Problem 1 (2.5 points)

A

B

C

D

E

Match the pictures of the ball parks shown above to the team names listed below.

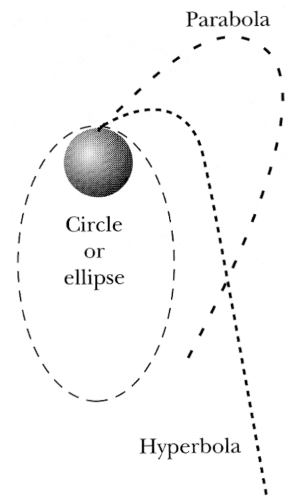
1. Yankees
2. White Sox
3. Mets
4. Cubs
5. Red Sox

ABCDE =

1. 12345
2. 21453
3. 21534
4. **31542**
5. 31254
6. 42351

Problem 2 (2.5 points)

The Figure on the right shows various possible trajectories of an object launched from the surface of the earth. What can you say about the total energy of the object for the various trajectories shown in the Figure?



1. $E_{\text{elliptical trajectory}} > 0$ J, $E_{\text{hyperbolic trajectory}} = 0$ J, and $E_{\text{parabolic trajectory}} < 0$ J.
2. $E_{\text{elliptical trajectory}} > 0$ J, $E_{\text{hyperbolic trajectory}} < 0$ J, and $E_{\text{parabolic trajectory}} = 0$ J.
3. $E_{\text{elliptical trajectory}} = 0$ J, $E_{\text{hyperbolic trajectory}} > 0$ J, and $E_{\text{parabolic trajectory}} < 0$ J.
4. $E_{\text{elliptical trajectory}} = 0$ J, $E_{\text{hyperbolic trajectory}} < 0$ J, and $E_{\text{parabolic trajectory}} > 0$ J.
5. $E_{\text{elliptical trajectory}} < 0$ J, $E_{\text{hyperbolic trajectory}} = 0$ J, and $E_{\text{parabolic trajectory}} > 0$ J.
6. **$E_{\text{elliptical trajectory}} < 0$ J, $E_{\text{hyperbolic trajectory}} > 0$ J, and $E_{\text{parabolic trajectory}} = 0$ J.**

Problem 3 (2.5 points)

Two satellites A and B of the same mass are going around Earth in concentric orbits. The distance of satellite B from Earth's center is twice that of satellite A. What is the ratio of the centripetal force acting on B to that acting on A?

1. 1/8
2. **1/4**
3. 1/2
4. $1/\sqrt{2}$
5. 1

Problem 4 (2.5 points)

A sports car accelerates from zero to 30 mph in 1.5 s. How long does it take for it to accelerate from zero to 60 mph, assuming the power of the engine to be independent of velocity and neglecting friction?

1. 2 s
2. 3 s
3. 4.5 s
4. **6 s**
5. 9 s
6. 12 s

Problem 5 (2.5 points)

When a small ball collides elastically with a more massive ball initially at rest, the massive ball tends to remain at rest, whereas the small ball bounces back at almost its original speed. Now consider a massive ball of inertial mass M moving at speed v and striking a small ball of inertial mass m initially at rest. The change in the small ball's momentum is

1. mv
2. **$2mv$**
3. Mv
4. $2Mv$
5. none of the above

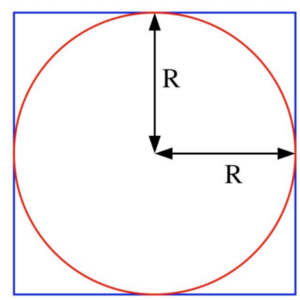
Problem 6 (2.5 points)

As you hold the string, a yoyo is released from rest so that gravity pulls it down, unwinding the string. What is the angular acceleration of the yoyo, in terms of the string radius R, the moment of inertia I, and the mass M?

1. **$gMR/(I + MR^2)$**
2. g/R
3. $g/(R + 2I/(MR))$
4. gMR/I

Problem 7 (2.5 points)

The moment of inertia of a square plate of area $4R^2$ and mass M, with respect to an axis through its center and perpendicular to the plate, is equal to $(2/3)MR^2$. A disk of radius R is removed from the center of the plate (see Figure). What is the moment of inertia of the remaining material with respect to the same axis?



1. $(1/2)MR^2$
2. **$(2/3 - \pi/8)MR^2$**
3. $(1/6)MR^2$
4. $(2/3 - \pi/4)MR^2$

Problem 8 (2.5 points)

Which of the following statements is **false**?

