Quantum Mechanics
Physics 237

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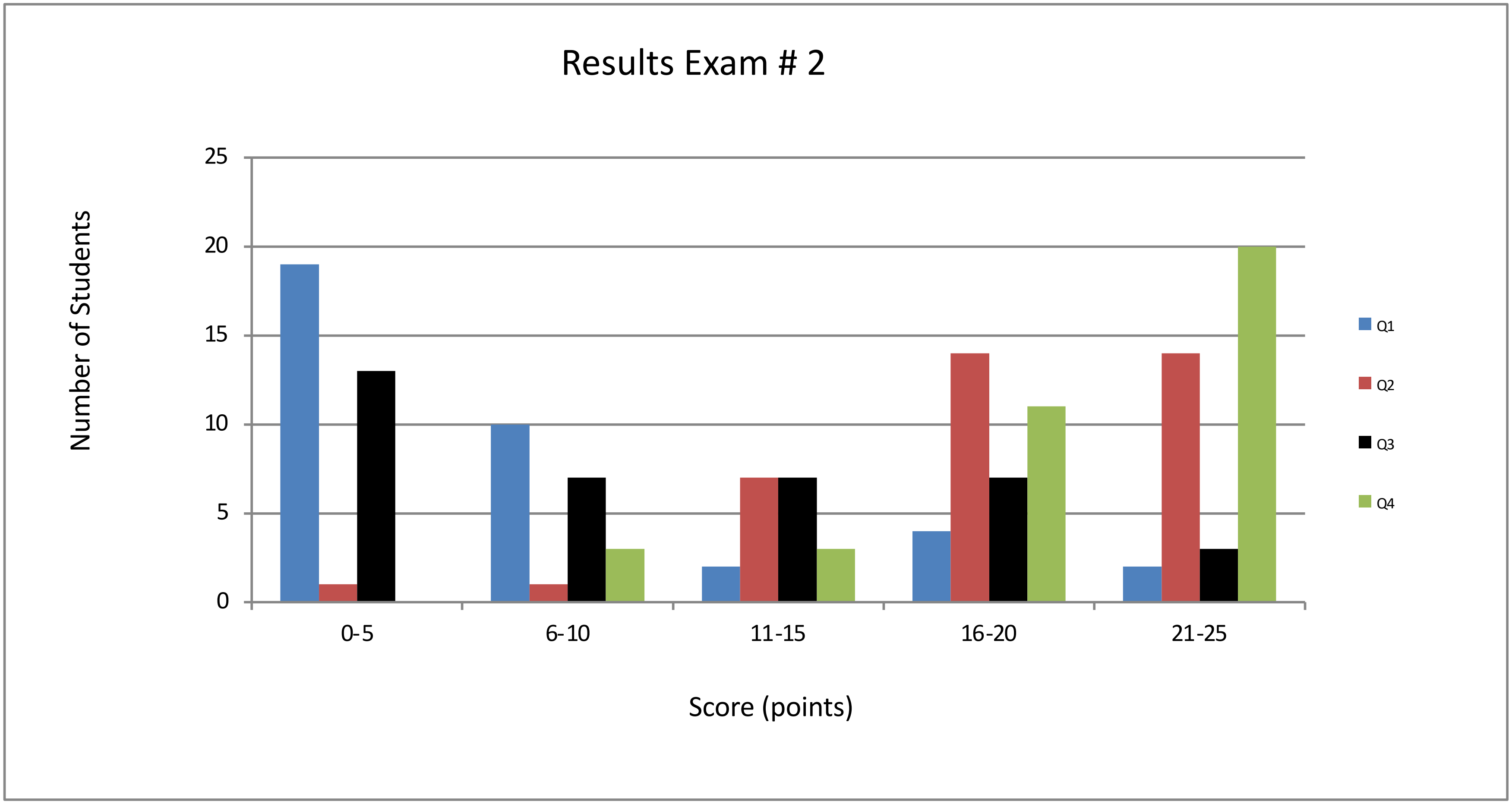
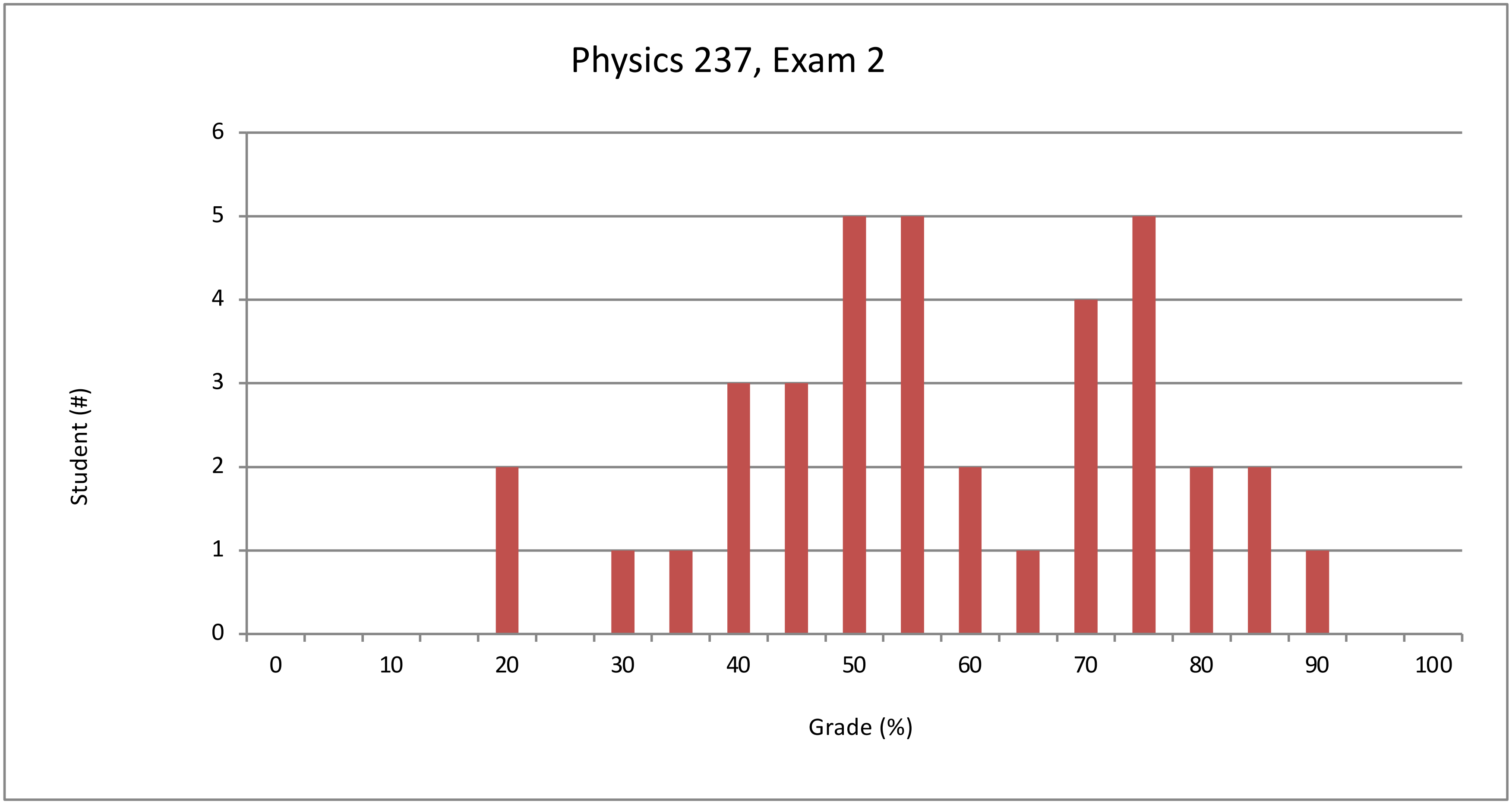
Announcements

