Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use **only** a writing instrument while taking this test. You may **not** consult any calculators, computers, books, nor each other.

Problems 1, 2, and 3 must be answered in exam booklet 1. Problems 4, 5, and 6 must be answered in exam booklet 2, and Problems 7 and 8 must be answered in exam booklet 3. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you must hand in your exam, the blue exam booklets, and the equation sheet. All items must be clearly labeled with your name, your student ID number, and the day/time of your recitation. **If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

You are required to complete the following *Honor Pledge for Exams*. Copy and sign the pledge before starting your exam.

"I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own."

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Name: ________________________________

Signature: ________________________________

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Good Luck !

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Table 17-1. Particles that are Stable or Decay either Weakly or Electromagnetically

Generic Name	Particle Symbol	Rest Mass (MeV/c^2)	Lifetime (sec)	Charge Q	Intrinsic Spin s	Lepton Number L_e,	L_μ,	or L_τ	Baryon Number B	Intrinsic Parity P	Isospin T	Isospin z component T_z	Strangeness S
Photon	γ	0	stable	0	1		0		0	Odd	0, 1	0	0
Leptons	ν_e	0	stable	0	1/2	+1			0				
	ν_μ	0	stable	0	1/2		+1		0				
	ν_τ	0	stable	0	1/2			+1	0				
	e^-	0.511	stable	−1	1/2	+1			0				
	μ^-	105.7	2.2×10^{-6}	−1	1/2		+1		0				
	τ^-	1784	5×10^{-13}	−1	1/2			+1	0				
Mesons	π^+	139.6	2.6×10^{-8}	+1	0		0		0	Odd	1	+1	0
	π^0	135.0	8×10^{-17}	0	0		0		0	Odd	1	0	0
	π^-	139.6	2.6×10^{-8}	−1	0		0		0	Odd	1	−1	0
	K^+	493.8	1.2×10^{-8}	+1	0		0		0	Odd	1/2	+1/2	+1
	K^0	497.8	(8.9×10^{-11}	0	0		0		0	Odd	1/2	−1/2	+1
			and										
	$\overline{K^0}$	497.8	5.2×10^{-8})	0	0		0		0	Odd	1/2	+1/2	−1
	K^-	493.8	1.2×10^{-8}	−1	0		0		0	Odd	1/2	−1/2	−1
	η^0	549	8×10^{-19}	0	0		0		0	Odd	0	0	0
	η'	958	2×10^{-21}	0	0		0		0	Odd	0	0	0
Baryons	p	938.3	stable	+1	1/2		0		+1	Even	1/2	+1/2	0
	n	939.6	925	0	1/2		0		+1	Even	1/2	−1/2	0
	Λ^0	1116	2.6×10^{-10}	0	1/2		0		+1	Even	0	0	−1
	Σ^+	1189	8.0×10^{-11}	+1	1/2		0		+1	Even	1	+1	−1
	Σ^0	1192	6×10^{-20}	0	1/2		0		+1	Even	1	0	−1
	Σ^-	1197	1.5×10^{-10}	−1	1/2		0		+1	Even	1	−1	−1
	Ξ^0	1315	2.9×10^{-10}	0	1/2		0		+1	Even	1/2	+1/2	−2
	Ξ^-	1321	1.6×10^{-10}	−1	1/2		0		+1	Even	1/2	−1/2	−2
	Ω^-	1672	8.2×10^{-11}	−1	3/2		0		+1	Even	0	0	−3

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Table 17-3. Applicability of the Conservation Laws to the Observed Interactions ("yes" Means Conserved; "no" Means Not Conserved)