1. The buoyant force in liquid is much larger than the buoyant force in air.
2. The buoyant force is a result of small differences in molecular density of the air/liquid across the surface of the object.
3. **The buoyant force is a result of differences in the average molecular velocity of the air/liquid molecules across the surface of the object.**
4. The magnitude of the buoyant force increases with increasing temperature, due to the corresponding increase in the average molecular velocities.

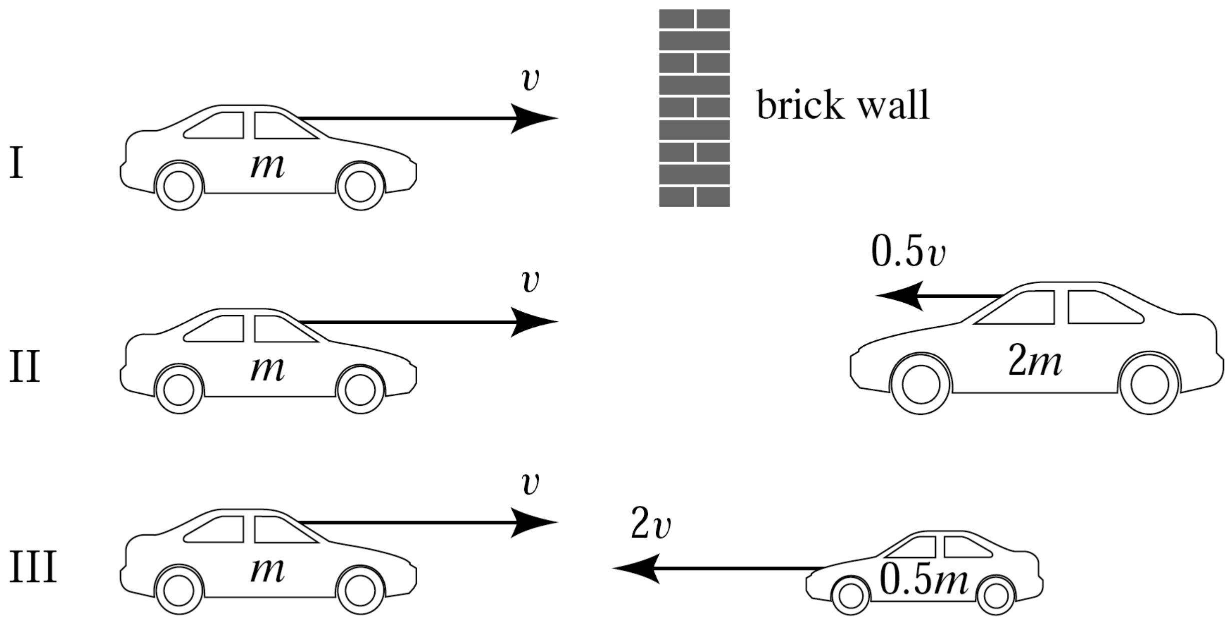
Problem 9 (2.5 points)

A solid disk and a ring roll down an incline. The ring is slower than the disk if

1. $m_{ring} = m_{disk}$, where m is the mass.
2. $r_{ring} = r_{disk}$, where r is the radius.
3. $m_{ring} = m_{disk}$ and $r_{ring} = r_{disk}$.
4. **The ring is always slower, regardless of the relative values of *m* and *r*.**

Problem 10 (2.5 points)

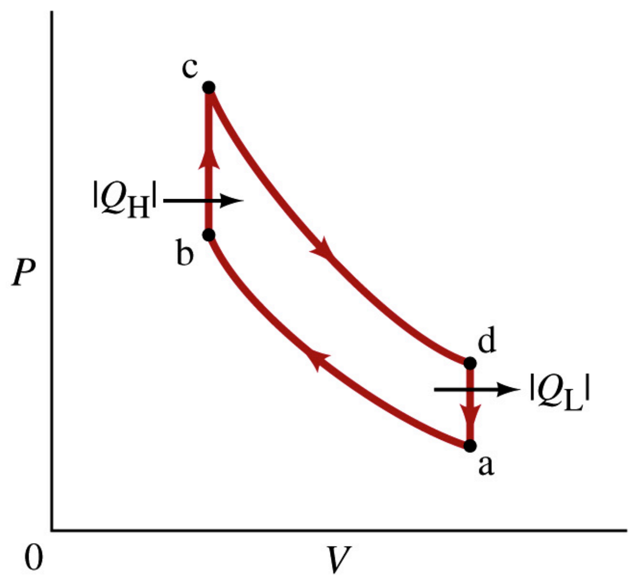
If all three collisions in the figure shown below are totally inelastic, which collisions cause(s) the most damage?