- Homework # 8 is due on Friday April 1.
- Exam # 2 is/will be returned this week during recitations.
- Reminder:
  - Requests to regrade certain parts of Exam # 2 will need to be submitted via email to Prof. Wolfs in writing (with a copy of the graded exam) by Thursday April 7.

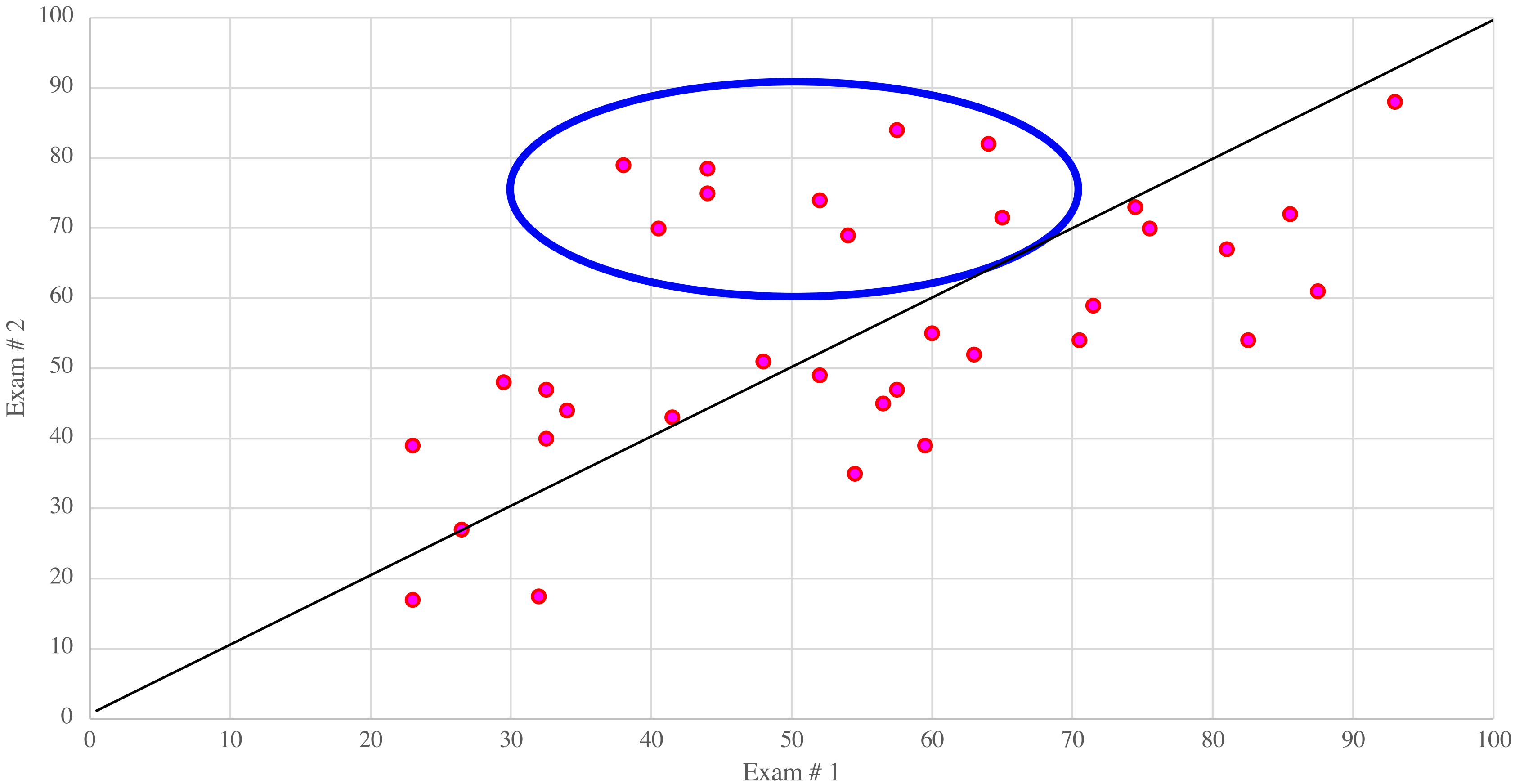
Results Exam # 2.



Results Exam # 2.



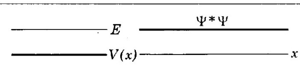
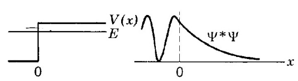
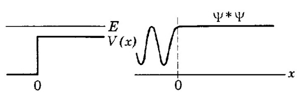
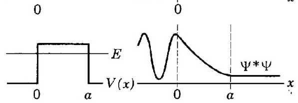
Some dramatic improvements on Exam # 2.



Sometimes I give useful hints …..

Exam # 2: knowing the wavefunction in different regions ($V > E$ and $V < E$) is important.

Table 6-2. A Summary of the Systems Studied in Chapter 6

Name of System	Physical Example	Potential and Total Energies	Probability Density	Significant Feature
Zero potential	Proton in beam from cyclotron			Results used for other systems
Step potential (energy below top)	Conduction electron near surface of metal			Penetration of excluded region
Step potential (energy above top)	Neutron trying to escape nucleus			Partial reflection at potential discontinuity
Barrier potential (energy below top)	α particle trying to escape Coloumb barrier			Tunneling

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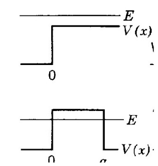
But what happens when $E = V$?

- When $E = V$, the Schrödinger equation reduces to

$$(d^2/dx^2)\ \psi = 0$$

and the solution is

$$\psi = Cx + D$$



Exam # 2: one more comment.

