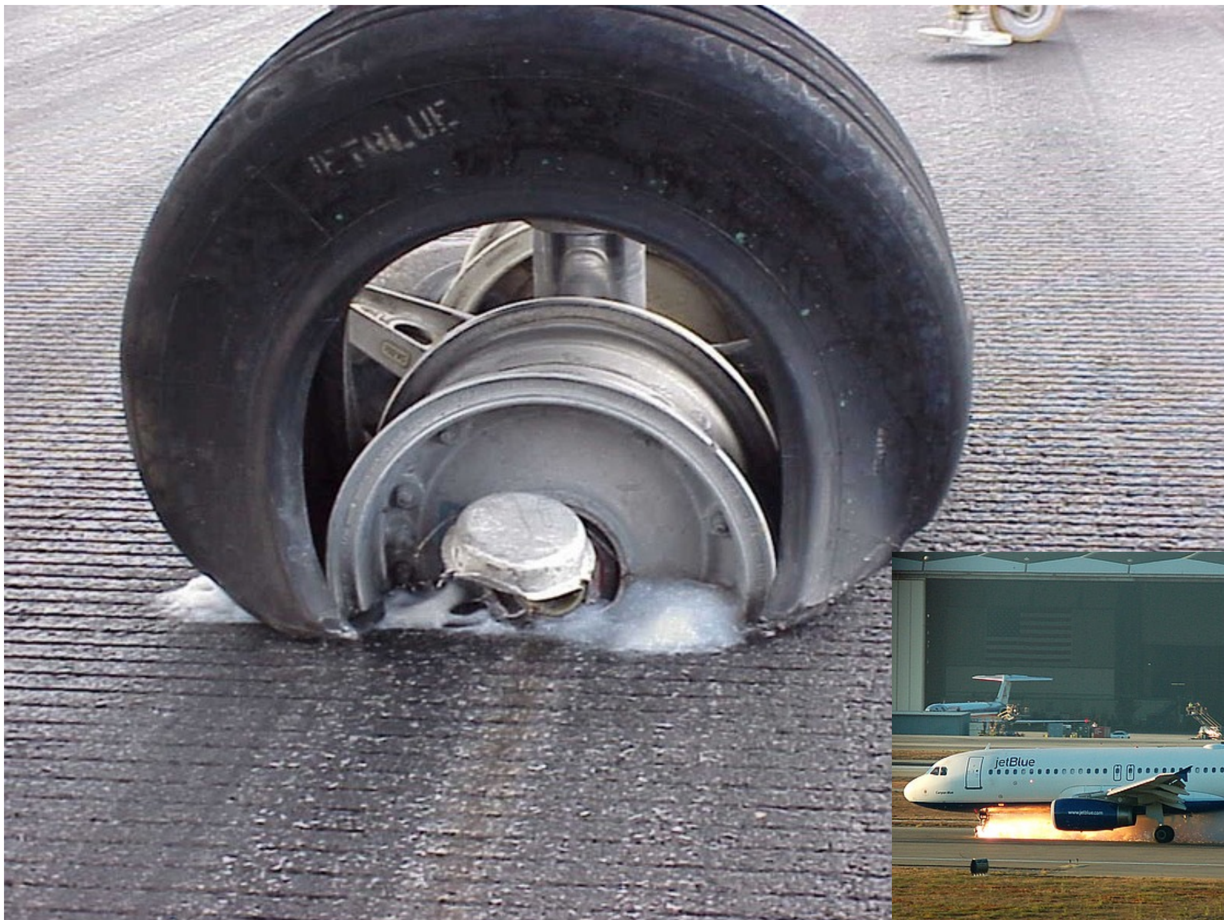
Physics 141.
Lecture 9.



Outline.

- Course information:
  - Homework set #4.
  - Lab report # 2.
  - Exam 1.
- Continue our discussion of Chapter 6:
  - Energy
  - Potential energy

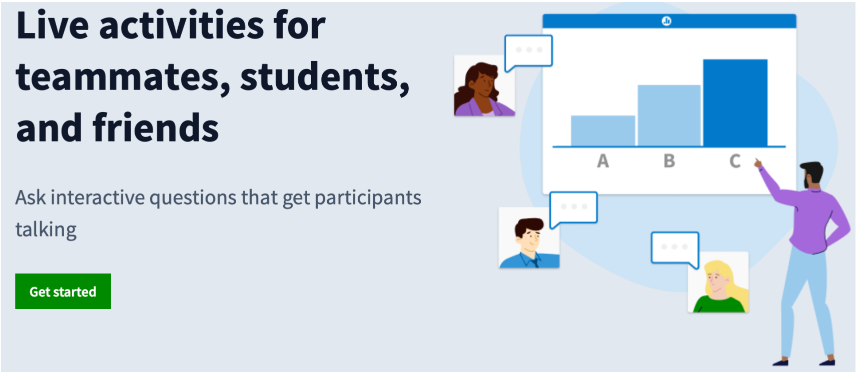
Course information.
Homework, Labs, and Exams.

