Quantum Mechanics
Physics 237

Frank L. H. Wolfs
Department of Physics and Astronomy
University of Rochester

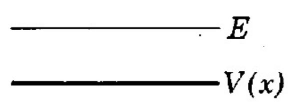
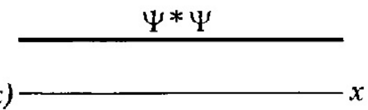
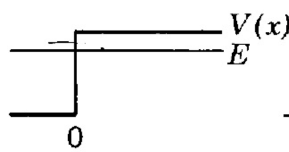
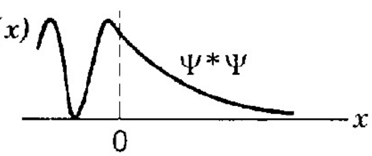
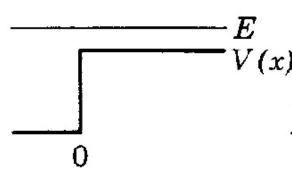
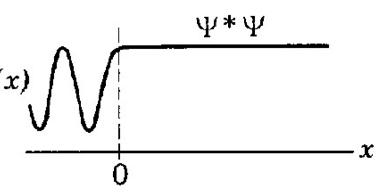
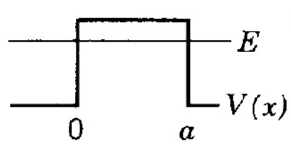
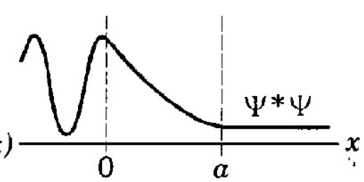
Announcements

- Midterm Exam # 2 will take place on Tuesday March 22 between 8 am EST and 9.30 EST.
- The material covered are Chapters 5 – 8.
- You will receive an equation sheet with the most important equations we discussed in these four Chapters.
- There will be extra office hours on Sunday March 20 and Monday March 21. Monday's recitation will a QA session on the material to be covered on the exam.

- **There will be no recitations and office hours on Wednesday March 23 and on Thursday March 24.**

Exam # 2: knowing the wavefunction in different regions ($V > E$ and $V < E$) is important.

Table 6-2. A Summary of the Systems Studied in Chapter 6

Name of System	Physical Example	Potential and Total Energies	Probability Density	Significant Feature
Zero potential	Proton in beam from cyclotron			Results used for other systems
Step potential (energy below top)	Conduction electron near surface of metal			Penetration of excluded region
Step potential (energy above top)	Neutron trying to escape nucleus			Partial reflection at potential discontinuity
Barrier potential (energy below top)	α particle trying to escape Coloumb barrier			Tunneling

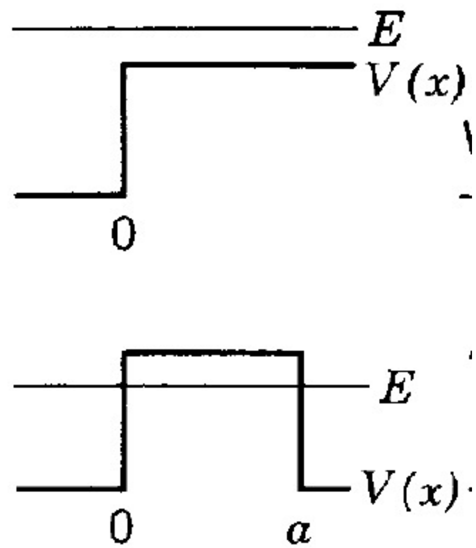
But what happens when $E = V$?

- When $E = V$, the Schrödinger equation reduces to

$$(d^2/dx^2)\ \psi = 0$$

and the solution is

$$\psi = Cx + D$$



Exam # 2: one more comment.

- Transitions between states are possible when the expectation value of the dipole moment is none zero:

$$\langle \vec{p}_{fi} \rangle = e \langle \vec{r} \rangle_{fi}$$

- This requires you to evaluate the expectation value of the vector r, **not the expectation value of the radial distance *r*.**

F.3 Spherical Coordinates

Refer to Figures F-3 and F-4

$$x_1 = r \sin\theta \cos\phi, \quad x_2 = r \sin\theta \sin\phi, \quad x_3 = r \cos\theta \qquad \text{(F.13)}$$

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2}, \quad \theta = \cos^{-1}\frac{x_3}{r}, \quad \phi = \tan^{-1}\frac{x_2}{x_1} \qquad \text{(F.14)}$$

$$ds^2 = dr^2 + r^2\,d\theta^2 + r^2 \sin^2\theta\,d\phi^2 \qquad \text{(F.15)}$$