- Transitions between states are possible when the expectation value of the dipole moment is none zero:

$$\langle \vec{p}_{fi} \rangle = e \langle \vec{r} \rangle_{fi}$$

- This requires you to evaluate the expectation value of the vector r, **not the expectation value of the radial distance *r*.**

F.3 Spherical Coordinates

Refer to Figures F-3 and F-4

$$x_1 = r \sin\theta \cos\phi, \quad x_2 = r \sin\theta \sin\phi, \quad x_3 = r \cos\theta \qquad \text{(F.13)}$$

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2}, \quad \theta = \cos^{-1}\frac{x_3}{r}, \quad \phi = \tan^{-1}\frac{x_2}{x_1} \qquad \text{(F.14)}$$

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \qquad \text{(F.15)}$$

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi \qquad \text{(F.16)}$$

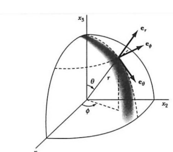


FIGURE F-3

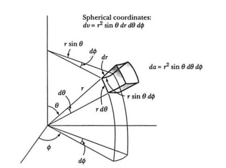


FIGURE F-4

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Digital Obsolescence.
It can happen quickly!!!

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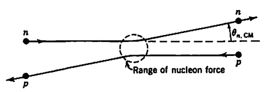
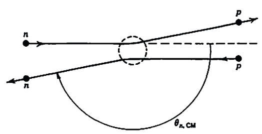
$$V_{scatter} = \frac{1}{2}\left(V(r) + V(r)P\right)$$



The scattering cross section is proportional to $\psi_f^* V_{scatter} \psi_i$. The product of the scattering potential and the initial wavefunction can be written as



$$V_{scatter}\psi_i = \frac{1}{2}\left(V(r) + V(r)P\right)\psi_i = \frac{1}{2}V(r)\psi_i + \frac{1}{2}V(r)P\psi_i$$

The exchange operator changes the proton into a neutron and vice versa. As a consequence, the effect of the exchange operator for a two-nucleon system with one proton and one neutron

$$P\psi_i = (-1)^\ell \psi_i \quad \Rightarrow \quad V_{scatter} = \frac{1}{2}\left(V(r) + V(r)P\right) = \frac{1}{2}V(r)\left(1 + (-1)^\ell\right)$$

This potential is also called the **Serber potential**. We note that when the orbital angular momentum is odd, the scattering potential is 0; when the orbital angular momentum is even, the scattering potential is *V*. The nucleon potential thus depends on the orbital angular momentum of the interacting nucleons.

We can use a classical picture to connect a certain kinetic energy *K* to a certain orbital angular momentum. Consider a state with an orbital angular momentum ℓ. If we look at this system in the center-of-mass reference frame of the two nucleons, we must require that each nucleon has a linear momentum *p* obtained in the following manner:

$$L = \sqrt{\ell(\ell+1)}\hbar \approx p\left(\frac{r}{2}\right) + p\left(\frac{r}{2}\right) = pr \quad \Rightarrow \quad p = \frac{\sqrt{\ell(\ell+1)}\hbar}{r}$$

where *r* is the largest distance at which the strong force acts. The kinetic energy of the two nucleons is thus be equal to

$$K_p + K_n = 2\frac{p_n^2}{2m_n} = \frac{\ell(\ell+1)\hbar^2}{m_n r^2}$$

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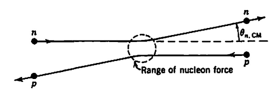
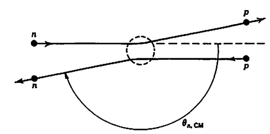


The scattering cross section is proportional to . The product of the scattering potential and the initial wavefunction can be written as



The exchange operator changes the proton into a neutron and vice versa. As a consequence, the effect of the exchange operator for a two-nucleon system with one proton and one neutron

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I started the conversion to latex. A slow process but it will allow me fix mistakes!

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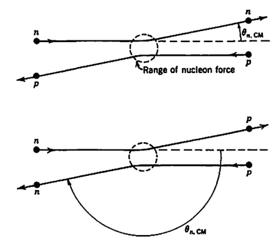


Figure 71: The exchange operator at work.

