

Physics 141.

Lecture 4.



Randy Johnson strikes again

$$\vec{p}_{\text{new}} = \vec{p}_{\text{old}} + \vec{F}_{\text{net}} \Delta t$$

Physics 141.

Lecture 4.

- Today's Topics:
 - Course information (HW # 2).
 - A brief review of the material covered in Lecture 3.
 - Poll Everywhere: two simple questions that show a lot about your understanding of linear motion.
 - The momentum principle.
 - Using VPython to solve dynamics problem: a demonstration.

Physics 141.

Course Information.

- Homework set # 2 is now available.
- This set has three components:
 - A WeBWorK component that focuses on typical mechanics problems and will develop your problem-solving skills
 - A written component, requiring you to work out a few problems in detail and develop numerical models for realistic systems.
 - The optional VPython component. This component needs to be emailed to me (see instructions for detail).

A quick review.

Linear motion in one dimension.

Parameters define initial conditions!

$$x(t) \qquad x(t) = \int_{t_0}^t v(t') dt' \qquad x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = \frac{dx}{dt} \qquad v(t) = \int_{t_0}^t a(t') dt' \qquad v(t) = v_0 + at$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} \qquad a(t) \qquad a(t) = a = \text{constant}$$

↑
The same for different observers!

A quick review.

Motion in three dimensions: constant a .

$$\vec{\mathbf{r}}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \quad \vec{\mathbf{v}}(t) = \begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix} \quad \vec{\mathbf{a}}(t) = \begin{pmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{pmatrix} \quad \text{where}$$

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \quad z(t) = z_0 + v_{0z}t + \frac{1}{2}a_z t^2$$

$$v_x(t) = v_{0x} + a_x t \quad v_y(t) = v_{0y} + a_y t \quad v_z(t) = v_{0z} + a_z t$$

$$a_x(t) = a_x = \text{constant} \quad a_y(t) = a_y = \text{constant} \quad a_z(t) = a_z = \text{constant}$$

Note: A non-zero acceleration in one direction only affects motion in that direction.

A quick review.

Projectile motion in two dimensions.

$$\begin{aligned}x(t) &= x_0 + v_{0x}t & y(t) &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \\v_x(t) &= v_{0x} = \text{constant} & v_y(t) &= v_{0y} - gt \\a_x(t) &= 0 & a_y(t) &= -g = \text{constant}\end{aligned}$$

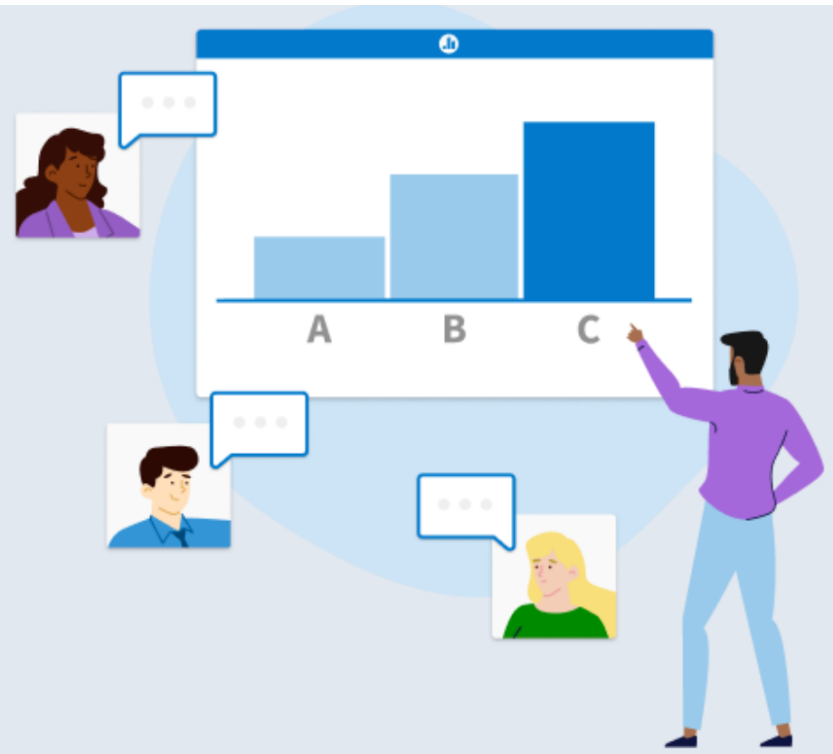
Note: The non-zero gravitational acceleration only affects motion in the vertical direction; not in the horizontal direction.