Quantity Conserved	Strong	Electro-magnetic	Weak
Energy	yes	yes	yes
Linear momentum	yes	yes	yes
Angular momentum	yes	yes	yes
Charge	yes	yes	yes
Electronic lepton number	yes	yes	yes
Muonic lepton number	yes	yes	yes
Tauonic lepton number	yes	yes	yes
Baryon number	yes	yes	yes
Isospin magnitude	yes	no	no ($\Delta T = 1/2$ for nonleptonic)
Isospin z component	yes	yes	no ($\Delta T_z = 1/2$ for nonleptonic)
Strangeness	yes	yes	no ($\Delta S = 1$)
Parity	yes	yes	no
Charge conjugation	yes	yes	no
Time reversal (or CP)	yes	yes	yes (But 10^{-3} violation in K^0 decay)

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Problem 1 (25 points) **Answer in booklet 1**

Consider the ground-state wave function of the hydrogen atom.

(a) **(15 points)** What is the expectation value $< V >$ of the potential energy of the hydrogen atom when it is in its ground state?

(b) **(5 points)** Express the energy of the ground state of the hydrogen atom in terms of the expectation value $< V >$ of the potential energy.

(c) **(5 points)** What is the expectation value of the kinetic energy of the electron when the hydrogen atom is in its ground state?

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 2 (25 points) **Answer in booklet 1**

Consider a system of N distinguishable atoms, maintained at a temperature T, which are distributed over two energy levels $\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$.

(a) **(15 points)** What is the total energy of this system?

(b) **(10 points)** What is c_V for this system?

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 3 (25 points) **Answer in booklet 1**

Consider the three lowest energy levels in Na, shown in Fig. 1.

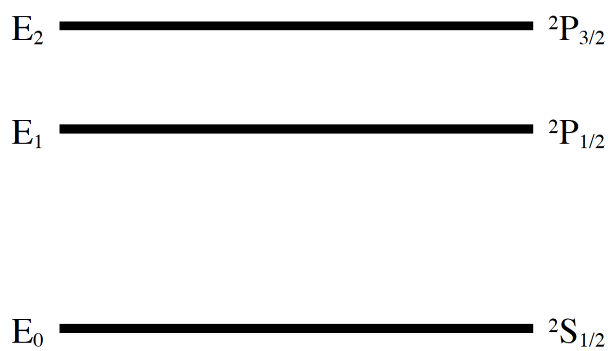

Figure 1: The three lowest energy levels of ^{22}Na.

(a) **(5 points)** What are the Landé g factors for these levels?

(b) **(10 points)** When the atom is placed in a weak magnetic field B, the energy levels split. Draw an energy level diagram showing the energy levels and determine the corresponding energies.

(c) **(5 points)** Which transitions between the ^{2}P and the ^{2}S energy levels are possible? **Explain why you selected these transitions.**

(d) **(5 points)** How many different photon energies will be observed if all 2P states are initially populated with equal probability?

Your answers need to be well motivated and expressed in terms of the variables provided.

Problem 4 (25 points) **Answer in booklet 2**

Consider the following eigenfunction of the electron in a one-electron atom with atomic number Z:

$$\psi = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}\sin\theta\,\cos\theta e^{-i\varphi} \tag{1}$$

(a) **(10 points)** What is the magnetic quantum number m_ℓ of this eigenfunction? **Note: you will not receive any credit for a correct answer if it is not properly justified. Referring to the equation sheet is not considered proper justification.**

(b) **(10 points)** What is the azimuthal quantum number ℓ of this eigenfunction? **Note: you will not receive any credit for a correct answer if it is not properly justified. Referring to the equation sheet is not considered proper justification.**

(c) **(5 points)** How would you calculate the energy of the electron? **Note: you will not receive any credit for a correct answer if it is not properly justified. Referring to the equation sheet is not considered proper justification.**

