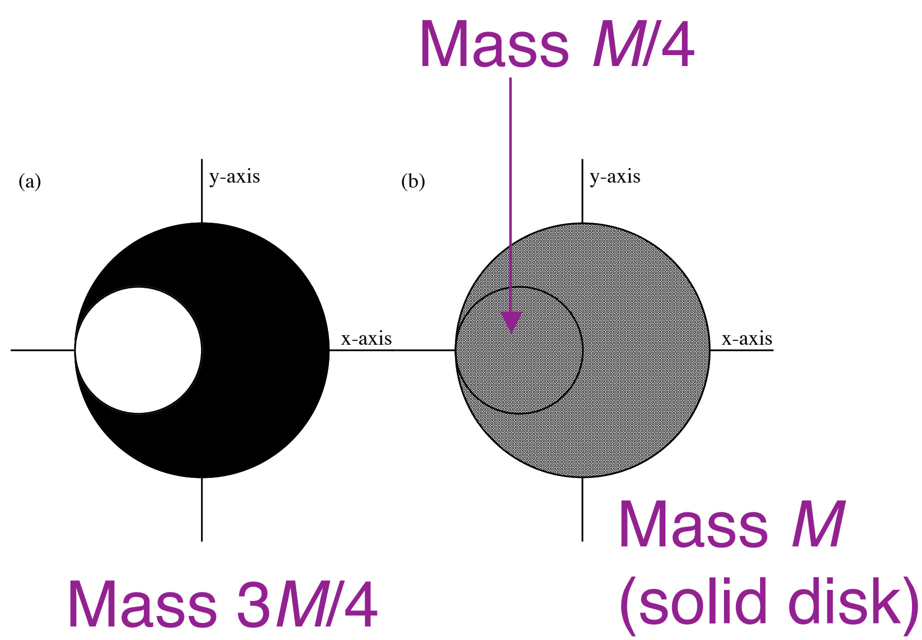
- Note: the center of mass may be located outside the object.



Calculating the position of the center of mass. Example problem.

- Consider a circular metal plate of radius 2*R* from which a disk if radius *R* has been removed. Let us call it object X. Locate the center of mass of object X.

- Since this object is complicated we can simplify our life by using the principle of superposition:
  - If we add a disk of radius *R*, located at (-*R*/2, 0) we obtain a solid disk of radius *R*.
  - The center of mass of this disk is located at (0, 0).



Calculating the position of the center of mass. Example problem.

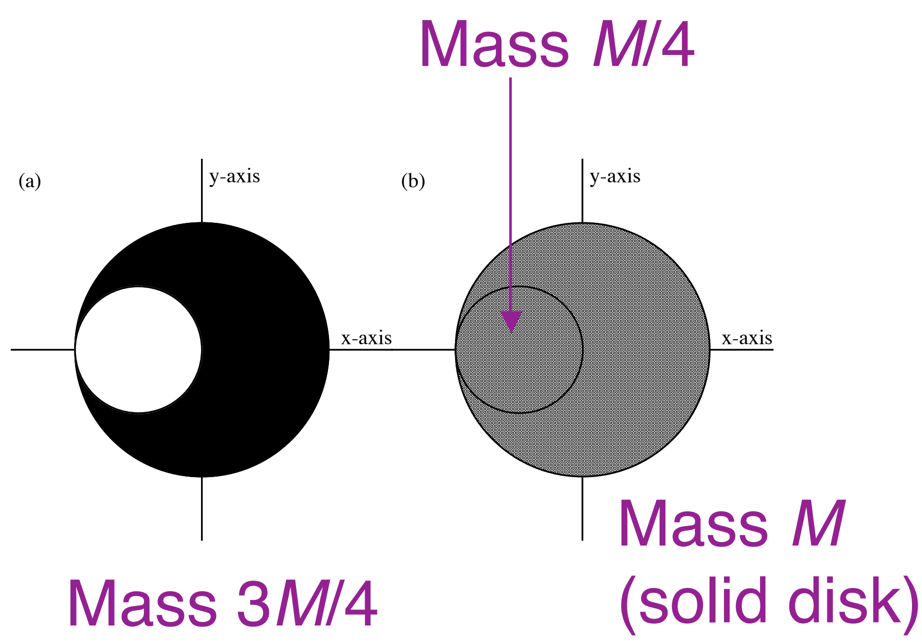
- The center of mass of the solid disk can be expressed in terms of the disk X and the disk D we used to fill the hole in disk X:

$$x_{cm,C} = \frac{x_{cm,X} m_X + x_{cm,D} m_D}{m_X + m_D} = 0$$

- This equation can be rewritten as

$$x_{cm,X} = -\frac{x_{cm,D} m_D}{m_X} = \frac{R m_D}{m_X}$$

- Where we have used the fact that the center of mass of disk D is located at (-R/2, 0).
- Thus, $x_{\mathrm{cm,X}} = R/3$.