- Homework # 4 is due on Friday October 6 at 12 pm (noon). **Note: written solutions dropped into the homework locker after 12 pm will not be graded.**
- Lab report # 2 is due on Wednesday October 4 at 12 pm (noon). **Note: pdf files submitted after 12 pm will not be graded.**
- If you feel that you deserve more points on certain questions on Exam 1, you need to return your exam to me with a note describing why you feel you deserve more points. You have until Thursday October 5 to make such a request. **Your TAs cannot change your exam grade.**

End of Course Information - back to Physics!

Quiz lecture 09.
PollEv.com/frankwolfs050

- The quiz today will have four questions.
- I will collect your answers electronically using the Poll Everywhere system.
- The answers for each question will be entered in sequence (first 30 s for question 1, followed by 30 s for question 2, etc.).



The energy principle.

- The energy principle states that the change in energy of a system (ΔE_{system}) is equal to the work done by the surroundings (W_{surr}) and the energy flow (Q) between the system and surroundings due to a difference in temperature:

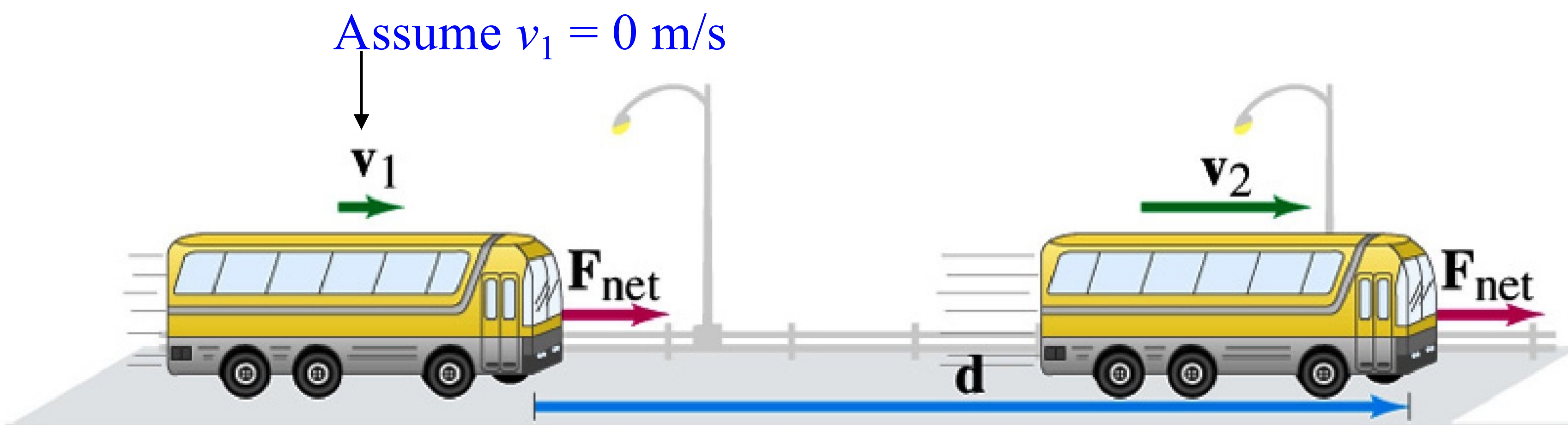
$$\Delta E_{\text{system}} = W_{\text{surr}} + Q$$

- The work W done by the force F is defined as

$$W = \vec{F} \bullet \vec{d} = Fd\cos\theta$$

  where θ is the angle between the force F and the displacement d.
- If $Q > 0$ J, energy flows into the system (e.g. $T_{\text{sys}} < T_{\text{surr}}$). If $Q < 0$ J, energy flows out of the system (e.g. $T_{\text{sys}} > T_{\text{surr}}$).