Problem 5 (25 points) **Answer in booklet 2**

A particle of mass m and energy $E = 2V_0$ is approaching $x = 0$ from the left, as shown in Fig. 2. The potential seen by the particle, shown in Fig. 2, can be described by the following function of x:

$$V(x) = \begin{cases} V_0 & x < 0 \\ 0 & x > 0 \end{cases} \tag{2}$$

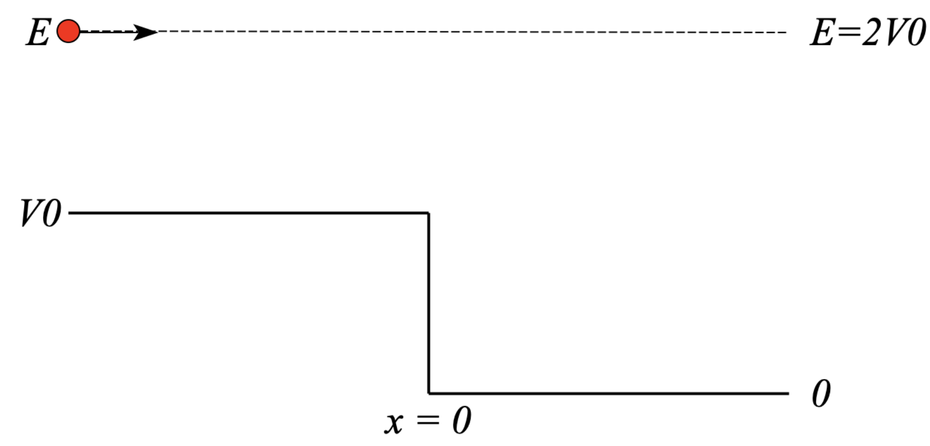

Figure 2: A particle of mass m approaching a potential step from the left.

(a) **(5 points)** What is the wave function in the region $x < 0$? **Note**: you do no yet have to determine the value of the constants that appear in the wavefunction. **Note 2**: you can set the amplitude of the incident wave to 1.

(b) **(5 points)** What is the wave function in the region $x \geq 0$? **Note**: you do no yet have to determine the value of the constants that appear in the wavefunction.

(c) **(15 points)** Calculate the transmission coefficient.

Your answers need to be well motivated and expressed in terms of the variables provided. You can use the constants $\hbar$ and c in your answers. You can treat this problem non-relativistically.

Problem 6 (25 points) **Answer in booklet 2**

Consider the Compton scattering of a photon with wavelength λ from a free electron of mass m, initially at rest, as shown schematically in Fig. 3.

