Slide 1

Physics 141, Lecture 7.

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Slide 2

New York Times 9/21/2023.
I directed the author the lecture 5.
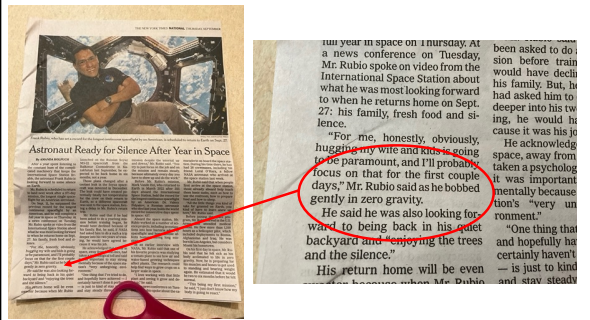
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Slide 3

Outline.

- Course information:
  - Exam # 1

- Quiz.

- Continuation of the discussion of Chapter 4:
  - Simple harmonic motion.
  - Damped harmonic motion.
  - Driven harmonic motion.

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Exam 1.

- The results of exam # 1 have been distributed via email.
- The exam will be returned this week during recitations.
- Compare your solutions with the posted solutions.
- Things to improve:
  - About 20% did not put their student ID on the scantron form.
  - One student did not enter a student ID and name on the scantron form.
  - One student put the wrong student ID and no name on the scantron form.
  - About 10% put all analytical answers in one booklet.

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Results Exam # 1.

5
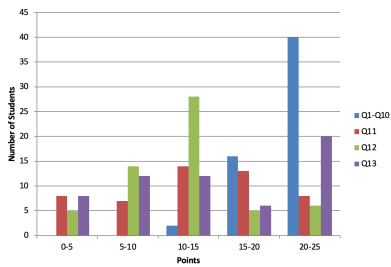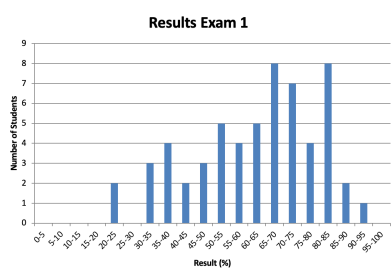
Results Exam # 1.

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Results Exam # 1.
How well did you think you did?
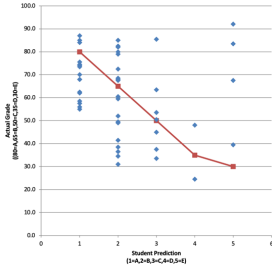
7

Quiz lecture 07.
PollEv.com/frankwolfs050

• The quiz today will have three questions.

• I will collect your answers electronically using the Poll Everywhere system.

Live activities for teammates, students, and friends

Ask interactive questions that get participants talking

Get started

• The answers for each question will be entered in sequence (first 30 s for question 1, followed by 30 s for question 2, etc.).

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A quick review of the material discussed in Lecture 6.

• We can visualize a solid as a collection of atoms of mass m, interconnected by springs.
• The atoms are not at rest in a solid, but continuously vibrate around an equilibrium position.
• The temperature of the solid is a measure of the kinetic energy associated with the motion of the atoms.
• This simple model can explain many important properties of matter, but many others can only be explained in terms of quantum mechanics.
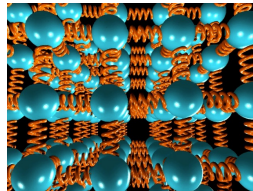
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3

The spring-mass system.

- The key to the understanding of the atomic model of matter is the understanding of the spring-like interaction between the atoms.

- Since matter will never be at the absolute zero temperature, the atoms will have an non-zero average kinetic energy (proportional to the temperature of the matter).

- Since the atoms will move, the "springs" in our model will carry out a dynamic motion which we will need to understand in more detail.

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The spring-mass system.

- For the spring force we know:

  - Its direction is opposite to the displacement.

  - Its magnitude is $k |x|$.

- Consider the force acting on mass m when it is located at position x:

  - $F = -kx$

  - But we also know that $F = ma$

  - Thus ……. $a = d^2x/dt^2 = -(k/m)x$
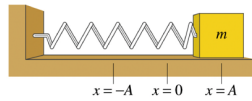
$x = -A \quad x = 0 \quad x = A$

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The spring-mass system.