Motion of the center of mass.

- To examine the motion of the center of mass we start with its position and then determine its velocity and acceleration:

$$M\vec{r}_{cm} = \sum_i m_i \vec{r}_i$$

$$M\vec{v}_{cm} = \sum_i m_i \vec{v}_i$$

$$M\vec{a}_{cm} = \sum_i m_i \vec{a}_i$$

Motion of the center of mass.

- The expression for Ma_{cm} can be rewritten in terms of the forces on the individual components:

$$M\vec{a}_{cm} = \frac{d}{dt}\left(M\vec{v}_{cm}\right) = \frac{d\vec{P}_{cm}}{dt} = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- We conclude that the motion of the center of mass is only determined by the external forces. Forces exerted by one part of the system on other parts of the system are called internal forces. According to Newton's third law, the sum of all internal forces cancel out (for each interaction there are two forces acting on two parts: they are equal in magnitude but pointing in an opposite direction and cancel if we take the vector sum of all internal forces).

Motion of the center of mass and linear momentum.

- Now consider the special case where there are no external forces acting on the system. In this case, $Ma_{\mathrm{cm,x}} = Ma_{\mathrm{cm,y}} = Ma_{\mathrm{cm,z}} = 0$ N.

- In this case, $Mv_{\mathrm{cm,x}}$, $Mv_{\mathrm{cm,y}}$, and $Mv_{\mathrm{cm,z}}$ are constant.

- The product of the mass and velocity of an object is called the linear momentum p of that object.

- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

Conservation of linear momentum.

- The change in the linear momentum of the system can now be calculated:

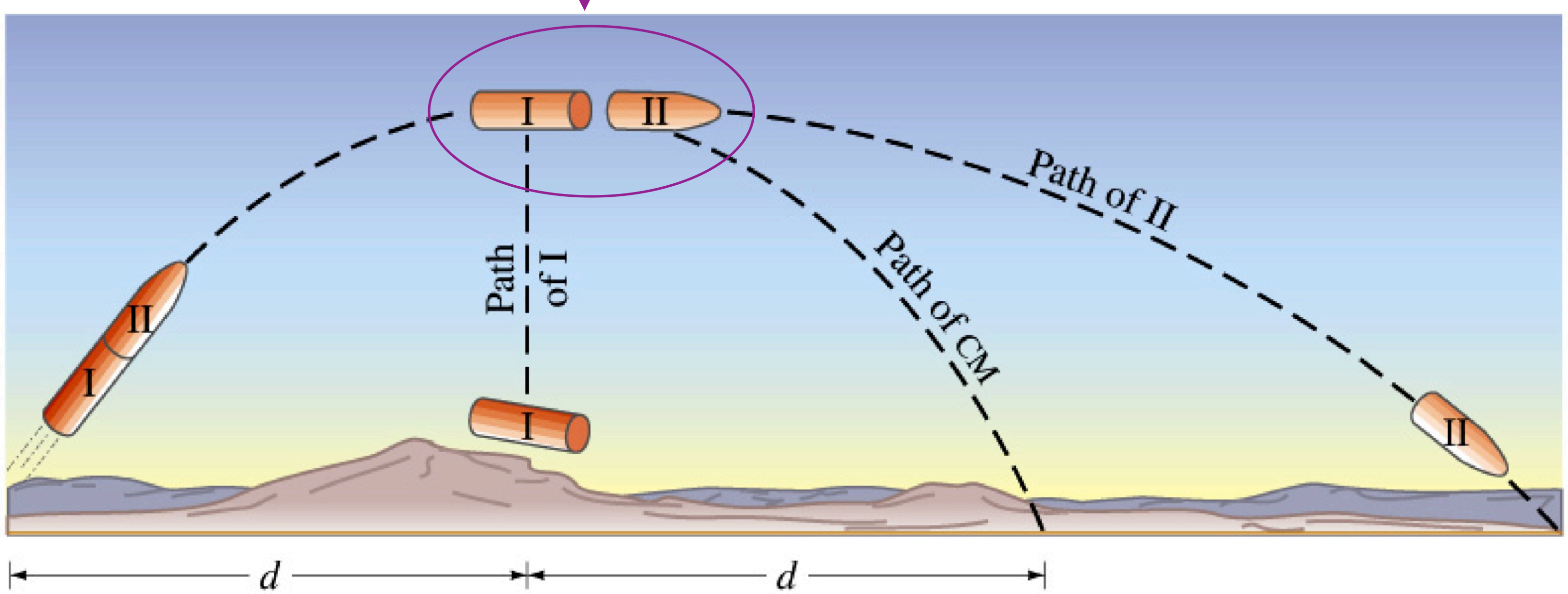
$$\frac{d\vec{P}_{cm}}{dt} = \frac{d}{dt}\left(M\vec{v}_{cm}\right) = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \sum_i m_i \dot{\vec{v}}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- This relations shows us that if there are no external forces, the total linear momentum of the system will be constant (independent of time).