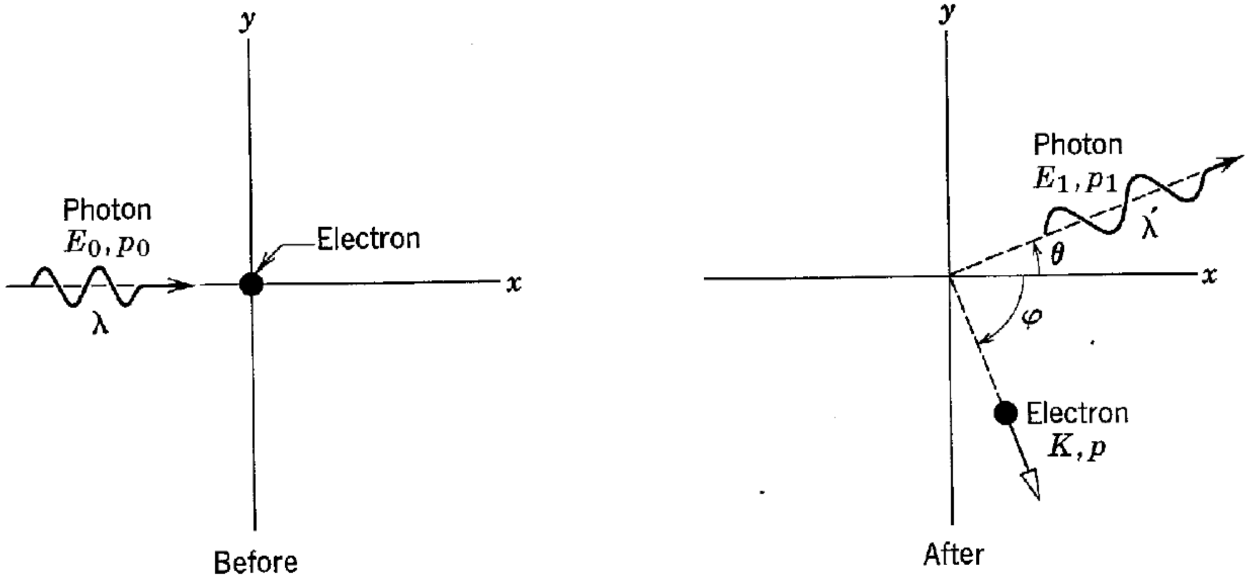

Figure 3: Compton scattering of a photon from an electron.

After the scattering, the wavelength of the photon has increased to λ'.

(a) **(10 points)** Use conservation of linear momentum in the x and y directions to obtain a relation between p_0, p_1, p, and θ.

(b) **(5 points)** Use conservation of kinetic energy to obtain a relation between K, m, and p.

(c) **(10 points)** Use the relations derived in part (a) and (b) to calculate the difference in the wavelength $\Delta\lambda$ of the scattered photon where $\Delta\lambda = \lambda' - \lambda$ in terms of m and θ.

Your answers need to be well motivated and expressed in terms of the variables provided. You can use the constants h and c in your answers. It is up to you to determine if you need to use relativity to answer these questions.

Problem 7 (25 points) **Answer in booklet 3**

Each answer must be well motivated

(a) **(10 points)** Figure 4 shows the ratio of the cross section for $e^+ + e^- \to$ hadrons to $e^+ + e^- \to \mu^+ + \mu^-$, plotted as function of the center-of-mass energy of the $e^+ + e^-$ system. At energies above 12.5 GeV the ratio is around 11/3. Explain the significance of this observation.

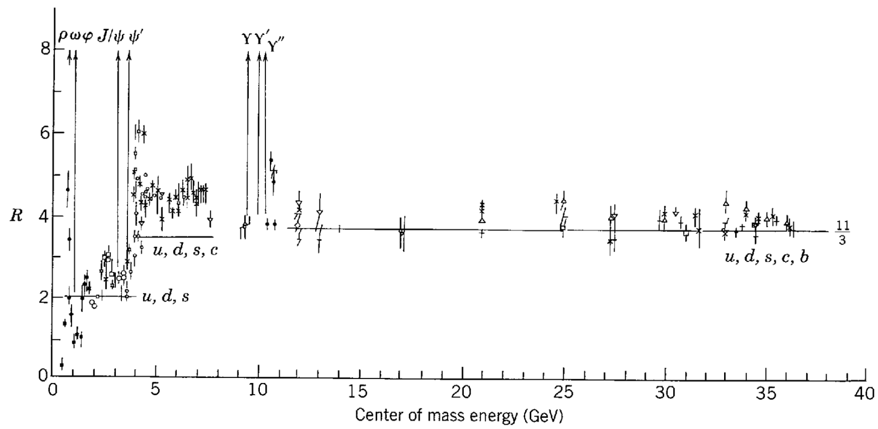

Figure 4: The ratio R of the cross sections for $e^+ + e^- \to$ hadrons to $e^+ + e^- \to \mu^+ + \mu^-$, plotted as function of the center-of-mass energy of the $e^+ + e^-$ system.

(b) **(10 points)** Figure 5 shows the even parity, spin 3/2 baryon decuplet. The mass information shown in this Figure is expressed in units of MeV. Based on the information provided in the Figure, what is your best estimate of the mass of the strange quark?