The exchange operator changes the proton into a neutron and vice versa. As a consequence, the effect of the exchange operator for a two-nucleon system with one proton and one neutron

$$P\psi_i = (-1)^{\ell}\psi_i \quad \Rightarrow \quad V_{scatter} = \frac{1}{2}\left(V\left(r\right) + V\left(r\right)P\right) = \frac{1}{2}V\left(r\right)\left(1 + (-1)^{\ell}\right) \tag{17.5}$$

This potential is also called the **Serber potential**. We note that when the orbital angular momentum is odd, the scattering potential is 0; when the orbital angular momentum is even, the scattering potential is V. The nucleon potential thus depends on the orbital angular momentum of the interacting nucleons.
We can use a classical picture to connect a certain kinetic energy K to a certain orbital angular momentum. Consider a state with an orbital angular momentum ℓ. If we look at this system in the center-of-mass reference frame of the two nucleons, we must require that each nucleon has a linear momentum p obtained in the following manner:

$$L = \sqrt{\ell\left(\ell+1\right)} \approx p\left(\frac{r}{2}\right) + p\left(\frac{r}{2}\right) = pr \quad \Rightarrow \quad p = \frac{\sqrt{\ell\left(\ell+1\right)}}{r} \tag{17.6}$$

where r is the largest distance at which the strong force acts. The kinetic energy of the two nucleons is thus be equal to

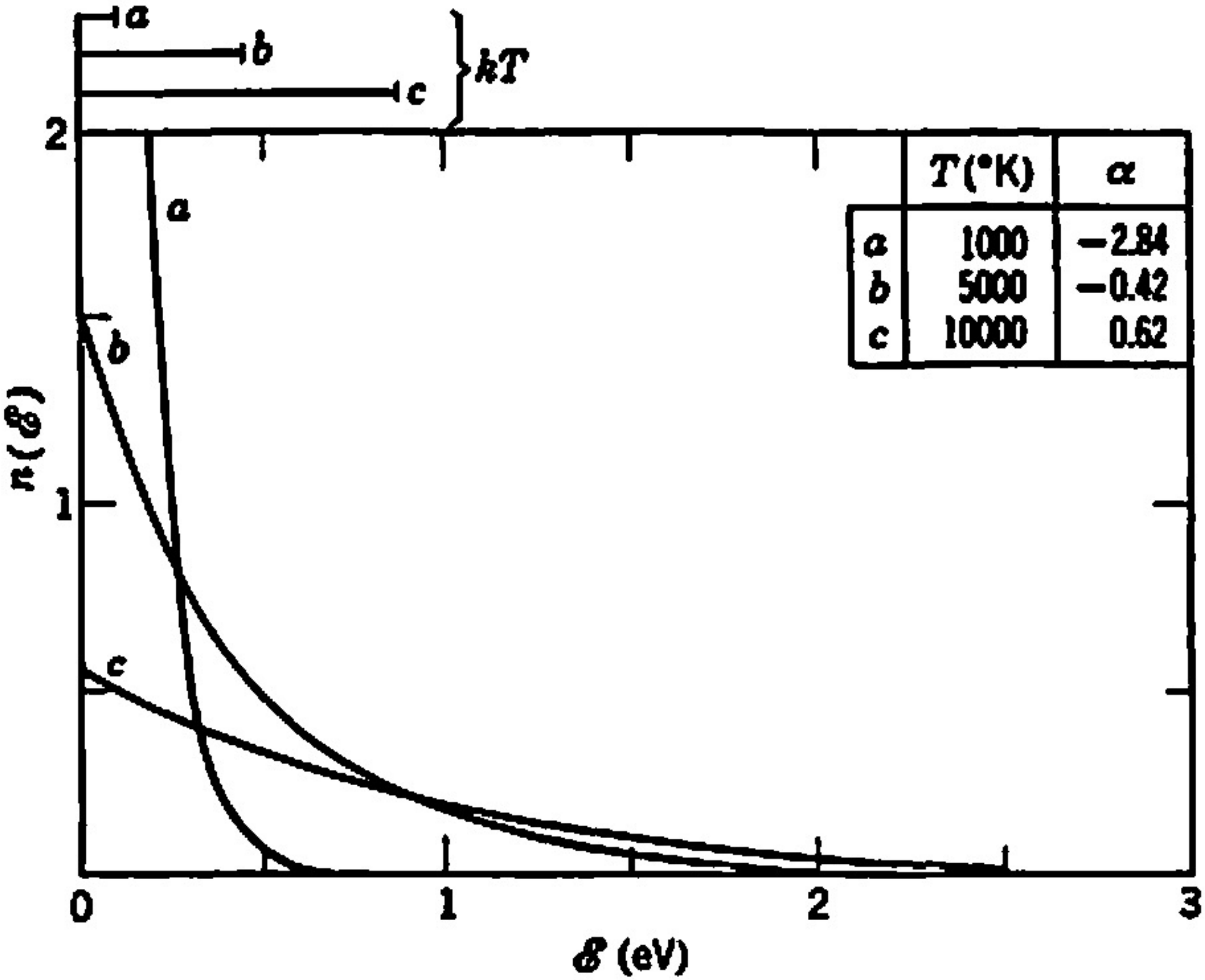
$$K_p + K_n = 2\frac{{p_n}^2}{2m_n} = \frac{\ell\left(\ell+1\right)^2}{m_n r^2} \tag{17.7}$$