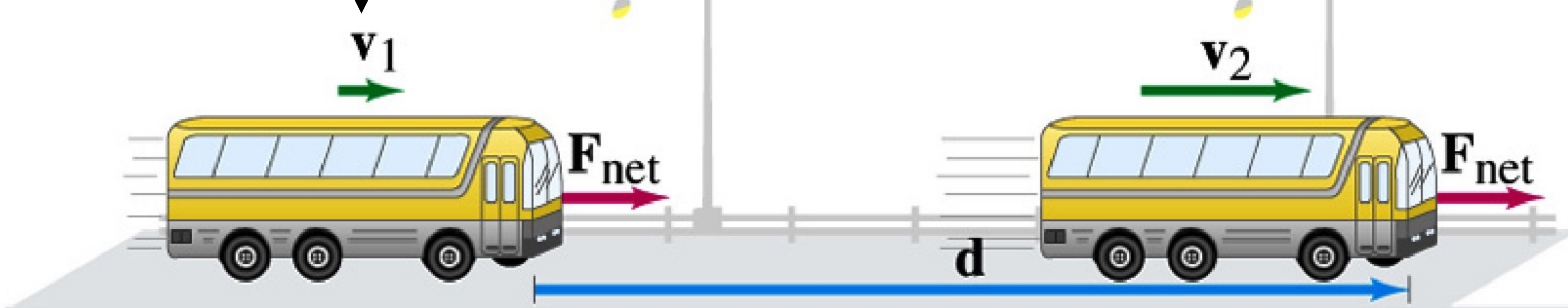
The energy principle.



- We have already seen that there is a connection between the work done by a force and the change in the speed of the object:
  - If $W > 0$ J: speed increases
  - If $W = 0$ J: speed remains constant
  - If $W < 0$ J: speed decreases

The energy principle.

Assume $v_1 = 0$ m/s



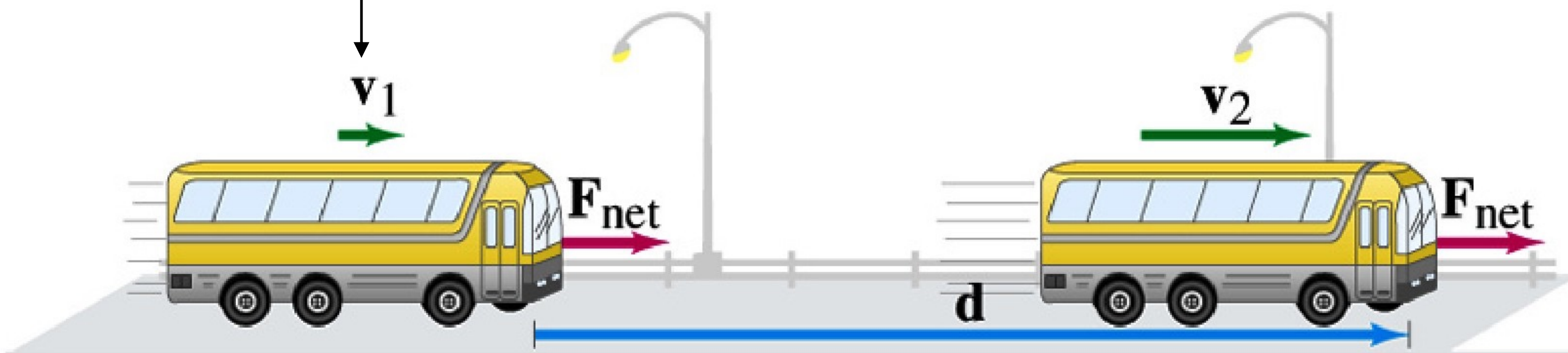
- Consider the bus starting from rest ($v_1 = 0$ m/s) and having an acceleration $a = F/m$ (assume $v << c$). The velocity at a later time t will be equal to

$$v(t) = v_0 + at = at$$

- This relation can be used to determine the time t at which the bus reaches a certain velocity v_2: $t = v_2/a$.

The energy principle.

Assume $v_1 = 0$ m/s



- The displacement at this time t is equal to

$$d = x\left(t = \frac{v_2}{a}\right) = x_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2}a\left(\frac{v_2}{a}\right)^2 = \frac{1}{2}\frac{v_2^{\ 2}}{a}$$