1. I.
2. II.
3. **III.**
4. I and II.
5. I and III.
6. II and III.
7. All three.

Problem 11 (12.5 points)

The operation of an automobile internal combustion engine can be approximated by a reversible cycle known as the Otto cycle, whose *PV* diagram is shown in the Figure below. The gas in cylinder at point *a* is compressed adiabatically to point *b*. Between point *b* and point *c*, heat is added to the gas, and the pressure increases at constant volume. During the power stroke, between point *c* and point *d*, the gas expands adiabatically. Between point *d* and point *a*, heat is removed from the system, and the pressure decreases at constant volume. Assume the gas is an ideal monatomic gas.



a. Assuming there are *N* molecules of gas in the system, what are the heats $|Q_H|$ and $|Q_L|$? Express your answer in terms of *N*, *k*, T_a, T_b, T_c, and T_d.

Answer:

The heat exchange takes place at constant volume and we thus must use the specific heat for constant volume C_V. For an ideal monatomic gas, $C_V = (3/2)k$ per molecule. The heat added or removed is proportional to the temperature difference. For the Otto engine, we find the following values:

$$|Q_H| = NC_V\left(T_c - T_b\right) = \frac{3}{2}Nk\left(T_c - T_b\right)$$

$$|Q_L| = NC_V\left(T_d - T_a\right) = \frac{3}{2}Nk\left(T_d - T_a\right)$$

b. What is the efficiency of the Otto cycle? Express your answer in terms of T_a, T_b, T_c, and T_d.

Answer:

The efficiency of the heat engine is defined as

$$e = \frac{|W|}{|Q_H|}$$

Using the results of part (a) we can rewrite this in terms of the temperature:

$$e = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{\frac{3}{2}Nk\left(T_d - T_a\right)}{\frac{3}{2}Nk\left(T_c - T_b\right)} = 1 - \frac{\left(T_d - T_a\right)}{\left(T_c - T_b\right)}$$

c. Express the efficiency of the Otto cycle in terms of just the compression ratio V_a/V_b and γ.

Answer:

During the adiabatic compression and expansion, the pressures and volumes are related in the following manner:

$$p_a V_a^{\gamma} = p_b V_b^{\gamma}$$

$$p_c V_c^{\gamma} = p_d V_d^{\gamma}$$

Using the ideal gas law we can replace the pressure by the temperature ($p = NkT/V$) and rewrite these relations as

$$T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$$

$$T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1}$$

The efficiency can now be rewritten as

$$e = 1 - \frac{T_c\left(\frac{V_c}{V_d}\right)^{\gamma-1} - T_b\left(\frac{V_b}{V_a}\right)^{\gamma-1}}{T_c - T_b} = 1 - \frac{T_c - T_b}{T_c - T_b}\left(\frac{V_b}{V_a}\right)^{\gamma-1} = 1 - \left(\frac{V_b}{V_a}\right)^{\gamma-1} = 1 - \left(\frac{V_a}{V_b}\right)^{1-\gamma}$$

In this step we have used the fact that $V_c = V_b$ and $V_d = V_a$.

d. How does the efficiency change when we replace the monatomic gas with a diatomic gas?

Answer:

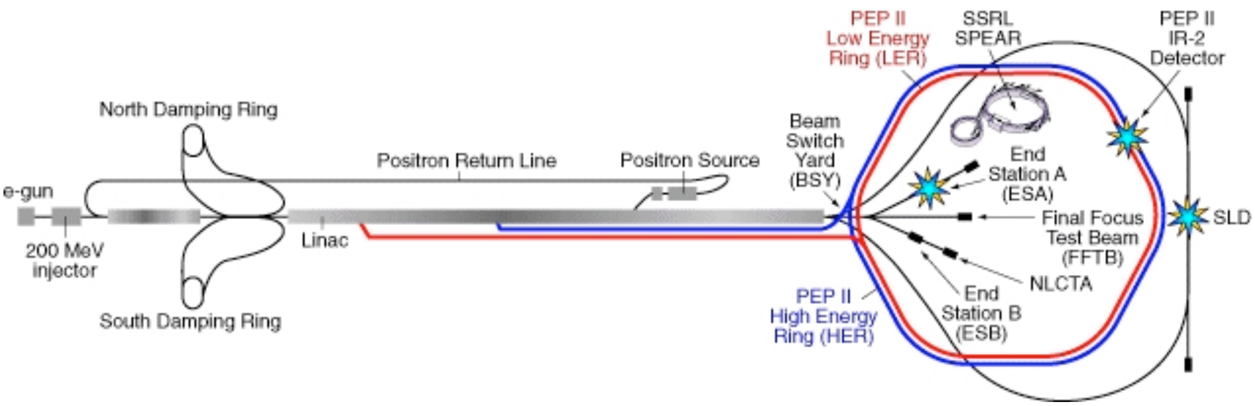
The constant γ is related to the molar specific heats:

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + k}{C_V} = 1 + \frac{k}{C_V}$$

We see that when C_V increases, γ decreases. The molecular specific heat at constant volume for a monatomic gas is smaller than the molecular specific heat at constant volume for a diatomic gas since the diatomic gas has more degrees of freedom. As a result, γ decreases when we replace the monatomic gas with a diatomic gas. Since the compression factor is larger than 1, a decrease in γ will result in an increase in $(V_a/V_b)^{1-\gamma}$ and a decrease in the efficiency e.

Problem 12 (25 points)

The Stanford Linear Accelerator Center (SLAC), located at Stanford University in Palo Alto, California, accelerates electrons through a vacuum tube of length *L*.



Electrons of mass *m*, which are initially at rest, are subjected to a continuous force *F* along the entire length of the tube and reach speeds very close to the speed of light.

a. Calculate the final energy of the electrons.

Answer:

The work done by the force *F* is *FL*, where *L* is the length of the accelerator. Using the work energy theorem, we can express the final energy of the electron in terms of its initial energy and the work done by *F*:

$$E_f = E_i + W = mc^2 + FL$$

b. Calculate the final momentum of the electrons.

Answer:

The final momentum of the electron can be obtained using the following relation:

$$E^2 - p^2c^2 = \left(mc^2\right)^2$$

or

$$p_f = \frac{1}{c}\sqrt{E_f^{\,2} - \left(mc^2\right)^2} = \frac{1}{c}\sqrt{\left(mc^2 + FL\right)^2 - \left(mc^2\right)^2} = \frac{1}{c}\sqrt{F^2L^2 + 2mc^2FL} = \frac{FL}{c}\sqrt{1 + 2\frac{mc^2}{FL}}$$

c. Calculate the final speed of the electrons.

Answer:

The final velocity of the electron can be found using the following relation:

$$E_f = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or

$$v = c\sqrt{1 - \left(\frac{mc^2}{E_f}\right)^2} = c\sqrt{1 - \left(\frac{mc^2}{mc^2 + FL}\right)^2} = \frac{c}{mc^2 + FL}\sqrt{\left(mc^2\right)^2 + 2mc^2FL} = \frac{c}{\left(1 + \frac{FL}{mc^2}\right)}\sqrt{1 + 2\frac{FL}{mc^2}}$$

d. Calculate the time required travel the distance L.

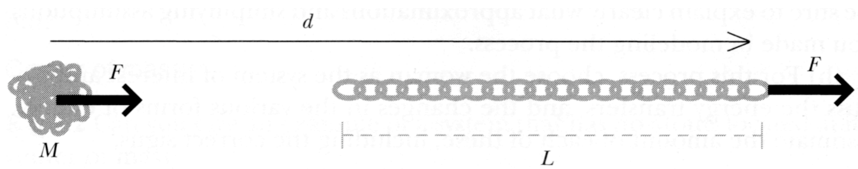
Answer:

The time required to complete the trip through the accelerator can be found by using the momentum principle:

$$\Delta t = \frac{\Delta p}{F} = \frac{p_f - p_i}{F} = \frac{p_f}{F} = \frac{L}{c}\sqrt{1 + 2\frac{mc^2}{FL}}$$

Problem 13 (25 points)

A chain of metal links is coiled up in a tight ball on a frictionless table. The mass of the chain is M. You pull on a link at one end of the chain with a constant force F. Eventually the chain straightens out to its full length L, and you keep pulling until you have pulled your end of the chain a total distance d.



a. What is the speed of the chain at this instant?