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi \qquad \text{(F.16)}$$

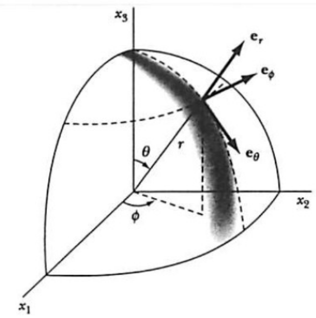


FIGURE F-3

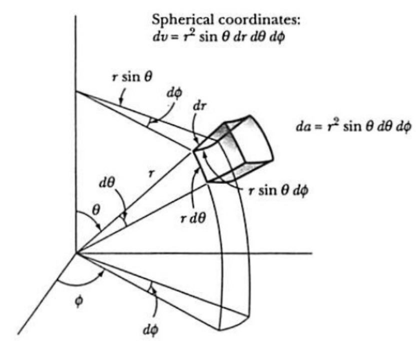


FIGURE F-4

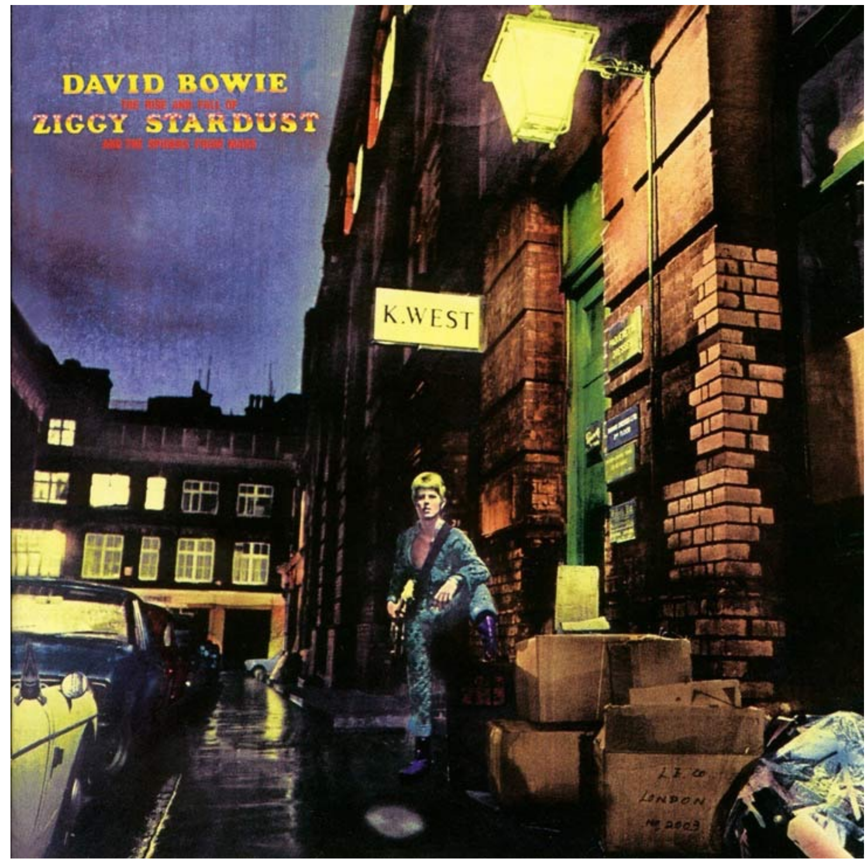
Chapter 9.
Multi-electron Atoms.

- We start our study of multi-electron atoms by looking at an atom with two electrons.
  - We assume no mutual interactions between the atoms.
  - We assume that we cannot distinguish the two electrons.
  - The requirement that we cannot distinguish the two electrons requires that the probability density distribution of the wavefunction does not change when we exchange particle 1 and particle 2.
- We conclude:
  - If the wavefunction of the two-electron system is asymmetric, the particles cannot have the same quantum numbers.
  - If the wavefunction of the two-electron system is symmetric, the particles can have the same quantum numbers.