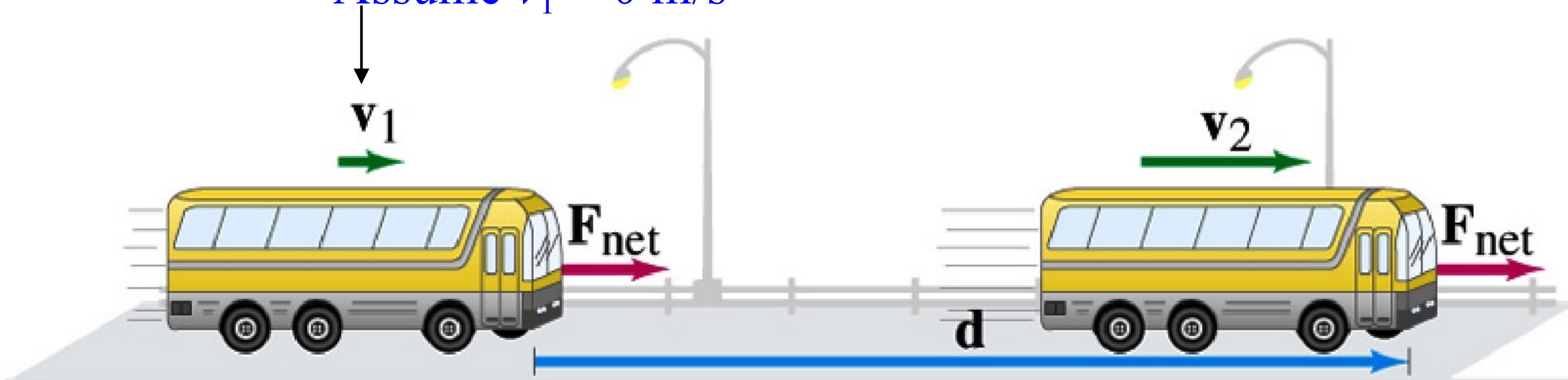
- The work done by the force F during this period is equal to

$$W = \vec{F} \bullet \vec{d} = ma\left(\frac{1}{2}\frac{v_2^{\ 2}}{a}\right) = \boxed{\frac{1}{2}mv_2^{\ 2}}$$

← Kinetic Energy K

The energy principle.

Assume $v_1 = 0$ m/s



- Assuming that there is no heat flow in or out of the bus, we conclude that for low velocities (non-relativistic velocities):

The net work done on an object is equal to the change in its kinetic energy.

- In the case of the bus: $F_{net}d = \Delta E = (1/2)mv_2^2 - (1/2)mv_1^2$

The energy principle.

- The relation between the change in the energy E and the work W (in the absence of heat flow) can be taken as the definition of energy:

$$\Delta E = W$$

- Consider that the particle, subjected to a force F, moves a distance dx along the x axis. The change in energy as a result of this motion will be

$$\Delta E = \vec{F} \bullet d\vec{r} = F_x \Delta x = \left(\frac{\Delta p_x}{\Delta t} \right) \Delta x$$

The energy principle.

- The previous relation can be rewritten as

$$\frac{\Delta E}{\Delta x} = \frac{\Delta p_x}{\Delta t} \quad \text{or, if } \Delta t \to 0, \quad \frac{dE}{dx} = \frac{dp_x}{dt}$$

- We know how *p* is defined, and we now have to determine what expression for *E* will satisfy the energy principle.

$$\frac{dp_x}{dt} = \frac{d}{dt}\left(\frac{mv}{\sqrt{1-v^2/c^2}}\right) = \frac{m}{\left(1-v^2/c^2\right)^{3/2}}\frac{dv}{dt}$$

- Since *E* is differentiated with respect to *x* we want to rewrite *dv*/*dt* in the following manner:

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} \quad \Rightarrow \quad \frac{dE}{dx} = \frac{dp_x}{dt} = \frac{mv}{\left(1-v^2/c^2\right)^{3/2}}\frac{dv}{dx}$$

The energy principle.

- By differentiating the following expression for E with respect x we can show that this expression satisfies the work-energy theorem:

$$E = \frac{mc^2}{\sqrt{1 - v^2 / c^2}}$$

- The is the energy of a particle of rest mass m, moving with velocity v.
- We note that even if the particle is not moving, it will have a non-zero energy, equal to mc^2. This energy is called the **rest energy** of the particle.

The energy principle.

- When the particle is moving, its energy is larger than its rest energy, and this excess of energy is called the **kinetic energy *K*** of the particle:

$$K = E - mc^2 = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)$$

- In the non-relativistic limit, $v \ll c$, we find for K

$$K \simeq mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right) = \frac{1}{2}mv^2$$

Relativistic energy and momentum. A few remarks.

- Using our definitions of energy and momentum we conclude that

$$E^2 = \frac{\left(mc^2\right)^2}{1 - v^2 / c^2} \qquad p^2 = \frac{\left(mv\right)^2}{1 - v^2 / c^2}$$

- These two relations can be combined in the following way

$$E^2 - p^2c^2 = \frac{\left(mc^2\right)^2}{1 - v^2 / c^2}\left\{1 - v^2 / c^2\right\} = \left(mc^2\right)^2$$

Relativistic energy and momentum.
A few remarks.