Answer:

During the motion depicted in the Figure the center of mass moves a distance d - L/2. The work done by the force on the center of mass during this motion is (d - L/2)F. Since the initial kinetic energy of the center of mass is 0, the final kinetic energy can be found by using the work-energy theorem

$$K_{cm,f} = \left(d - \frac{1}{2}L\right)F$$

The final velocity of the center of mass is thus equal to

$$v_{cm,f} = \sqrt{\frac{2K_{cm,f}}{M}} = \sqrt{(2d - L)\frac{F}{M}}$$

b. When the chain straightens out, the links of the chain bang against each other, and their temperature rises. Calculate the increase in thermal energy of the chain, assuming that the process is so fast that there is insufficient time for the chain to lose much thermal energy to the table.

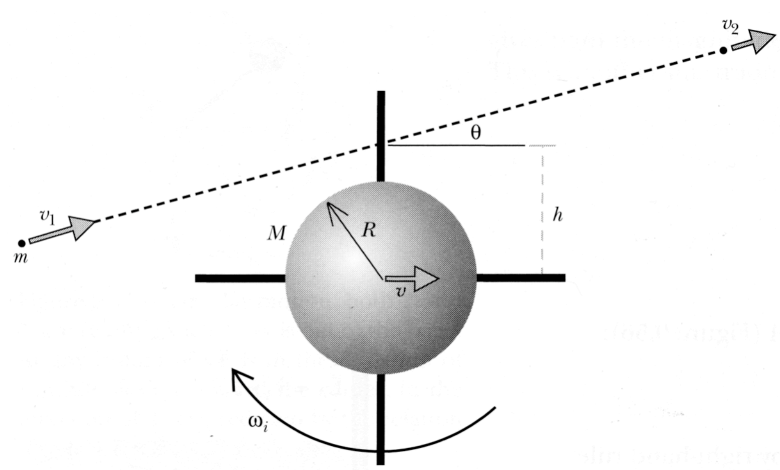
Answer:

The total work done by the force F is Fd. Part of the work is converted into kinetic energy of the center of mass (see part a). The remainder is converted into thermal energy of the chain (note there is no relative motion with respect to the center of mass after the chain has been unrolled). The thermal energy is thus equal to the difference between Fd and the work on the center of mass calculated in part a:

$$E_{thermal} = Fd - \left(d - \frac{1}{2}L\right)F = \frac{1}{2}LF$$

Problem 14 (25 points)

A satellite has four low-mass solar panels sticking out as shown in the Figure below. The satellite can be considered to be a uniform solid sphere of mass M and radius R. Initially the satellite is traveling to the right with speed v and rotating clockwise with angular speed ω_i.



A tiny meteor of mass m, traveling at speed v_1, rips through one of the solar panels and continues in the same direction, but at reduced speed v_2.

a. Calculate the x and y components of the motion of velocity of the center of mass of the satellite after it is hit by the meteor. The positive x direction is taken to be the direction of the original motion of the satellite. The positive y direction is the direction from the center of mass of the satellite towards the solar panel that is hit by the meteor.

Answer:

Since there are no external forces acting on the system (meteor + satellite), linear momentum will be conserved. The initial liner momentum is equal to

$$\vec{\mathbf{p}}_i = Mv\hat{\mathbf{x}} + \left\{ mv_1 \cos\theta\hat{\mathbf{x}} + mv_1 \sin\theta\hat{\mathbf{y}} \right\} = \left(Mv + mv_1 \cos\theta \right)\hat{\mathbf{x}} + mv_1 \sin\theta\hat{\mathbf{y}}$$

The final linear momentum is equal to

$$\vec{\mathbf{p}}_f = M\vec{\mathbf{v}}_f + \left\{ mv_2 \cos\theta \hat{\mathbf{x}} + mv_2 \sin\theta \hat{\mathbf{y}} \right\} = \left(Mv_{f,x} + mv_2 \cos\theta \right) \hat{\mathbf{x}} + \left(Mv_{f,y} + mv_2 \sin\theta \right) \hat{\mathbf{y}}$$

Conservation of linear momentum requires that

$$Mv_{f,x} + mv_2 \cos\theta = Mv + mv_1 \cos\theta$$

$$Mv_{f,y} + mv_2 \sin\theta = mv_1 \sin\theta$$

or

$$v_{f,x} = v + \frac{m}{M}(v_1 - v_2)\cos\theta$$

$$v_{f,y} = \frac{m}{M}(v_1 - v_2)\sin\theta$$

b. What is the angular velocity of the satellite after it is hit by the meteor? Specify both magnitude and direction.