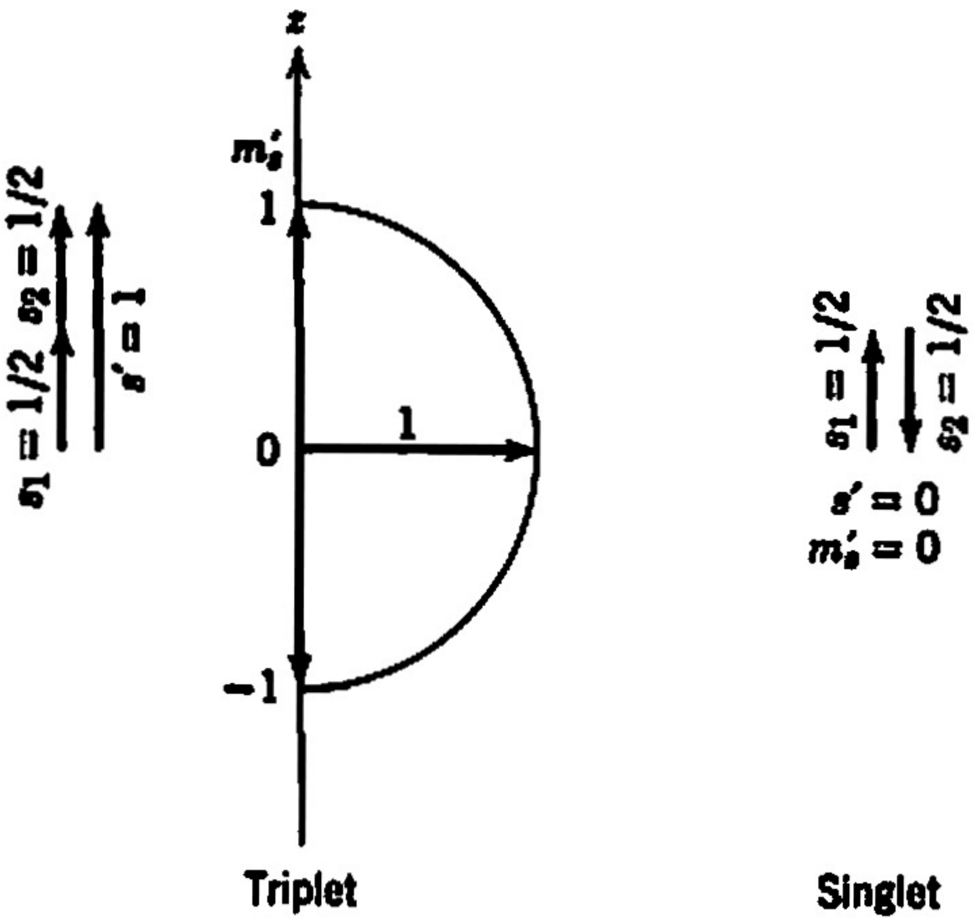
3 Minute 12 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 12 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

