Physics 237, Final Exam

Wednesday May 5, 2010

7.15 pm – 10.15 pm

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument and your equation sheet while taking this test. You may *not* consult any calculators, computers, books, or each other.

1. Problems 1, 2, and 3 must be answered in booklet # 1.

2. Problems 4, 5, 6, and 7 must be answered in booklet # 2.

3. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. **Answers that are not motivated will not receive any credit, even if correct**.

At the end of the exam, you need to hand in your exam, your equation sheet, and the three blue exam booklets. All items must be clearly labeled with your name, your student ID number, and the day/time of your workshop.

Name: ____

ID number: _______________________________________

Workshop Day/Time: _________________________________

Useful Constants and Conversion Factors

Quoted to a useful number of significant figures.

Speed of light in vacuum	$c = 2.998 \times 10^{8}$ m/sec
Electron charge magnitude	$e = 1.602 \times 10^{-19}$ coul
Planck's constant	$h = 6.626 \times 10^{-34}$ joule-sec
	$\hbar = h/2\pi = 1.055 \times 10^{-34}$ joule-sec
	$= 0.6582 \times 10^{-15}$ eV-sec
Boltzmann's constant	$k = 1.381 \times 10^{-23}$ joule/°K
	$= 8.617 \times 10^{-5}$ eV/°K
Avogadro's number	$N_0 = 6.023 \times 10^{23}$/mole
Coulomb's law constant	$1/4\pi\epsilon_0 = 8.988 \times 10^{9}$ nt-m²/coul²
Electron rest mass	$m_e = 9.109 \times 10^{-31}$ kg $= 0.5110$ MeV/c^2
Proton rest mass	$m_p = 1.672 \times 10^{-27}$ kg $= 938.3$ MeV/c^2
Neutron rest mass	$m_n = 1.675 \times 10^{-27}$ kg $= 939.6$ MeV/c^2
Atomic mass unit ($C^{12} \equiv 12$)	$u = 1.661 \times 10^{-27}$ kg $= 931.5$ MeV/c^2
Bohr magneton	$\mu_b = e\hbar/2m_e = 9.27 \times 10^{-24}$ amp-m² (or joule/tesla)
Nuclear magneton	$\mu_n = e\hbar/2m_p = 5.05 \times 10^{-27}$ amp-m² (or joule/tesla)
Bohr radius	$a_0 = 4\pi\epsilon_0\hbar^2/m_e e^2 = 5.29 \times 10^{-11}$ m $= 0.529$ Å
Bohr energy	$E_1 = -m_e e^4/(4\pi\epsilon_0)^2 2\hbar^2 = -2.17 \times 10^{-18}$ joule $= -13.6$ eV
Electron Compton wavelength	$\lambda_C = h/m_e c = 2.43 \times 10^{-12}$ m $= 0.0243$ Å
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c = 7.30 \times 10^{-3} \simeq 1/137$
kT at room temperature	$k300°\text{K} = 0.0258$ eV $\simeq 1/40$ eV

1 eV $= 1.602 \times 10^{-19}$ joule   1 joule $= 6.242 \times 10^{18}$ eV

1 Å $= 10^{-10}$ m   1 F $= 10^{-15}$ m   1 barn (bn) $= 10^{-28}$ m²

Table 7-2 Some Eigenfunctions for the One-Electron Atom

Quantum Numbers n	l	m_l	Eigenfunctions
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(2-\frac{Zr}{a_0}\right) e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}\cos\theta$
2	1	± 1	$\psi_{21\pm 1} = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}\sin\theta\, e^{\pm i\varphi}$
3	0	0	$\psi_{300} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(27-18\frac{Zr}{a_0}+2\frac{Z^2r^2}{a_0^2}\right) e^{-Zr/3a_0}$
3	1	0	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6-\frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0}\cos\theta$
3	1	± 1	$\psi_{31\pm 1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6-\frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0}\sin\theta\, e^{\pm i\varphi}$
3	2	0	$\psi_{320} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}(3\cos^2\theta-1)$
3	2	± 1	$\psi_{32\pm 1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}\sin\theta\cos\theta\, e^{\pm i\varphi}$
3	2	± 2	$\psi_{32\pm 2} = \frac{1}{162\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}\sin^2\theta\, e^{\pm 2i\varphi}$