- Consider the relation between E and p:

$$E^2 - p^2c^2 = \left(mc^2\right)^2$$

- The right-hand side depends only on the rest mass of the particle, which is the same in each reference frame.
- Since the velocity of the particle is a relative parameter, both **energy and momentum will change when we move to different references frames**.
- However, the previous discussion shows that the difference between E^2 and $(pc)^2$ is an **invariant quantity** (independent of the reference frame being used).
- If the particle has no mass (e.g. a photon) then the right-hand side is zero, we we conclude that $E = pc$. We thus see that we can still assign a linear momentum to a massless particle: $p = E/c$.

2 Minute 33 Second Intermission

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 33 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Go asleep, as long as you wake up in 2 minutes and 32 seconds.

The energy principle ($Q = 0$). Multi-particle systems.

- When we are dealing with multi-particle systems we have to separate the forces acting on the system into two groups:
  - **Internal forces**: forces that act between the particles that make up the system. The work done by the internal forces is W_{int}.
  - **External force**: forces that are generated due to interactions between the system and its surroundings. The work done by the external forces is W_{ext}.
- The change in the energy of the particles that make up the system is equal to

$$\sum_i \Delta E_i = W_{ext} + W_{\text{int}}$$

- The opposite of the work done by the internal forces on the system results in **a change in the potential energy *U*** of the system

$$\sum_i \Delta E_i + \left(-W_{\text{int}}\right) = \sum_i \Delta E_i + \Delta U = W_{ext}$$

The energy principle ($Q = 0$). Multi-particle systems.

- The **total energy of the system (E_{system})** is defined as

$$E_{system} = \sum_i E_i + U$$

- The work-energy theorem for the system, expressed in terms of the total energy of the system, is thus equal to

$$\Delta E_{system} = W_{ext}$$

- The total energy (the energy of the system and the energy of its surroundings) is a conserved quantity:

$$\Delta E_{system} + \Delta E_{surroundings} = 0$$

The potential energy *U*.

- The potential energy *U* is the energy associated with the interaction between the constituents of the system.
- The change in the potential energy of a pair of particles with an interaction force *F* acting between them is given by

$$\Delta U = -\left(\vec{F}_{1,2} \bullet \Delta\vec{r}_1 + \vec{F}_{2,1} \bullet \Delta\vec{r}_2\right) = -\vec{F}_{2,1} \bullet \left(\Delta\vec{r}_2 - \Delta\vec{r}_1\right) = -\vec{F}_{2,1} \bullet \Delta\vec{r}_{21}$$

- If the configuration of the system does not change (e.g. a rigid object), its potential energy will also not change.

Calculating the potential energy U. One dimension.

- Per definition, the change in potential energy is related to the work done by the force:

$$\Delta U = -W = -\int_{x_0}^{x} F(x)\,dx$$

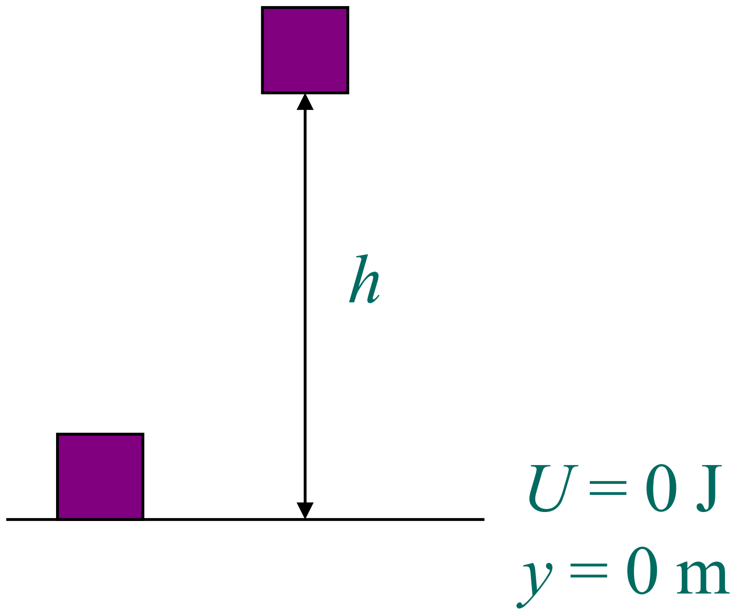
- The potential energy at x can thus be related to the potential energy at a point x_0:

$$U(x) = U(x_0) + \Delta U = U(x_0) - \int_{x_0}^{x} F(x)\,dx$$

- When we apply conservation of energy, we are in general only concerned with changes in the potential energy, ΔU, and not the actual value of U.
- We are free to assign a value of 0 J to the potential energy when the system is in its reference configuration.