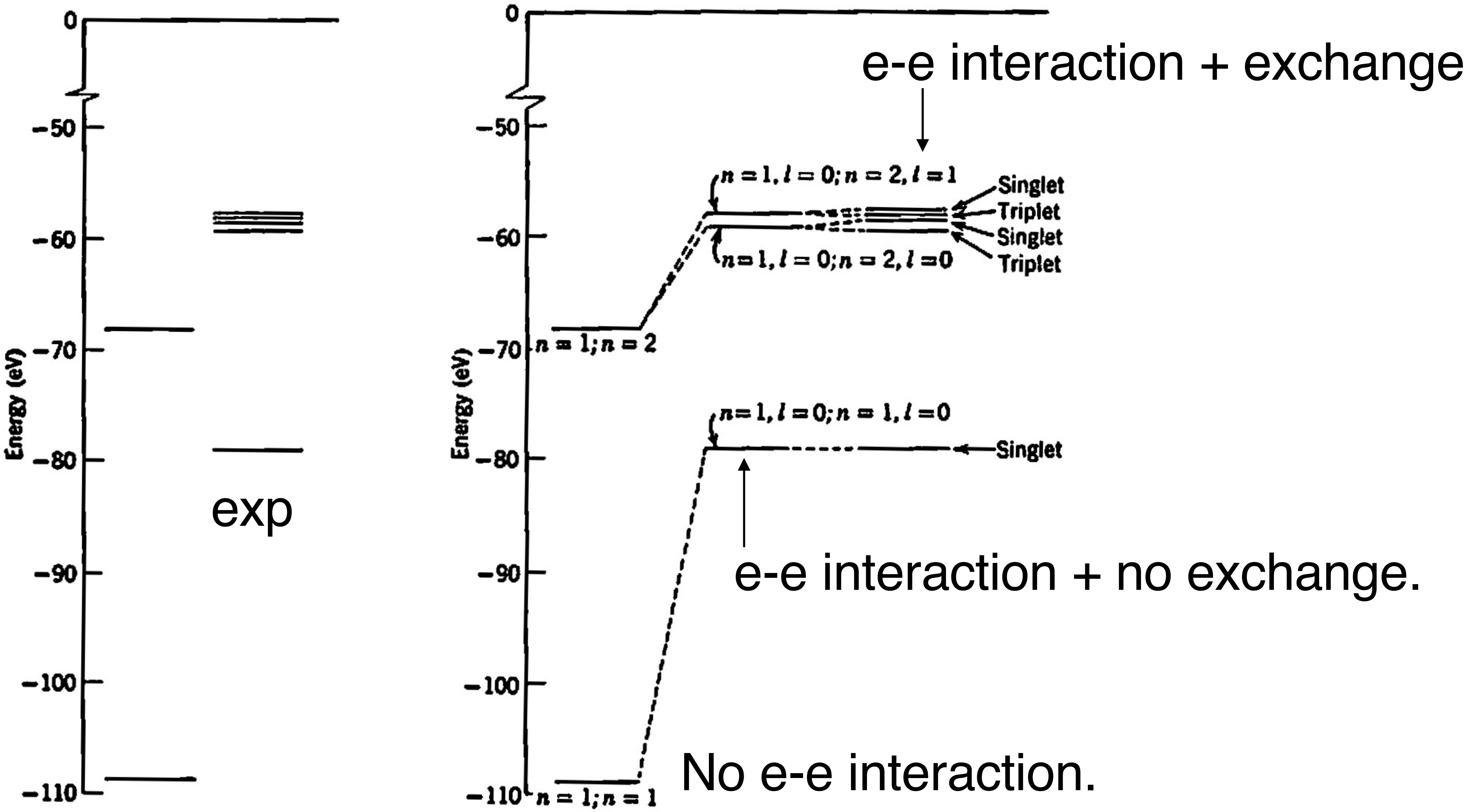


Adding spins.



The exchange force.

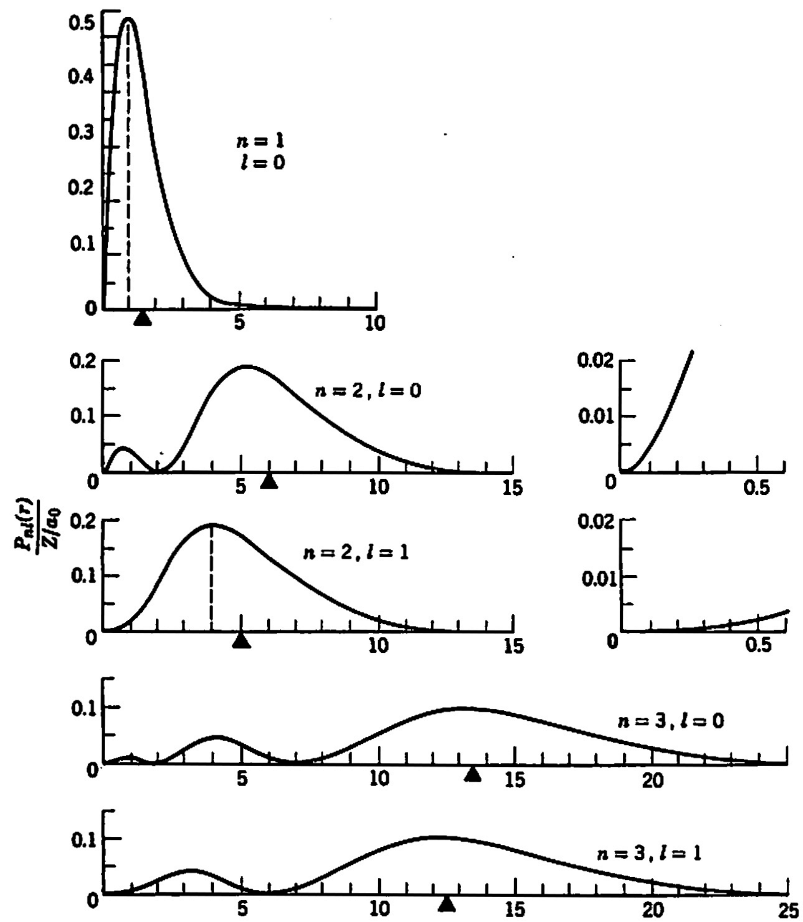
Energy levels in helium.



Solutions single-electron atom.

Table 7-2 Some Eigenfunctions for the One-Electron Atom

Quantum Numbers n	l	m_l	Eigenfunctions
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(2 - \frac{Zr}{a_0}\right) e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \cos\theta$
2	1	±1	$\psi_{21\pm1} = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \sin\theta\, e^{\pm i\varphi}$
3	0	0	$\psi_{300} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(27 - 18\frac{Zr}{a_0} + 2\frac{Z^2r^2}{a_0^2}\right) e^{-Zr/3a_0}$
3	1	0	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6 - \frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0} \cos\theta$
3	1	±1	$\psi_{31\pm1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6 - \frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0} \sin\theta\, e^{\pm i\varphi}$
3	2	0	$\psi_{320} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}(3\cos^2\theta - 1)$
3	2	±1	$\psi_{32\pm1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0} \sin\theta\cos\theta\, e^{\pm i\varphi}$
3	2	±2	$\psi_{32\pm2} = \frac{1}{162\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0} \sin^2\theta\, e^{\pm 2i\varphi}$



ENOUGH FOR TODAY?