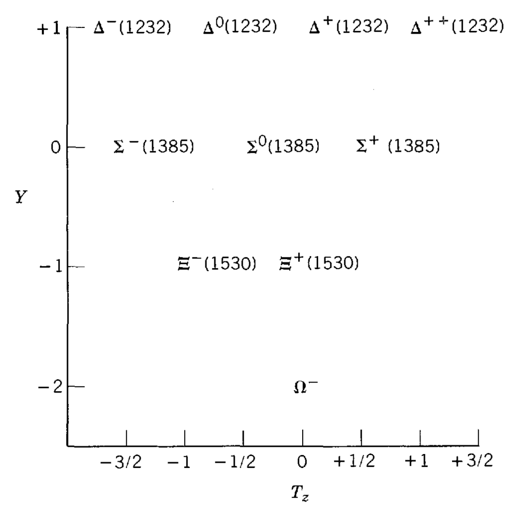

Figure 5: The even parity, spin 3/2 baryon decuplet.

(c) **(5 points)** Figure 6 shows the differential cross section $d\sigma/d\Omega$ for the scattering of 90-MeV neutrons by protons as function of the center-of-mass scattering angle of the neutron.

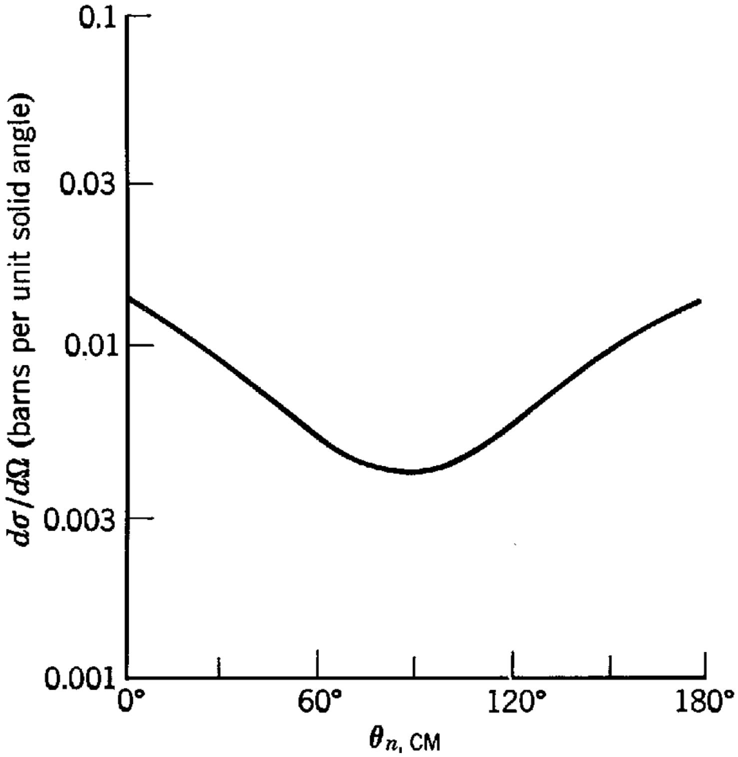

Figure 6: Measured differential cross section $d\sigma/d\Omega$ for scattering of 90 MeV neutrons by protons.

i. What reaction mechanism dominates at large scattering angles?
ii. On what basis did one conclude that this reaction mechanism dominates the scattering process at these scattering angles?

Your answers need to be well motivated.

Problem 8 (25 points) **Answer in booklet 3**

Short answer questions. No derivations are needed

(a) **(5 points)**
Consider the experiment shown Fig. 7 where monochromatic X rays fall on a graphite scatterer. The distribution of scattered wavelengths as function of the scattering angle is measured using Bragg reflection from the crystal shown on the right-hand side in Fig. 7.

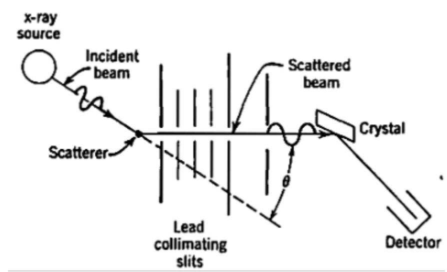

Figure 7: X-ray scattering experiment with monochromatic x rays.