Calculating the potential energy. The gravitational potential energy - Part I.

- The gravitational force close to the surface of the Earth is $-mg$.
- When we move a box from the surface (y = 0 m) to a height h ($y = h$) the work done by the force is equal to $-(mg)h$. Note: the work is negative since the force and the displacement are pointing in opposite directions.
- The potential energy associated with the position of the box is $U(y) = U(0) - W = mgh$.

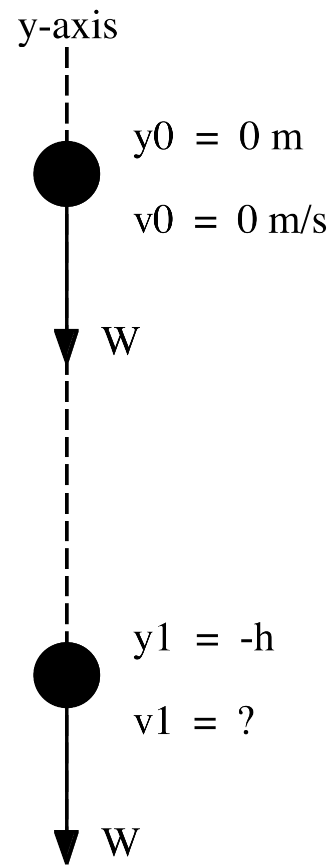


Note: reference position is the surface.
The potential energy at this position is 0 J.

The energy principle. Example 1.

- An object with mass m is at rest at time $t = 0$ s. It falls under the influence of gravity through a distance h. What is its velocity at that point?

- Solution:
  - Assume $Q = 0$.
  - Work done by the gravitational force = mgh.
  - Change in energy = change in kinetic energy = $(1/2)mv_1^2$.
  - The Energy Principle ($Q = 0$): $mgh = (1/2)mv_1^2$ or $v_1 = \sqrt{(2gh)}$



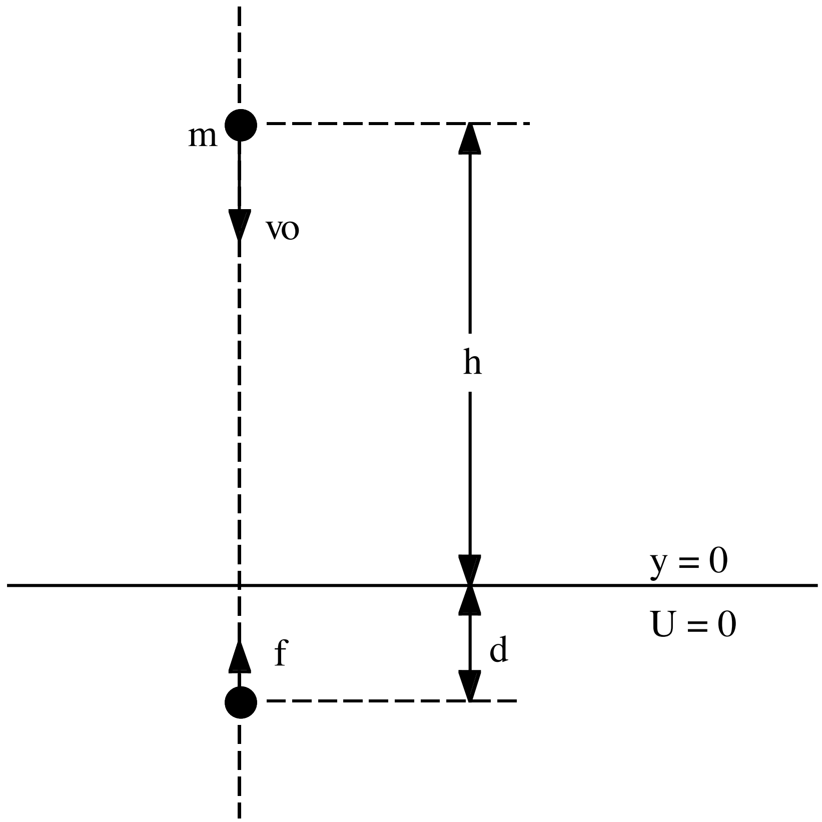
Conservation of energy. Dissipative forces.

- When dissipative forces, such as friction forces, are present, the total energy of a system will no longer be conserved and energy will be exchanged between the system and its surroundings.
- The amount of energy being exchanged will depend on the path along which the system evolved, and we can not assign a potential energy with these forces (nor will a motion along the path in opposite direction restore the energy that we moved from the system to its environment.
- The amount of energy dissipated by these non-conservative forces can be calculated if we know the magnitude and direction of these forces along the path followed by the object.