If the orbital angular momentum parameter is equal to 1, the total kinetic energy is 20 MeV. If $\ell = 2$, the total kinetic energy is 60 MeV, etc. If the kinetic energy is less than 20 MeV, the distance r must increase in order to achieve $\ell = 1$ but an increase in r creates a separation between the nucleons that is larger than the range of the strong force and as a result, the $\ell = 1$scattering process is not influenced by the strong force. Consider the following examples:

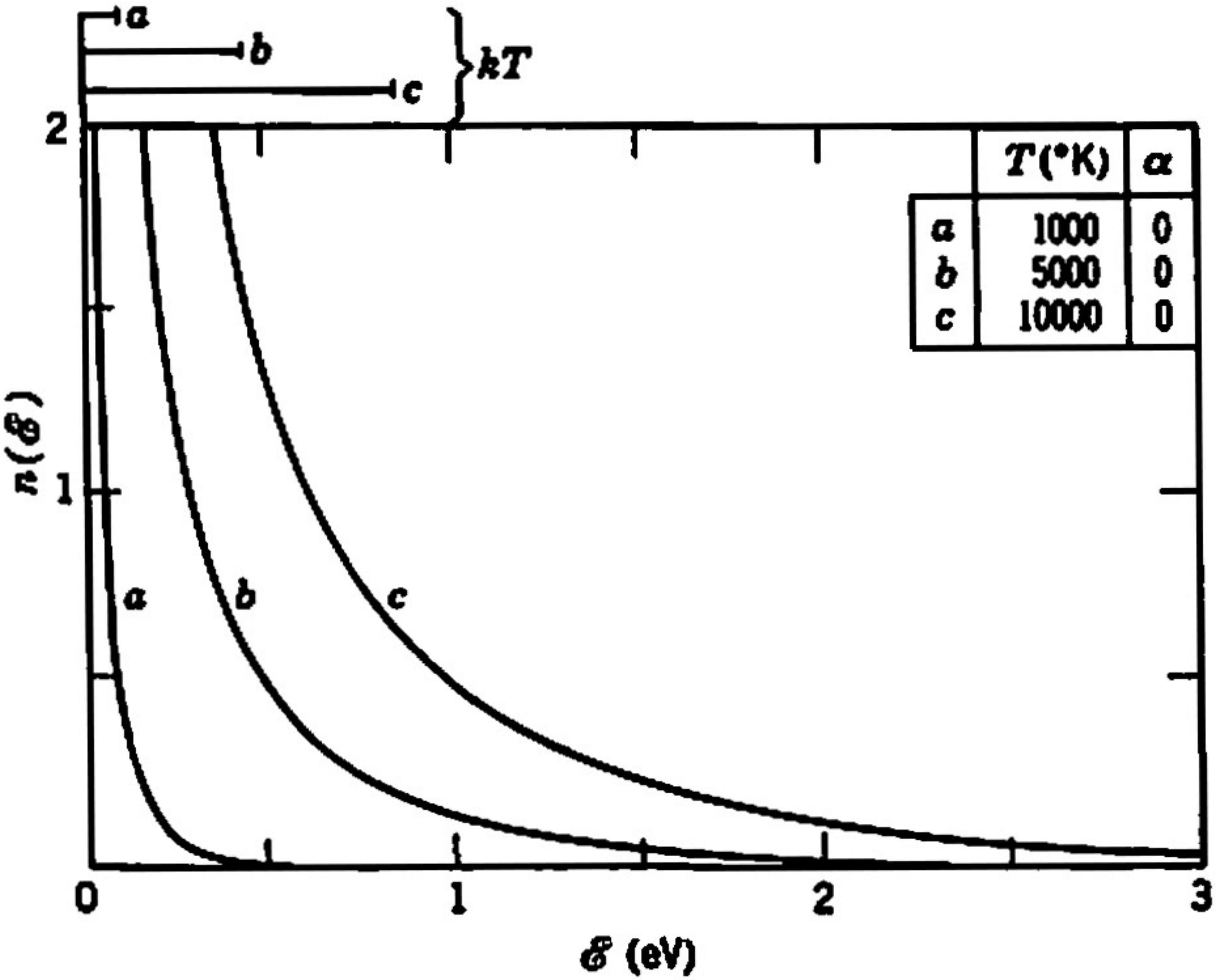
1. ***K* = 40 MeV**. The scattering process will only be influenced by $\ell = 0$ and $\ell = 1$ scattering. But, for $\ell = 1$ $V(r) = 0$ and the scattering process only involves $\ell = 0$ contributions. The wavefunctions associated with $\ell = 0$ have spherical symmetry and the scattering process is thus isotropic.