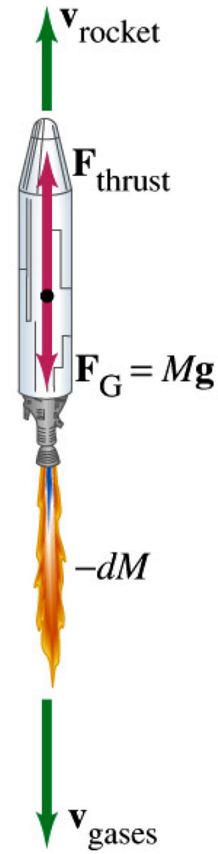
- Note: the system can change as a result of internal forces!

Conservation of linear momentum. Applications.

Internal forces are responsible for the breakup.

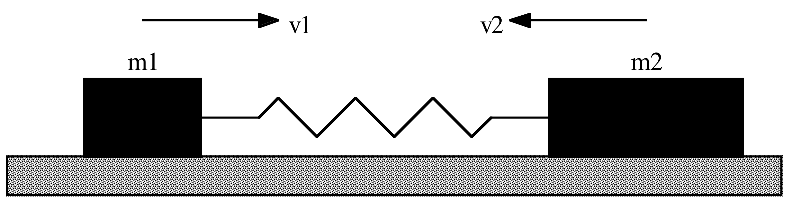


Conservation of linear momentum. Applications.



Conservation of linear momentum. An example.

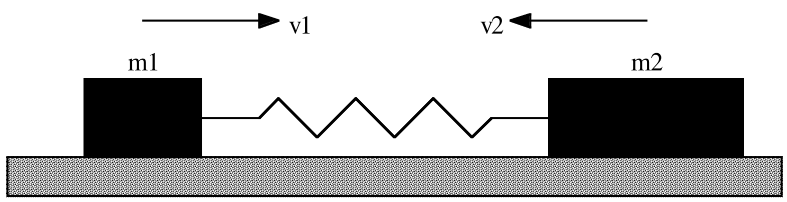
- Two blocks with mass m_1 and mass m_2 are connected by a spring and are free to slide on a frictionless horizontal surface. The blocks are pulled apart and then released from rest. What fraction of the total kinetic energy will each block have at any later time?



Conservation of linear momentum. An example.

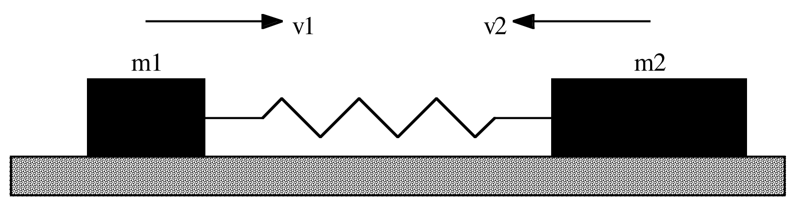
- The system of the blocks and the spring is a closed system, and the horizontal component of the external force is 0 N. The horizontal component of the linear momentum is thus conserved.
- Initially the masses are at rest, and the total linear momentum is thus 0 kg m/s.
- At any point in time, the velocities of block 1 and block 2 are related:

$$v_2 = -\frac{m_1}{m_2} v_1$$



Conservation of linear momentum. An example.