Problems with dissipative forces. Example 2.

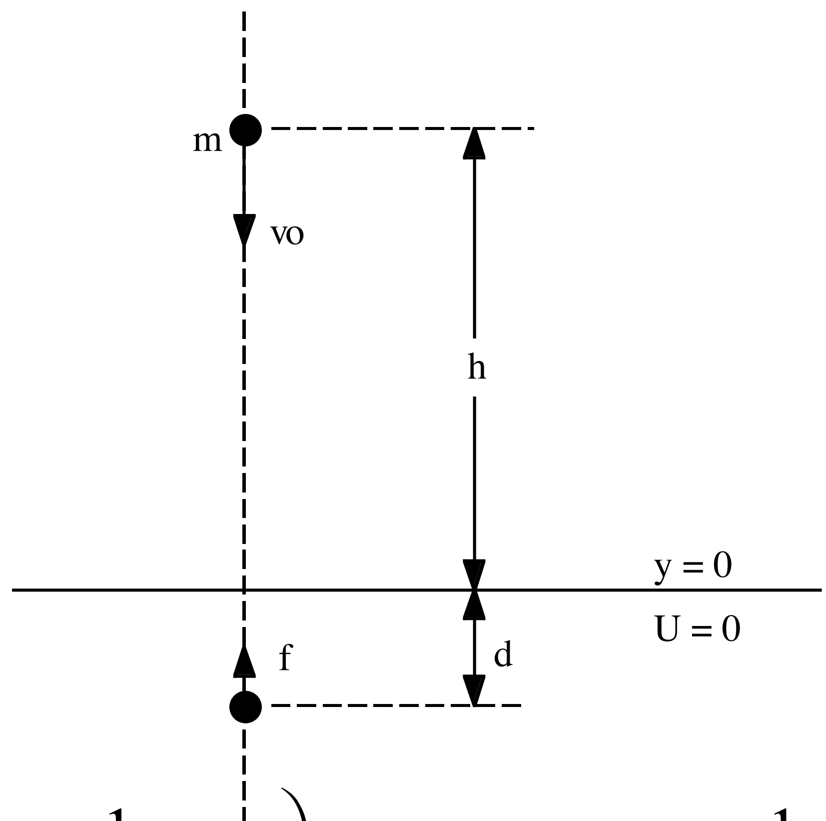
- A ball bearing whose mass is m is fired vertically downward from a height h with an initial velocity v_0. It buries itself in the sand at a depth d. What average upward resistive force f does the sand exert on the ball as it comes to rest?
- The initial total energy of the system (the bearing + the earth) is equal to

$$E_i = mgh + \frac{1}{2} m v_0^{\ 2}$$



Problems with dissipative forces. Example 2 (continued).

- The final energy of the system is equal to just the potential energy of the bearing which is equal to $E_f = -mgd$.
- The energy of the ball bearing is not conserved since the friction force between the bearing and its environment dissipates some of the system energy. The work done by the friction force is equal to $W_f = -fd$.
- The change in the energy of the system is



$$\Delta E = E_f - E_i = \left(-mgd\right) - \left(mgh + \frac{1}{2}mv_0^{\ 2}\right) = -mg\left(h+d\right) - \frac{1}{2}mv_0^{\ 2}$$

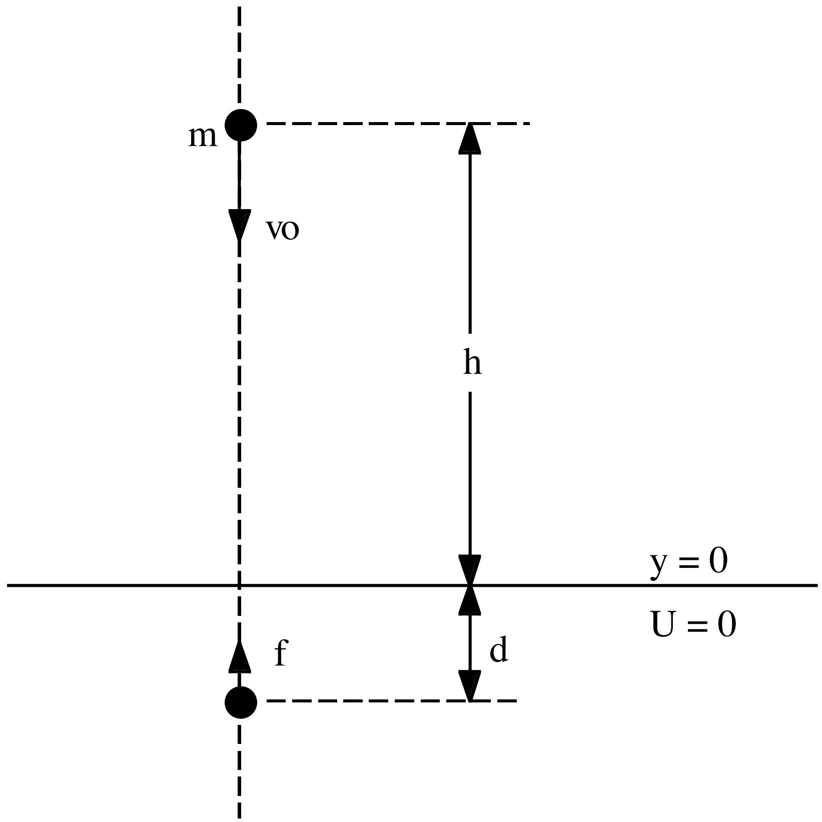
Problems with dissipative forces. Example 2 (continued).