2. ***K* = 330 MeV**. At this energy, the maximum orbital angular momentum parameter that can contribute is $\ell = 3$. Since for odd values of ℓ the scattering potential is 0, we only need to consider even values of ℓ. The scattering process is thus determined by the scattering associated with and $\ell = 2$.

The Boltzmann distribution. Particle distributions at constant density.



The Bose distribution.

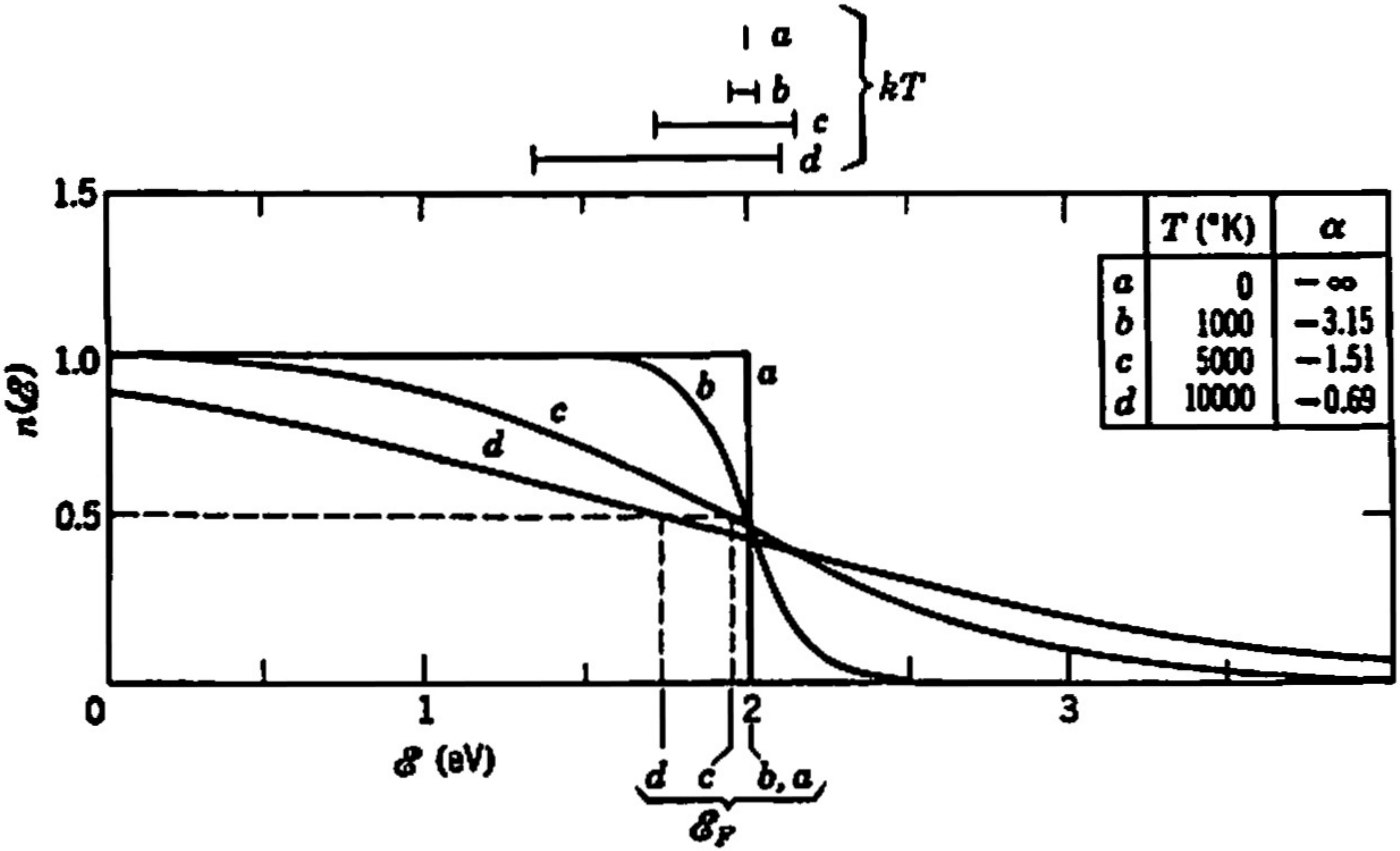


2 Minute 56 Second Intermission.

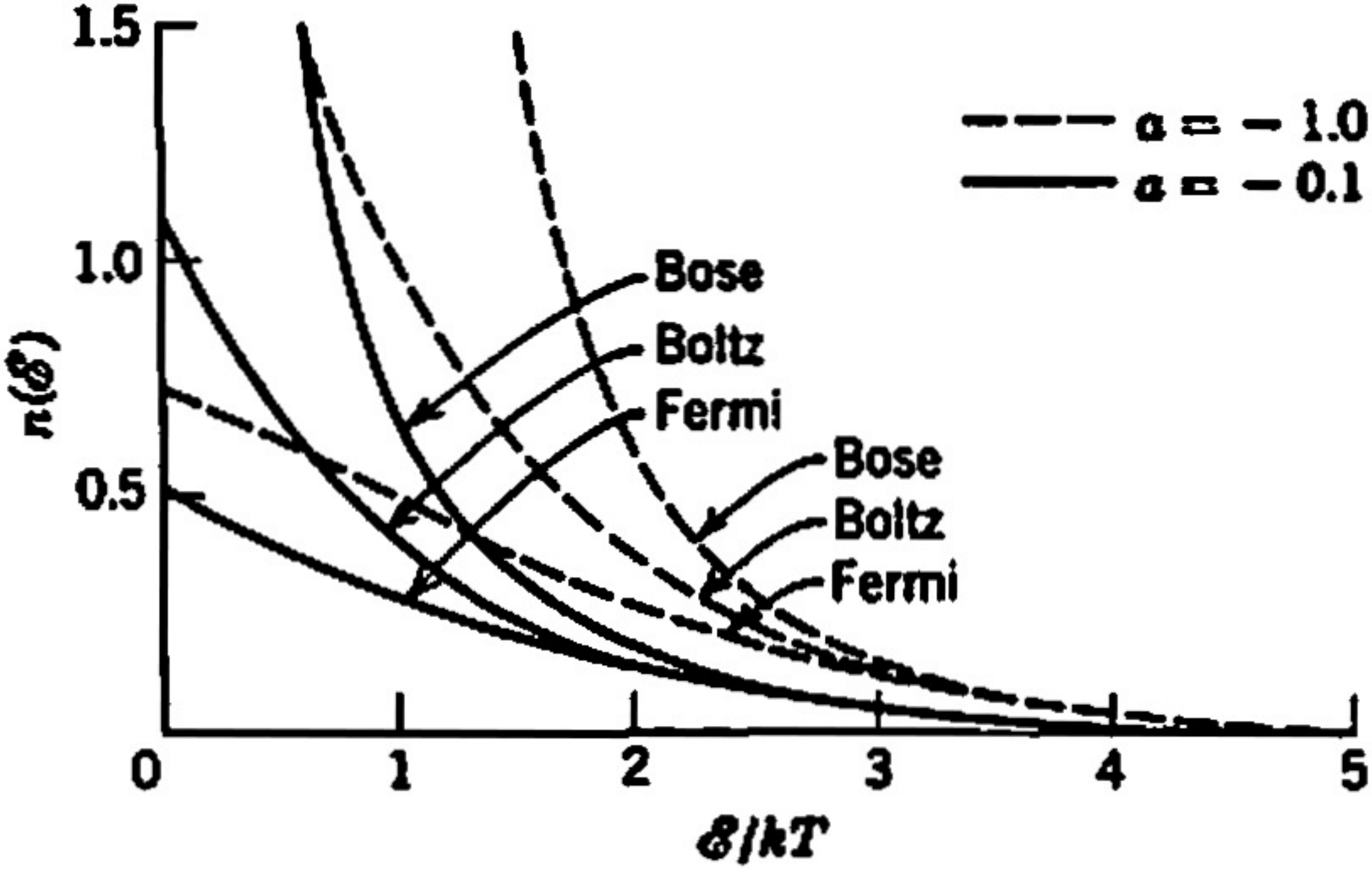
- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 56 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

The Fermi Distribution.



Comparing the distributions.



ENOUGH FOR TODAY?