Table 17-1. Particles that are Stable or Decay either Weakly or Electromagnetically

Generic Name	Particle Symbol	Rest Mass (MeV/c^2)	Lifetime (sec)	Charge Q	Intrinsic Spin s	Lepton Number L_e, L_μ, or L_τ	Baryon Number B	Intrinsic Parity P	Isospin T	Isospin z component T_z	Strangeness S
Photon	γ	0	stable	0	1	0	0	Odd	0, 1	0	0
Leptons	ν_e	0	stable	0	1/2	+1	0				
	ν_μ	0	stable	0	1/2	+1	0				
	ν_τ	0	stable	0	1/2	+1	0				
	e^-	0.511	stable	−1	1/2	+1	0				
	μ^-	105.7	2.2×10^{-6}	−1	1/2	+1	0				
	τ^-	1784	5×10^{-13}	−1	1/2	+1	0				
Mesons	π^+	139.6	2.6×10^{-8}	+1	0	0	0	Odd	1	+1	0
	π^0	135.0	8×10^{-17}	0	0	0	0	Odd	1	0	0
	π^-	139.6	2.6×10^{-8}	−1	0	0	0	Odd	1	−1	0
	K^+	493.8	1.2×10^{-8}	+1	0	0	0	Odd	1/2	+1/2	+1
	K^0	497.8	(8.9×10^{-11} and	0	0	0	0	Odd	1/2	−1/2	+1
	$\overline{K^0}$	497.8	5.2×10^{-8})	0	0	0	0	Odd	1/2	+1/2	−1
	K^-	493.8	1.2×10^{-8}	−1	0	0	0	Odd	1/2	−1/2	−1
	η^0	549	8×10^{-19}	0	0	0	0	Odd	0	0	0
	η'	958	2×10^{-21}	0	0	0	0	Odd	0	0	0
Baryons	p	938.3	stable	+1	1/2	0	+1	Even	1/2	+1/2	0
	n	939.6	925	0	1/2	0	+1	Even	1/2	−1/2	0
	Λ^0	1116	2.6×10^{-10}	0	1/2	0	+1	Even	0	0	−1
	Σ^+	1189	8.0×10^{-11}	+1	1/2	0	+1	Even	1	+1	−1
	Σ^0	1192	6×10^{-20}	0	1/2	0	+1	Even	1	0	−1
	Σ^-	1197	1.5×10^{-10}	−1	1/2	0	+1	Even	1	−1	−1
	Ξ^0	1315	2.9×10^{-10}	0	1/2	0	+1	Even	1/2	+1/2	−2
	Ξ^-	1321	1.6×10^{-10}	−1	1/2	0	+1	Even	1/2	−1/2	−2
	Ω^-	1672	8.2×10^{-11}	−1	3/2	0	+1	Even	0	0	−3

Table 17-3. Applicability of the Conservation Laws to the Observed Interactions ("yes" Means Conserved; "no" Means Not Conserved)

Quantity Conserved	Strong	Electro-magnetic	Weak
Energy	yes	yes	yes
Linear momentum	yes	yes	yes
Angular momentum	yes	yes	yes
Charge	yes	yes	yes
Electronic lepton number	yes	yes	yes
Muonic lepton number	yes	yes	yes
Tauonic lepton number	yes	yes	yes
Baryon number	yes	yes	yes
Isospin magnitude	yes	no	no ($\Delta T = 1/2$ for nonleptonic)
Isospin z component	yes	yes	no ($\Delta T_z = 1/2$ for nonleptonic)
Strangeness	yes	yes	no ($\Delta S = 1$)
Parity	yes	yes	no
Charge conjugation	yes	yes	no
Time reversal (or CP)	yes	yes	yes (But 10^{-3} violation in K^0 decay)