Answer:

Since there are no external forces acting on the system (meteor + satellite), the external torque is equal to zero, and angular momentum will be conserved. The initial angular momentum is equal to

$$\vec{\mathbf{L}}_i = \vec{\mathbf{L}}_{meteor,i} + \vec{\mathbf{L}}_{satellite,i} = \left(-hmv_1 \cos\theta - I\omega_i \right) \hat{\mathbf{z}}$$

The final angular momentum is equal to

$$\vec{\mathbf{L}}_f = \vec{\mathbf{L}}_{meteor,f} + \vec{\mathbf{L}}_{satellite,f} = -\left(hmv_2 \cos\theta \right) \hat{\mathbf{z}} + \vec{\mathbf{L}}_{satellite,f}$$

Since angular momentum is conserved, we can now determine the final angular momentum of the satellite:

$$\begin{aligned}\vec{\mathbf{L}}_{satellite,f} &= \vec{\mathbf{L}}_f + \left(hmv_2\cos\theta\right)\hat{\mathbf{z}} = \left(-hmv_1\cos\theta - I\omega_i\right)\hat{\mathbf{z}} + \left(hmv_2\cos\theta\right)\hat{\mathbf{z}} = \\ &= \left(-hm\left[v_1 - v_2\right]\cos\theta - I\omega_i\right)\hat{\mathbf{z}}\end{aligned}$$

The satellite will thus have an angular velocity that is directed along the negative z axis. The magnitude of the angular velocity is equal to

$$\omega_f = \frac{hm\left[v_1 - v_2\right]\cos\theta + I\omega_i}{I} = \frac{hm\left[v_1 - v_2\right]\cos\theta}{\frac{2}{5}MR^2} + \omega_i = \frac{5}{2}\frac{mh}{MR^2}\left[v_1 - v_2\right]\cos\theta + \omega_i$$

Problem 15 (25 points)

A simple model of the Earth assumes that the density is uniform throughout its interior.

a. If the total mass of the Earth is M and its radius is R, what is its density?

Answer:

The density of the Earth is the ratio of its mass and its volume:

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

b. Using the shell theorem, calculate the gravitational force the Earth exerts on a small particle of mass m located a distance r from the center of the Earth ($r < R$).

Answer:

The shell theorem tells us that the force on the particle is equal to the gravitational force due to he mass contained in the region with a radius r. The force can be calculated as if all this mass is located at the center of the sphere.

The mass contained in a sphere of radius r is equal to

$$M_r = \rho \frac{4}{3}\pi r^3 = M\frac{r^3}{R^3}$$

The magnitude of the gravitational force is thus equal to

$$F(r) = G\frac{M_r m}{r^2} = G\frac{Mm}{R^3}r$$

The force is directed towards the center of the Earth.

c. Use the result of part b to calculate the work required to move a particle of mass *m* from the center of the Earth to the surface. If you were not able to obtain the answer to part b, assume that the force in the interior of the Earth can be written as $\alpha\, r$ and adjust α to ensure that force at $r = R$ agrees with what you know about the gravitational force on the surface.

Answer:

The work done by the gravitational force when you move the particle from the center of the Earth to the surface is negative since the force and displacement are pointing in opposite directions. Using the results from part b we obtain

$$W = -\int_0^R G\frac{Mm}{R^3} r dr = -\frac{1}{2}G\frac{Mm}{R^3}r^2\bigg|_0^R = -\frac{1}{2}G\frac{Mm}{R^3}R^2 = -\frac{1}{2}G\frac{Mm}{R}$$

The work you need to do to move the particle from the center of the Earth to the surface must be positive and exceed the magnitude of the work calculated above.

d. Compare the work calculated in part c with the work required to move a mass *m* from the surface of the Earth to infinity ($r \rightarrow \infty$).

Answer:

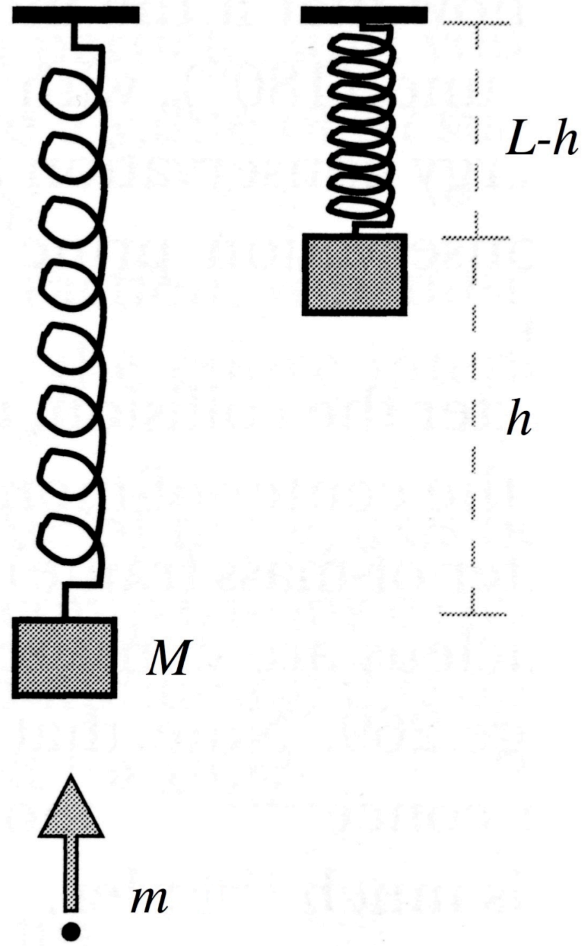
The work done by the gravitational force when you move the particle from the surface of the Earth to the infinity is negative since the force and displacement are pointing in opposite directions. Using the gravitational force law we obtain

$$W = -\int_R^\infty G\frac{Mm}{r^2} dr = G\frac{Mm}{r}\bigg|_R^\infty = -G\frac{Mm}{R}$$

The work you need to do to move the particle from the surface of the Earth to infinity must be positive and exceed the magnitude of the work calculated above. Comparing the results obtained here with the results obtained in part c we see that the work required to move the particle from the surface of the Earth to infinity is twice the amount of work required to move the particle from the center of the Earth to the surface of the Earth.

Problem 16 (25 points)

A block of mass M is attached to a spring with spring constant k. The spring-mass system is in equilibrium when is has a length L. A bullet of mass m is fired from below and buries itself in the block. The block reaches a maximum height h above its original position.



a. What is the rest length of the spring?

Answer:

In the equilibrium position, the net force on the block is 0 N, and the magnitude of the gravitational force is equal to the magnitude of the spring force. This requires that

$$Mg = k\left(L - L_0\right)$$

or

$$L_0 = L - \frac{Mg}{k}$$

where L_0 is the rest length of the spring.

b. What is the speed of the block right after the bullet hits?

Answer:

In order to determine the speed of the block right after the bullet hits we use conservation of energy. The energy of the block (and bullet) when it reaches its maximum height is all in the form of potential energy (gravitational and spring):

$$E_f = \left(M+m\right)gh + \frac{1}{2}k\left(L-h-L_0\right)^2 = \left(M+m\right)gh + \frac{1}{2}k\left\{\left(L-L_0\right)^2 - 2h\left(L-L_0\right) + h^2\right\}$$

The energy of the block (and bullet) right after the bullet hits is in the form of potential energy (spring) and kinetic energy (block and bullet):

$$E_i = \frac{1}{2}\left(M+m\right)v_{block}{}^2 + \frac{1}{2}k\left(L-L_0\right)^2$$

Note: we have assumed here that the gravitational potential energy is 0 at the equilibrium position of the block.

Applying conservation of energy we see that

$$\frac{1}{2}\left(M+m\right)v_{block}{}^2 + \frac{1}{2}k\left(L-L_0\right)^2 = \left(M+m\right)gh + \frac{1}{2}k\left\{\left(L-L_0\right)^2 - 2h\left(L-L_0\right) + h^2\right\}$$

or

$$v_{block} = \sqrt{\frac{2mgh + kh^2}{\left(M+m\right)}}$$

c. What is the speed of the bullet just before it hits the block?

Answer:

In order to determine the speed of the bullet just before it hits the block, we apply conservation of linear momentum. The final linear momentum of the bullet and the block is equal to

$$p_f = \left(M+m\right)\sqrt{\frac{2mgh+kh^2}{\left(M+m\right)}} = \sqrt{\left(M+m\right)\left(2mgh+kh^2\right)}$$