- The change in the mechanical energy of the system is must be equal to the work done by the friction force (assuming this is the only dissipative force acting on the system):

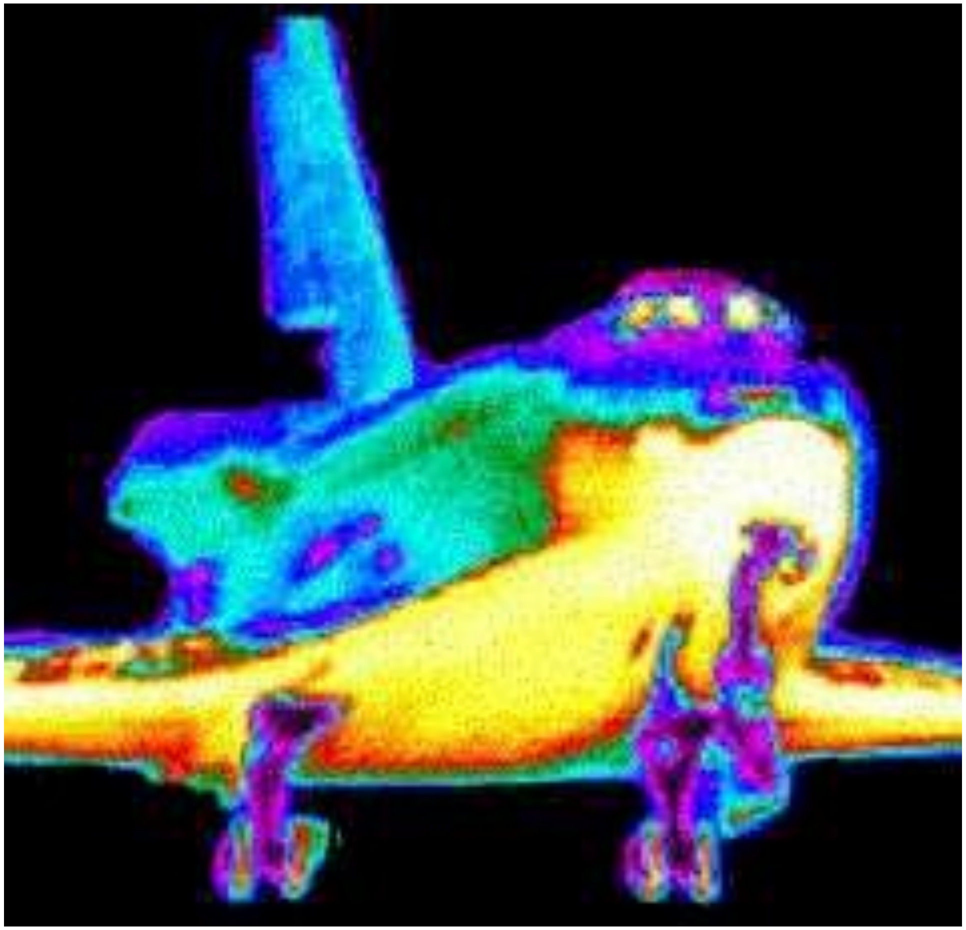
$$-fd = -mg\left(h+d\right) - \frac{1}{2}m{v_0}^2$$

- This relation can now be used to calculate the magnitude of the friction force f:

$$f = mg\left(1+\frac{h}{d}\right) + \frac{m{v_0}^2}{2d}$$



That is all for today.
More energy next lecture!



This object is not in a state of thermal equilibrium.