INTENTIONALLY LEFT BLANK

Problem 1 (35 points) **ANSWER IN BOOKLET 1**

Consider the motion of an electron of mass m under the influence of the following potential V:

$$V(x) = \infty \qquad x < 0 \text{ and } x > a$$
$$V(x) = -V_0 \quad 0 < x < a$$

a) What is the general solution of the time-independent Schrödinger equation for this system?

b) What is the energy of the solution obtained in part a)?

c) What is the zero-point energy of the system?

d) If at time $t = 0$, the electron is in a state corresponding to the first excited state, what is the probability that the electron will be in that same state at time $t = \lambda_C/c$ where λ_C is the Compton wavelength of the electron?

Problem 2 (30 points) **ANSWER IN BOOKLET 1**

Consider one electron in the n = 2 shell of a one-electron atom. If we ignore the spin-orbit coupling, then the wavefunctions in the n = 2 shell are degenerate. Assume that the electron can be found with equal probability in each of the degenerate n = 2 states.

a) Write down the wavefunction describing this electron in terms of the eigenfunctions of the one-electron atom. Make sure your wavefunction is properly normalized.

b) What is the shape of the probability density distribution of this electron? You will need to specify the r dependence, the θ dependence, and the ϕ dependence of the probability density distribution.

Problem 3 (35 points) **ANSWER IN BOOKLET 1**

Consider a He atom. When He is in its ground state, both electrons are in the 1*s* state.

a) What is the proper spectroscopic notation of the ground state of He?

If we shine ultraviolet light on the He atoms, we can excite the atom into its first few excited states. These excited states have one electron in the 1*s* state and one electron in the 2*s* or in the 2*p* state.

b) What is the proper spectroscopic notation of the excited states in He when the electrons are in a (1*s*)(2*s*) configuration?

c) What is the proper spectroscopic notation of the excited states in He when the electrons are in a (1*s*)(2*p*) configuration?

d) Draw an energy-level diagram, showing all the states of He discussed in parts a), b), and c). Label each state with the proper spectroscopic notation. **Do <u>not</u> ignore the spin-orbit coupling.**

e) In the diagram obtained in part d), indicate which transitions can occur between the excited states discussed in parts b) and c) and the ground state discussed in part a).

f) If we put the atom in an external magnetic field of strength B, we see an increase in the number of transitions we can observe. How many transitions will we be able to observe now?

Problem 4 (30 points) **ANSWER IN BOOKLET 2**

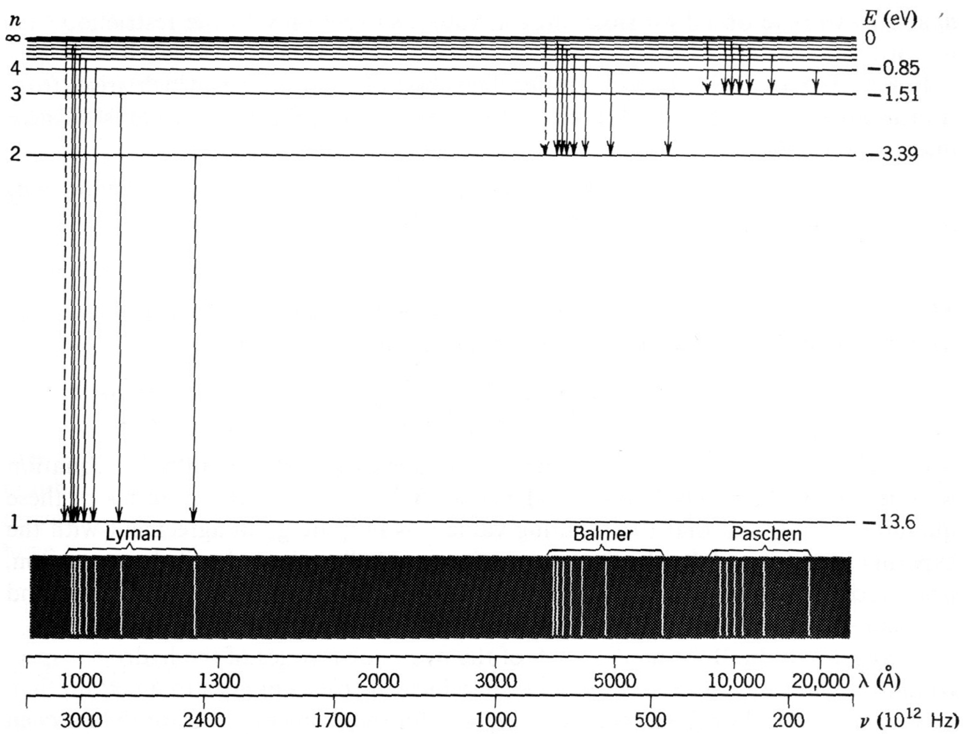
Consider a photon, moving in one dimension in a region where $V = 0$.