The measured wavelength distributions at various scattering angles are shown in Fig. 8.

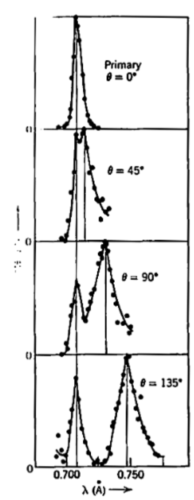

Figure 8: Results obtained with the apparatus shown in Fig. 7.

Explain the two components visible in each of the wavelength distributions in Fig. 8.

(b) **(5 points)**

Figure 9 shows the low-lying energy levels of helium, obtained based on calculations that may or may not include the several corrections for multi-electron atoms discussed in Chapter 9.

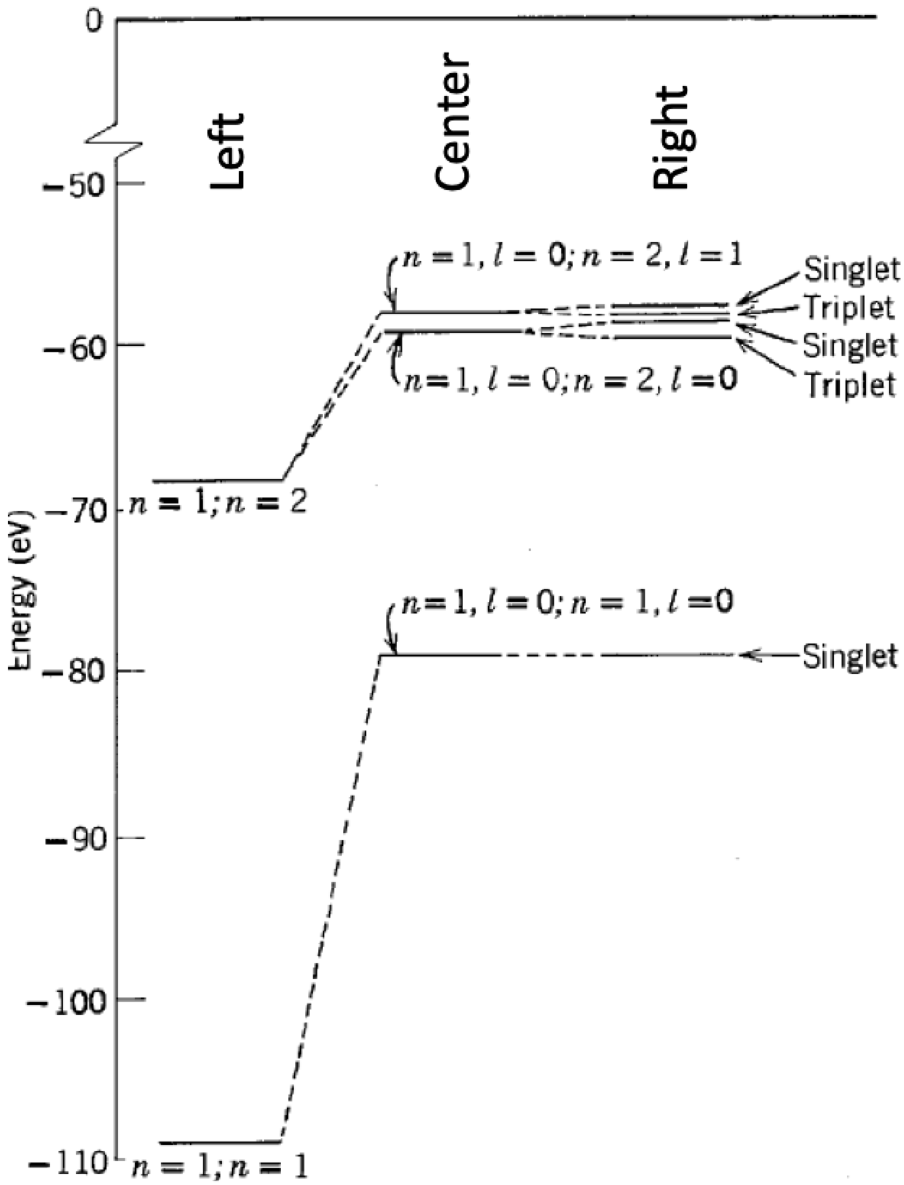

Figure 9: Low-lying calculated energy levels in helium.