- The displacement of the spring as function of time can thus be determine if we can solve the following equation:

  $d^2x/dt^2 + (k/m)x = 0$

- This equation has two possible solutions:

  $x(t) = A \cos(\omega t + \varphi)$
  $x(t) = A \sin(\omega t + \varphi)$

  where $\omega^2 = k/m$.

- This motion is an example of simple harmonic motion.

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Simple harmonic motion.
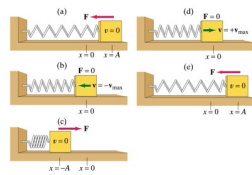
Phase Constant

Amplitude

$$x(t) = A\cos\left(\omega t + \phi\right)$$

Angular Frequency

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Simple harmonic motion.

• Instead of the angular frequency ω the motion can also be described in terms of its period T or its frequency v.
• The period T is the time required to complete one oscillation:

$$x(t) = x(t + T)$$

or

$$A\cos(\omega t + \varphi) = A\cos(\omega t + \omega T + \varphi)$$

• In order for this to be true, we must require that $\omega T = 2\pi$. The period T is thus equal to $2\pi/\omega$.
• The frequency v is the number of oscillations carried out per second $(v = 1/T)$. The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 s^{-1}.

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Simple harmonic motion.
What forces are required?

• Consider we observe simple harmonic motion.
• The observation of the equation of motion can be used to determine the nature of the force that generates this type of motion.
• In order to do this, we need to determine the acceleration of the object carrying out the harmonic motion:

$$x(t) = A\cos\left(\omega t + \phi\right)$$

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}\left(A\cos\left(\omega t + \phi\right)\right) = -\omega A\sin\left(\omega t + \phi\right)$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(-\omega A\sin\left(\omega t + \phi\right)\right) = -\omega^2 A\cos\left(\omega t + \phi\right) = -\omega^2 x(t)$$

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Simple harmonic motion.
What forces are required?

(a)

(b)

(c)

Note: maxima in displacement correlate with minima in acceleration.

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Simple harmonic motion.
What forces are required?

• Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

• We conclude:

Simple harmonic motion is the motion executed by a particle of mass m, subject to a force F that is proportional to the displacement of the particle, but opposite in sign.

• Any force that satisfies this criterion **can** produce simple harmonic motion. If more than one force is present, you need to examine the net force, and make sure that the net force is proportional to the displacements, but opposite in sign.

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Simple harmonic motion (SHM).
The simple pendulum.

• Consider a simple pendulum.
  • A simple pendulum is a pendulum for which all the mass is located at a single point at the end of a massless string.
  • There are two forces acting on the mass: the tension T and the gravitational force mg.
  • The tension T cancels the radial component of the gravitational force when the $|x|$ and $|\theta|$ reach their maxima. At all other positions, the net radial force is pointing in the same direction as the tension T and provides the required centripetal acceleration.
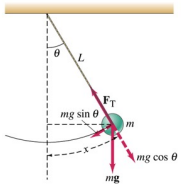
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Simple harmonic motion (SHM).
The simple pendulum.

- The net force acting on he mass is directed perpendicular to the string and is equal to

$$F = - mg \sin\theta$$

The minus sign indicates that the force is directed opposite to the angular displacement.

- When the angle θ is small, we can approximate $\sin\theta$ by θ:

$$F = - mg\theta = - mgx/L$$

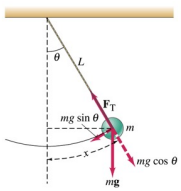- Note: the force is again proportional to the displacement.

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Simple harmonic motion (SHM).
The simple pendulum.

- The equation of motion for the pendulum is thus

$$F = m \, d^2x/dt^2 = -(mg/L)x$$

or

$$d^2x/dt^2 = - (g/L)x$$

- The equation of motion is the same as the equation of motion for a SHM, and the pendulum will thus carry out SHM with an angular frequency $\omega = \sqrt{(g/L)}$.

- The period of the pendulum is thus $2\pi/\omega = 2\pi\sqrt{(L/g)}$. Note: the period is independent of the mass of the pendulum.
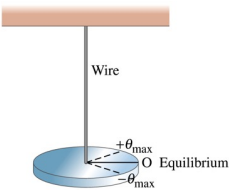
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Simple harmonic motion (SHM).
The torsion pendulum.

- What is the angular frequency of the SHM of a torsion pendulum:

- When the base is rotated, it twists the wire and a the wire generated a torque which is proportional to the the angular twist:

$$\tau = -K\theta$$

The torque generates an angular acceleration α:

$$\alpha = d^2\theta/dt^2 = \tau/I = -(K/I)\,\theta$$

The resulting motion is harmonic motion with an angular frequency $\omega = \sqrt{(K/I)}$.