a) Starting with the relativistic expression for total energy in terms of linear momentum and mass, formulate the Schrödinger equation for the photon.

b) Use separation of variables to solve the Schrödinger equation. What is the wavefunction that describes the photon?

Problem 5 (30 points) **ANSWER IN BOOKLET 2**

An energy diagram for Hydrogen is shown in the Figure below.

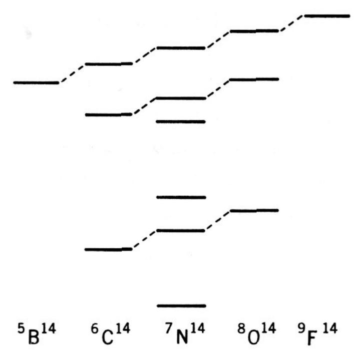


Two techniques can be used to confirm the energies of the levels of Hydrogen: studies of emission spectra and studies of absorption spectra.

At low temperatures, the absorption spectrum is dominated by transitions that are part of the Lyman series. Estimate the temperature at which Balmer lines will be observed in the absorption spectrum. Note: you do **not** need to evaluate the numerical expression you obtain for the temperature T.

Problem 6 (35 points) **ANSWER IN BOOKLET 2**

a) Consider the following energy-level diagram for atoms with 14 nucleons. Energy levels with similar properties are connected with dashed lines.



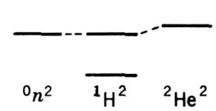
Identify the isospin and the z component of the isospin for all states shown in the diagram.

b) Consider the following reactions:

$$\Lambda^0 \to n + \gamma$$
$$\pi^+ + p \to p + p + \bar{n}$$
$$\bar{n} \to \bar{p} + e^+ + \nu_e$$
$$\pi^- + p \to \Sigma^+ + K^-$$

For each of these reactions, state the fastest interaction through which it can proceed. If the reaction is forbidden, indicate why.

c) Consider the 4 lowest energy levels observed for the two-nucleon system, shown in the Figure below.



For each of the four states shown in the Figure, determine the total spin. Which of these states are stable? What is the cause of the energy difference between the state shown for ^{2}He and the state shown for ^{0}n?

Problem 7 (5 points) **ANSWER IN BOOKLET 2**

Match the following players

a) Y Berra
b) B Williams
c) M Mantle
d) D Jeter
e) L Gehrig

to the following At Bat (AB) numbers of these players for the New York Yankees:

___ 8757
___ 8102
___ 8001
___ 7869
___ 7546

Note: these number were correct on May 1, 2010 but may have changed on the day of the exam.

Red Sox leadoff man Jacoby Ellsbury breaks his bat while hitting a pitch by Yankees lefty Andy Pettitte.
Brita Meng Outzen/MLB.com