For each of the three level schemes, indicate whether or not any corrections are included. If corrections are applied, indicate which corrections are applied.

1) Left: ....

2) Center: ....

3) Right: ....

(c) **(5 points)** Consider the four potential distributions shown on the left-hand side of Fig. 10. For each potential distribution shown in Fig. 10 the energy of the system is also indicated. Assume a particle of energy E is approaching the barrier from the left.

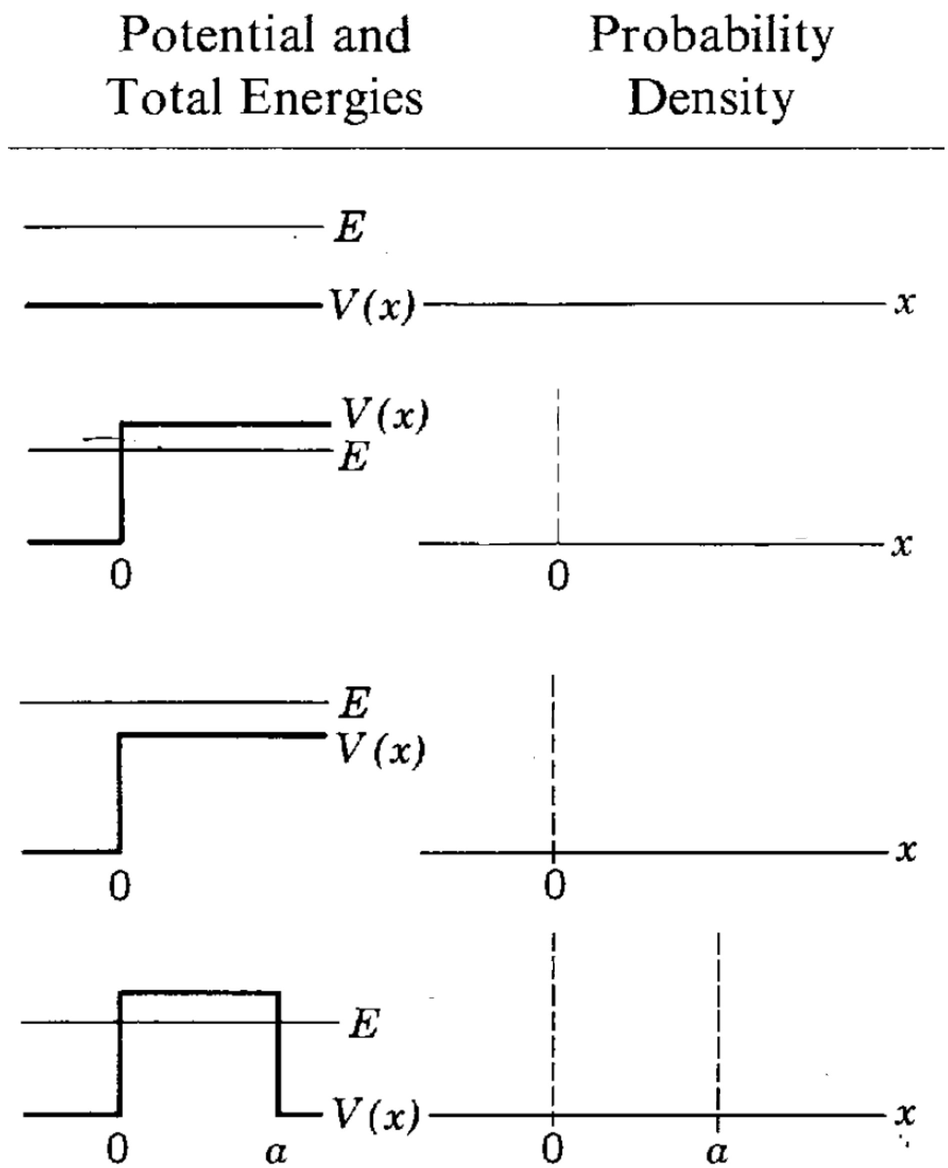

Figure 10: Various potentials and total energies.

For each potential distribution shown in Fig. 10, sketch the corresponding probability density distribution in your blue booklet. Make sure that your probability density distribution covers the same region in x that is covered by the potential distributions shown on the left-hand side in Fig. 10.