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3 Minute 35 Second Intermission

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 35 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.
  - Go asleep, as long as you wake up in 3 minutes and 35 seconds.

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Damped harmonic motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b\frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

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Damped harmonic motion.

- The general solution of this equation of motion is

$$x(t) = Ae^{i\omega t}$$

- If we substitute this solution in the equation of motion we find

$$-\omega^2 Ae^{i\omega t} + i\omega\frac{b}{m}Ae^{i\omega t} + \frac{k}{m}Ae^{i\omega t} = 0$$

- In order to satisfy the equation of motion, the angular frequency must satisfy the following condition:

$$\omega^2 - i\omega\frac{b}{m} - \frac{k}{m} = 0$$

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Damped harmonic motion.

• We can solve this equation and determine the two possible values of the angular velocity:

$$\omega = \frac{1}{2}\left(i\frac{b}{m} \pm \sqrt{4\frac{k}{m} - \frac{b^2}{m^2}}\right) \simeq \frac{1}{2}i\frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

• The solution to the equation of motion is thus given by

$$x(t) \simeq x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

↑   ↑

Damping Term   SHM Term

Frank L. H. Wolfs   Department of Physics and Astronomy, University of Rochester, Lecture 07, Page 25
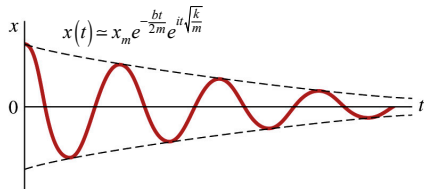
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Damped harmonic motion.

$$x(t) \simeq x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

The general solution contains a SHM term, with an amplitude that decreases as function of time

Frank L. H. Wolfs   Department of Physics and Astronomy, University of Rochester, Lecture 07, Page 26

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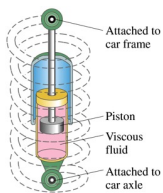## Damped harmonic motion has many practical applications.

Attached to car frame

Piston

Viscous fluid

Attached to car axle

Damping is not always a curse.

Frank L. H. Wolfs   Department of Physics and Astronomy, University of Rochester, Lecture 07, Page 27

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Driven harmonic motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0.

- Assume the external force $F(t) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + F_0 \sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.

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Driven harmonic motion.

- Consider the general solution

$$x(t) = A\cos(\omega t + \phi)$$

- The parameters in this solution must be chosen such that the equation of motion is satisfied. This requires that

$$-\omega^2 A\cos(\omega t + \phi) + \omega_0^2 A\cos(\omega t + \phi) - F_0 \sin(\omega t) = 0$$

- This equation can be rewritten as

$$(\omega_0^2 - \omega^2) A\cos(\omega t)\cos(\phi) -$$
$$(\omega_0^2 - \omega^2) A\sin(\omega t)\sin(\phi) - F_0 \sin(\omega t) = 0$$

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Driven harmonic motion.

- Our general solution must thus satisfy the following condition:

$$(\omega_0^2 - \omega^2) A\cos(\omega t)\cos(\phi) - \{(\omega_0^2 - \omega^2) A\sin(\phi) - F_0\}\sin(\omega t) = 0$$

- Since this equation must be satisfied at all time, we must require that the coefficients of $\cos(\omega t)$ and $\sin(\omega t)$ are 0. This requires that

$$(\omega_0^2 - \omega^2) A\cos(\phi) = 0$$

and

$$(\omega_0^2 - \omega^2) A\sin(\phi) - F_0 = 0$$

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Driven harmonic motion.

• The interesting solutions are solutions where $A \neq 0$ and $\omega \neq \omega_0$. In this case, our general solution can only satisfy the equation of motion if

$$\cos(\phi) = 0$$

and

$$\left(\omega_0^2 - \omega^2\right) A \sin(\phi) - F_0 = \left(\omega_0^2 - \omega^2\right) A - F_0 = 0$$

• The amplitude of the motion is thus equal to

$$A = \frac{F_0}{\left(\omega_0^2 - \omega^2\right)}$$

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Driven harmonic motion.

• If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
• In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.

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Driven harmonic motion.

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That's all for today!
Next lecture: force, motion, and energy.

"Rush Hour in Reno"
Credit and Copyright: John Endter of Minden, Nevada

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