The initial linear momentum of the bullet just before it hits the block is equal to

$$p_i = mv_{bullet}$$

Conservation of linear momentum requires that

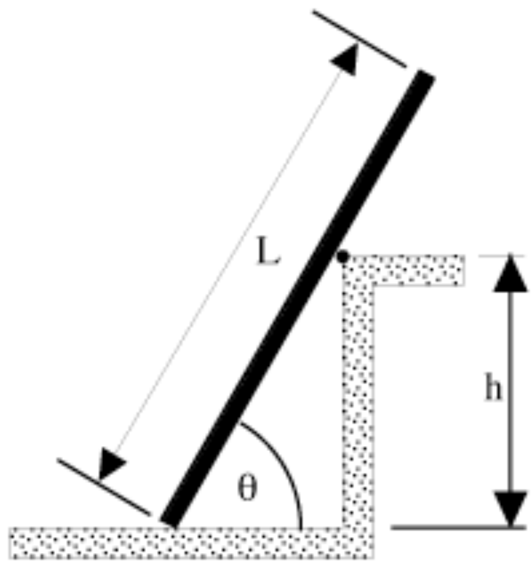
$$mv_{bullet} = \sqrt{\left(M+m\right)\left(2mgh+kh^2\right)}$$

or

$$v_{bullet} = \frac{\sqrt{\left(M+m\right)\left(2mgh+kh^2\right)}}{m} = \sqrt{\left(1+\frac{M}{m}\right)\left(2gh+\frac{kh^2}{m}\right)}$$

Problem 17 (25 points)

A plank, of length L and mass M, rests on the ground and on a frictionless roller at the top of a wall of height h (see Figure). The center of gravity of the plank is at its center. The plank remains in equilibrium for any value of $\theta \geq \theta_0$ but slips if $\theta < \theta_0$.



a. Calculate the magnitude of the force exerted by the roller on the plank when $\theta = \theta_0$.

Answer:

There are four forces acting on the ladder:

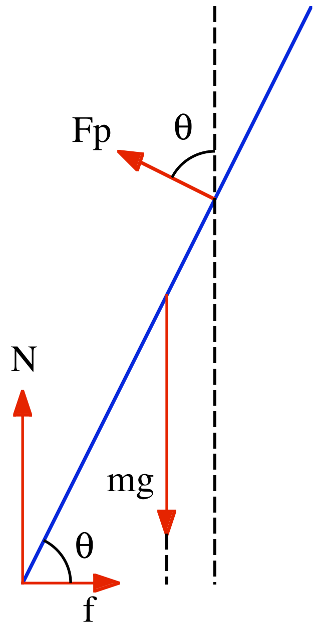
The friction force

The normal force

The weight

The force exerted by the pin.

These forces and their directions are schematically indicated in the following figure:



One of the conditions for equilibrium requires that the torque with respect to any reference point must be 0 Nm. If we calculate the torque with respect to the contact point on the ground, only the weight and the pin force will contribute to the torque. In that case, the net torque is equal to

$$\tau = F_p \frac{h}{\sin\theta_0} - Mg\frac{L}{2}\cos\theta_0 = 0$$

The only unknown in this equation is the pin force, which we can now easily calculate:

$$F_p = \frac{Mg\frac{L}{2}\cos\theta_0}{\frac{h}{\sin\theta_0}} = \frac{L}{2h}Mg\cos\theta_0\sin\theta_0$$

b. Calculate the magnitude of the normal force exerted by ground on the plank when $\theta = \theta_0$.

Answer:

A second requirement for equilibrium is that the net force on the plank must be zero. This requires that

$$\sum F_x = f - F_p \sin\theta_0 = 0$$

$$\sum F_y = N + F_p \cos\theta_0 - Mg = 0$$

The normal force exerted by the floor on the ladder can be obtained from the requirement that the vertical component of all the forces is equal to 0 N:

$$N = Mg - F_p \cos\theta_0 = Mg\left(1 - \frac{L}{2h}\cos^2\theta_0 \sin\theta_0\right)$$

c. Calculate the magnitude of the friction force between the ground and the plank when $\theta = \theta_0$.

Answer:

The friction force can be obtained from the requirement that the horizontal component of all the forces is equal to 0 N:

$$f = F_p \sin\theta_0 = \frac{L}{2h} Mg \cos\theta_0 \sin^2\theta_0$$

d. Calculate the coefficient of static friction.

Answer:

Since the ladder is at the critical angle, the static friction force has reached its maximum value:

$$f = \mu_s N$$

The static friction coefficient is thus equal to

$$\mu_s = \frac{f}{N} = \frac{\dfrac{L}{2h} Mg\cos\theta_0 \sin^2\theta_0}{Mg\left(1 - \dfrac{L}{2h}\cos^2\theta_0 \sin\theta_0\right)} = \frac{L\cos\theta_0 \sin^2\theta_0}{\left(2h - L\cos^2\theta_0 \sin\theta_0\right)}$$