- The kinetic energies of mass m_1 and m_2 are thus equal to



$$K_1 = \frac{1}{2} m_1 v_1^2$$

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\frac{1}{m_2}\left(m_2 v_2\right)^2 = \frac{1}{2}\frac{m_1}{m_2}\frac{\left(m_1 v_1\right)^2}{m_1} = \frac{m_1}{m_2} K_1$$

- The fraction of the total kinetic energy carried away by block 1 is equal to

$$f_1 = \frac{K_1}{K_t} = \frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + \frac{m_1}{m_2} K_1} = \frac{m_2}{m_1 + m_2}$$

Systems with variable mass.

- Rocket motion is an example of a system with a variable mass:

$$M(t+dt) = M(t) + dM$$

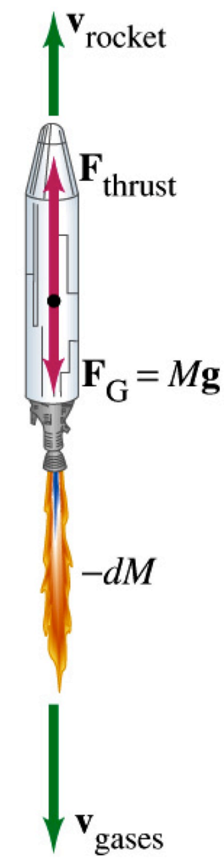
Mass of exhaust.
dM < 0 kg

- As a result of dumping the exhaust, the rocket will increase its velocity:

$$v(t+dt) = v(t) + dv$$

- Since this is an isolated system, linear momentum must be conserved. The initial momentum is equal to

$$p_i = M(t)v(t)$$



Systems with variable mass.

- The final linear momentum of the system is given by

$$p_f = \left(M(t) + dM\right)\left(v(t) + dv\right) + \left(-dM\right)U$$

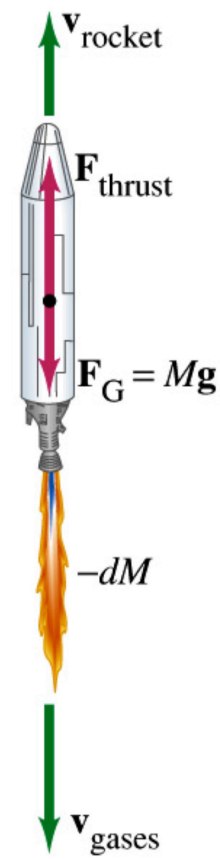
where U is the velocity of the exhaust.

- Conservation of linear momentum therefore requires that

$$M(t)v(t) = \left(M(t) + dM\right)\left(v(t) + dv\right) + \left(-dM\right)U$$

- The exhaust has a fixed velocity U_0 with respect to the engine. U_0 and U are related in the following way:

$$U - U_0 = v(t) + dv$$



Systems with Variable Mass.

- Conservation of linear momentum can now be rewritten as

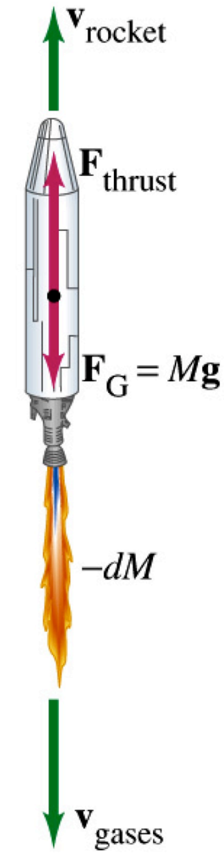
$$M(t)v(t) = \left(M(t)+dM\right)\left(v(t)+dv\right) + \left(-dM\right)\left(v(t)+dv+U_0\right)$$

or

$$M(t)v(t) = M(t)\left(v(t)+dv\right) - \left(dM\right)U_0$$

- We conclude

$$\left(dM\right)U_0 = M(t)\left(dv\right)$$



Systems with variable mass.

- The previous equation can be rewritten as

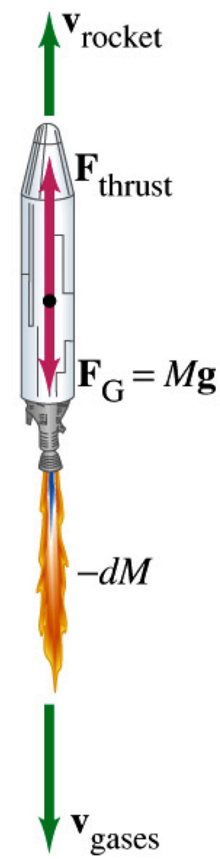
$$\left(\frac{dM}{dt}\right)U_0 = M(t)\frac{dv(t)}{dt}$$

