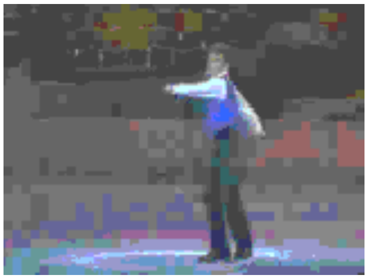
Physics 141.
Lecture 17.



Physics 141.
Lecture 17.

- Course information.
- Quiz
- Topics to be discussed today (Chapter 11):
  - Rotational Variables
  - Rotational Kinetic Energy
  - Torque

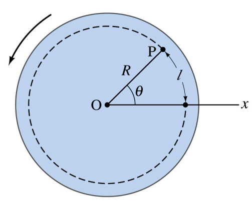
Physics 141.
Course information.

- Homework set # 7 is due on Friday 11/4 at noon.
- Homework set # 8 is due on Friday 11/11 at noon.
- Lab report # 4 is due on Wednesday 11/9 at noon.

The Personal Response System (PRS). Quiz.

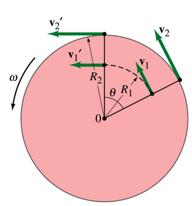
Rotational variables.

- The variables that are used to describe rotational motion are:
  - Angular position θ
  - Angular velocity $\omega = d\theta/dt$
  - Angular acceleration $\alpha = d\omega/dt$
- The rotational variables are related to the linear variables:
  - Linear position $l = R\theta$
  - Linear velocity $v = R\omega$
  - Linear acceleration $a = R\alpha$



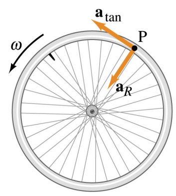
Rotational variables.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:
  - Each part of the rigid object has the same angular velocity.
  - Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
  - The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



Rotational variables.

- Note: the acceleration $a_t = r\alpha$ is only one of the two component of the acceleration of point P. The two components of the acceleration of point P are:
  - The **radial component**: this component is always present since point P carried out circular motion around the axis of rotation.
  - The **tangential component**: this component is present only when the angular acceleration is not equal to 0 rad/s^2.

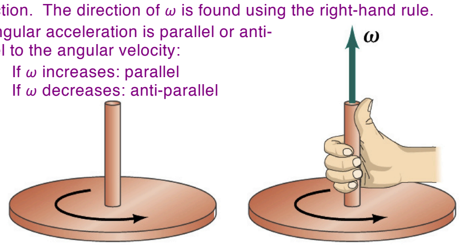


Rotational variables.

Angular velocity and acceleration are vectors! They have a magnitude and a direction. The direction of ω is found using the right-hand rule.

The angular acceleration is parallel or anti-parallel to the angular velocity:

If ω increases: parallel
If ω decreases: anti-parallel



Rotational kinetic energy.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2}\sum_i m_i (\omega r_i)^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I\omega^2$$

- The kinetic energy is proportional to the rotational velocity ω. Note: the equation is similar to the translational kinetic energy (1/2 mv^2) except that instead of being proportional to the the mass m of the object, the rotational kinetic energy is proportional to the **moment of inertia *I*** of the object:

$$I = \sum_i m_i r_i^2$$

Note: units of *I*: kg m^2

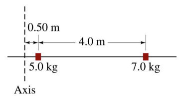
The moment of inertia I. Calculating I.

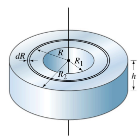
- The moment of inertia of an objects depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$

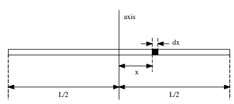




Calculating the moment of inertia. Sample problem.

- Consider a rod of length L and mass m. What is the moment of inertia with respect to an axis through its center of mass?
- Consider a slice of the rod, with width dx, located a distance x from the rotation axis. The mass dm of this slice is equal to

$$dm = \frac{m}{L} dx$$



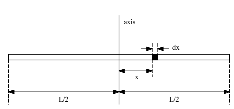
Calculating the moment of inertia. Sample problem.

- The moment of inertia dI of this slice is equal to

$$dI = x^2 dm = \frac{m}{L} x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:

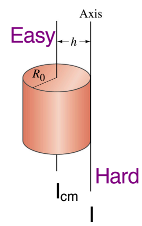
$$I = \int_{-L/2}^{L/2} \frac{m}{L} x^2 dx = \frac{m}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] = \frac{1}{12} mL^2$$



Calculating the moment of inertia. Parallel-axis theorem.

- Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:

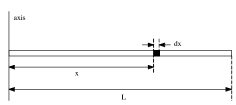
$$I = I_{cm} + Mh^2$$



Calculating the moment of inertia. Sample problem.