(d) **(5 points)**
Consider the potential energy distribution, shown in the Fig. 11.

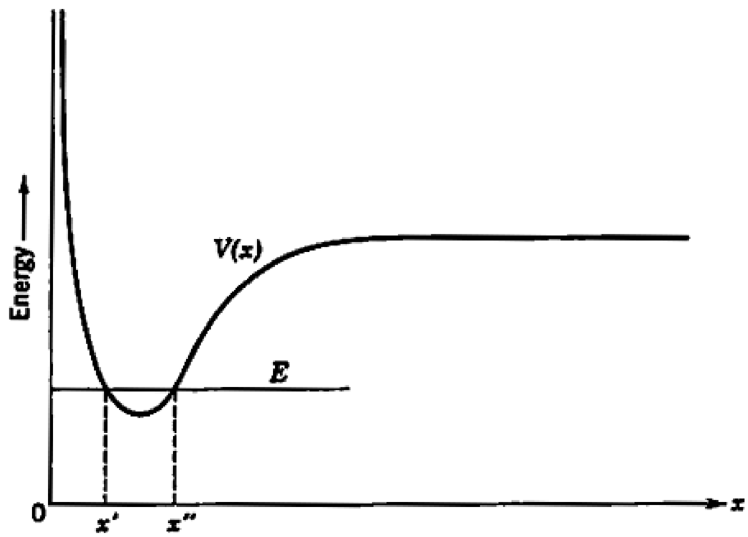

Figure 11: Potential energy distribution.

Three acceptable wave functions for this potential are shown in Fig. 12.

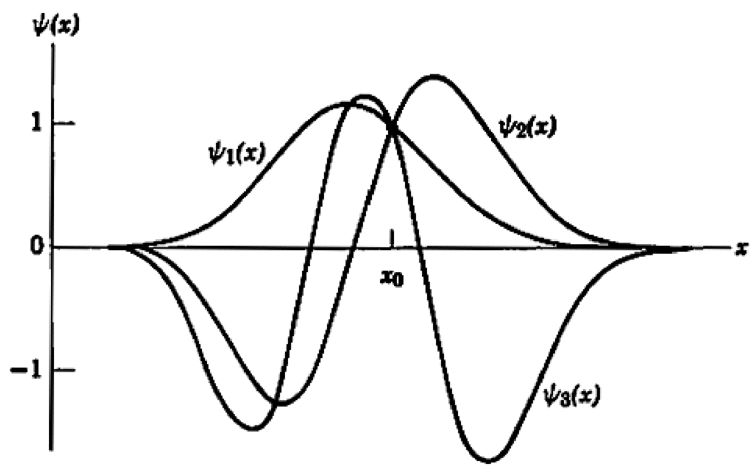

Figure 12: Three solutions.

The three wave functions have the same value at $x = x_0$. Use this information to rank the three wave functions in order of their energy (lowest energy, middle energy, highest energy).

(e) **(5 points)** Did this really happen? If so, in which country?

Figure 13: A finger in a dike.

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GOOD LUCK!!!