In this equation:

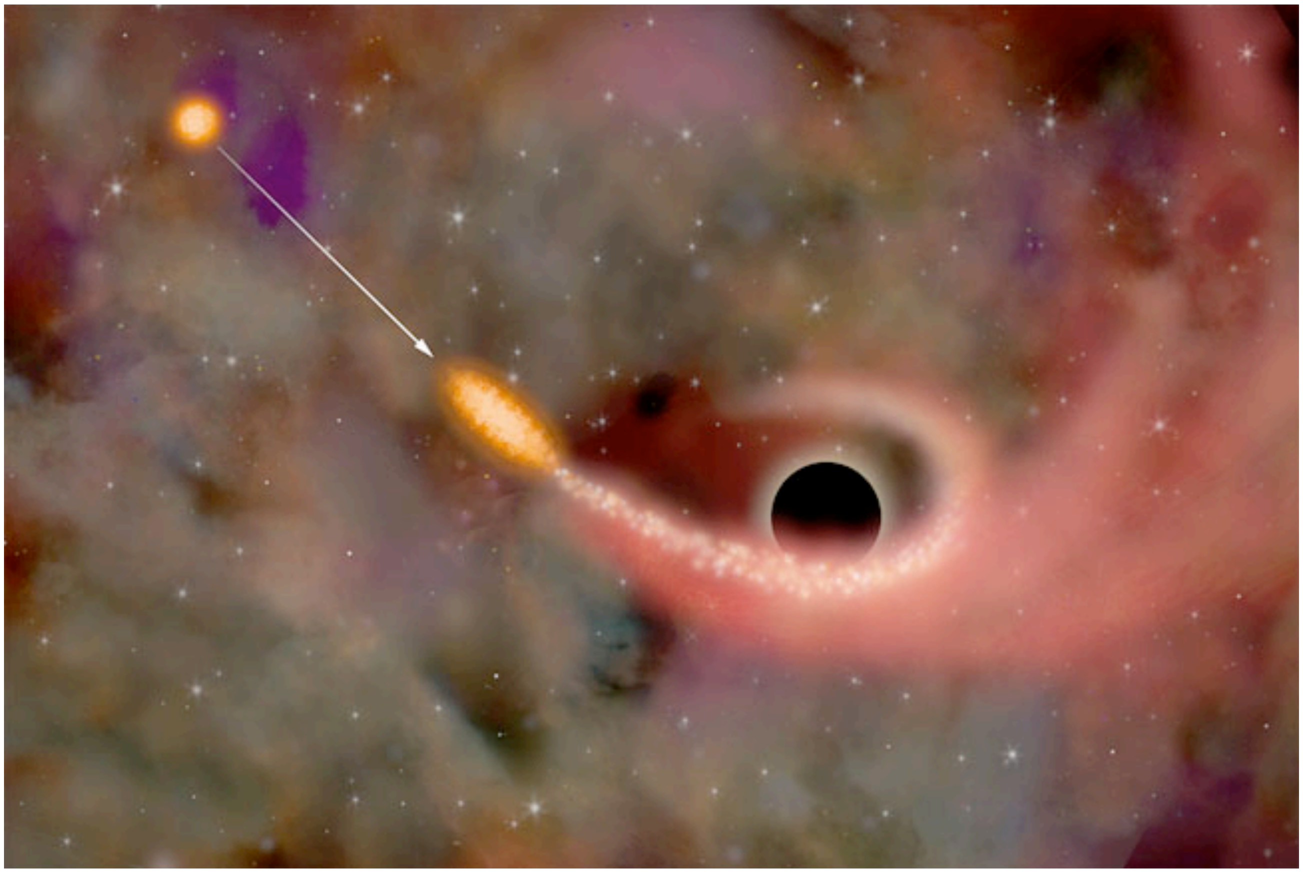
  - $dM/dt = -R$ where R is the rate of fuel consumption.
  - $U_0 = -u$ where u is the (positive) velocity of the exhaust gasses relative to the rocket.
  - dv/dt is the acceleration of the rocket.

- This equation can be rewritten as $RU_0 = Ma_{\text{rocket}}$ which is called the **first rocket equation**. This equation can be used to find the velocity of the rocket (**second rocket equation**):

$$v_f = v_i + u\ln\left(\frac{M_i}{M_f}\right)$$



Done for today!
Next week: collisions.



X-Rays Indicate Star Ripped Up by Black Hole. Illustration Credit: M. Weiss, CXC, NASA