- Consider a rod of length *L* and mass *m*. What is the moment of inertia with respect to an axis through its left corner?
- We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:

$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$$

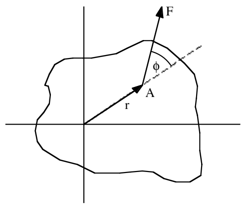


3 Minute 2 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 2 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Solve a WeBWorK problem.

Torque.

- Consider a force F applied to an object that can only rotate.
- The force F can be decomposed into two two components:
  - A **radial component** directed along the direction of the position vector r. The magnitude of this component is $F\cos\theta$. This component will not produce any motion.
  - A **tangential component**, perpendicular to the direction of the position vector r. The magnitude of this component is $F\sin\theta$. This component will result in rotational motion.



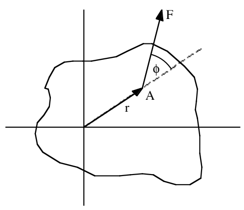
Torque.

- If a mass m is located at the position on which the force is acting (and we assume any other masses can be neglected), it will experience a linear acceleration equal to $F\sin\phi/m$.
- The corresponding angular acceleration α is equal to

$$\alpha = \frac{F\sin\phi}{mr}$$

- Since in rotation motion the moment of inertia plays an important role, we will rewrite the angular acceleration in terms of the moment of inertia:

$$\alpha = \frac{rF\sin\phi}{mr^2} = \frac{rF\sin\phi}{I}$$



Torque.

- Consider rewriting the previous equation in the following way:

$$rF\sin\phi = I\alpha$$

- The left-hand-side of this equation is called the torque τ of the force F:

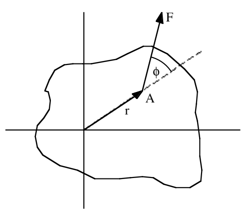
$$\tau = I\alpha$$

- This equation looks similar to Newton's second law for linear motion:

$$F = ma$$

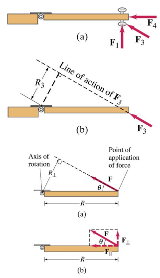
- Note:

linear	rotational
mass m	moment I
force F	torque τ



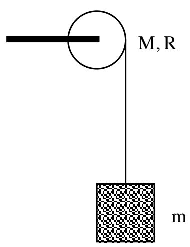
Torque.

- In general the torque associated with a force F is equal to
  $$\left|\vec{\tau}\right| = rF\sin\theta = \left|\vec{r}\times\vec{F}\right|$$
- The arm of the force (also called the moment arm) is defined as $r\sin\theta$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle θ is 90°.



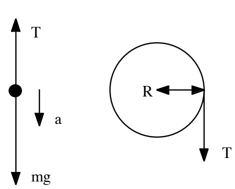
Rotational motion. Sample problem.

- Consider a uniform disk with mass M and radius R. The disk is mounted on a fixed axle. A block with mass m hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension of the cord.
- Expectations:
  - Linear acceleration should approach g when M approaches 0 kg.



Rotational motion. Sample problem.

- Start with considering the forces and torques involved.
- Define the sign convention to be used.
- The block will move down and we choose the positive and we choose the positive y axis in the direction of the linear acceleration.
- The net force on mass m is equal to
  $$ma = mg - T$$



Rotational motion.
Sample problem.

- The net torque on the pulley is equal to

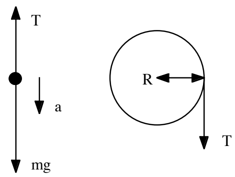
$$\tau = RT$$

- The resulting angular acceleration is equal to

$$\alpha = \frac{\tau}{I} = \frac{RT}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

- Assuming the cord is not slipping we can determine the linear acceleration:

$$a = \alpha R = 2\frac{T}{M}$$



Rotational motion.
Sample problem.

- We now have two expressions for a:

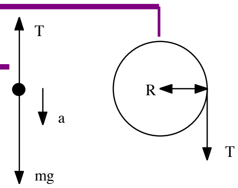
$$a = 2\frac{T}{M}$$

$$a = \frac{mg - T}{m} = g - \frac{T}{m}$$

- Solving these equations we find:

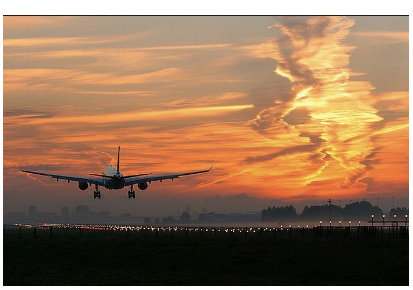
$$T = \frac{M}{M + 2m}mg$$

$$a = \frac{2m}{M + 2m}g$$



Note: $a = g$ when $M = 0$ kg!!!

Done for today!



Landing at Amsterdam Airport.