Do you really understand 1D/2D motion?

Live activities for teammates, students, and friends

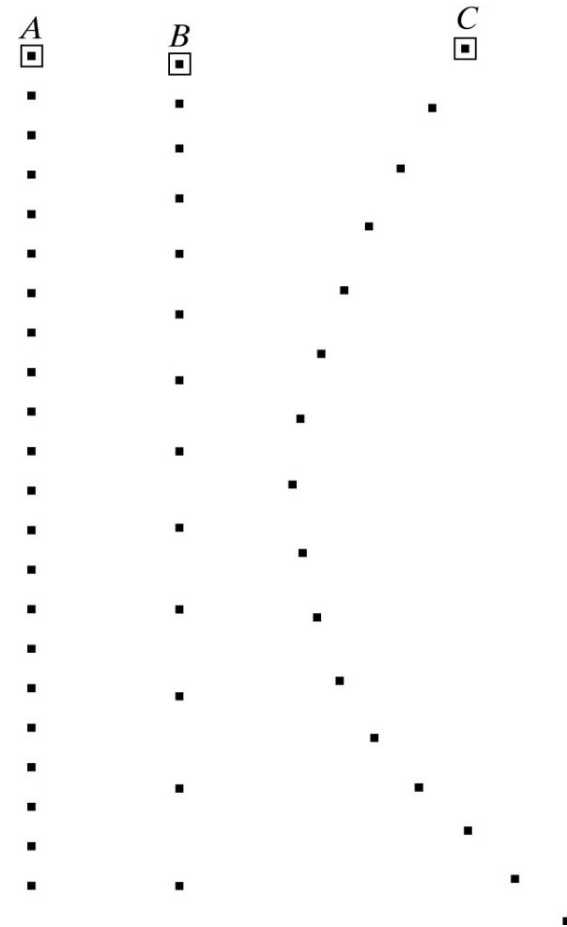
Ask interactive questions that get participants talking

Get started



Detecting interactions.

- A non-zero force acting on an object will accelerate it:
 - Change its direction
 - Change its speed
- The change in the direction and/or speed provides us with information about the magnitude and the direction of the interaction.
- If we know the interaction we can determine the change in the direction and/or speed.



Quantifying the extent of an interaction.

- The effect of an interaction will depend on both the velocity and the mass of the observed object:
 - It is easier to change the velocity of an object when it is moving slow compared to when it is moving fast.
 - It is easier to change the velocity of a light object compared to what is required for a massive object moving with the same velocity.
- It is observed that the change in the **linear momentum**

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is proportional to the "amount" of the interaction.

Quantifying the extent of an interaction.

- The change in the linear momentum of an object is proportional to the strength of the interaction and to the duration of the interaction. This principle is known as the **momentum principle**:

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

- If we do not know the interaction, but we measure the change in the linear momentum of an object, we can determine the extent of the interaction:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

2 Minute 18 Second Intermission



- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 2 minute 18 second intermission.
- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.
 - Go asleep, as long as you wake up in 2 minutes and 18 seconds.



Applying the momentum principle.

- When we apply the momentum principle we assume that the interaction is constant during the time interval Δt .

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

This will only be a good approximation if the time interval is small.

- Note: the requirement does not depend on whether the interaction has an explicit time dependence. If the interaction is position dependent, the force experienced by the object will also change as function of time due to the change in position.

Applying the momentum principle.

- Consider that an object is at position (x, y, z) at time $t = 0$ s. At this position and time, its linear momentum is given. In addition, we know the strength and direction of the interaction as function of both position and time.
- After a short time interval Δt the linear momentum of the object will have changed and will be equal to

$$\vec{p}_{new} = \vec{p}_{old} + \vec{F}_{net}\Delta t$$

- The new position of the object can be found if we know its velocity:

$$\vec{r}_{new} = \vec{r}_{old} + \vec{v}\Delta t$$

Applying the momentum principle.

- We conclude that:

- The force (interaction) defines the change in momentum.
- The velocity defines the change in position.
- We now need to express the velocity in terms of the momentum. This is easy in Newtonian mechanics ($v = p/m$) but more complicated in relativistic mechanics:

$$p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} mv \Rightarrow \frac{p^2}{m^2} = \frac{1}{1 - \frac{v^2}{c^2}} v^2 \Rightarrow \frac{p^2}{m^2} - \frac{p^2}{m^2 c^2} v^2 = v^2 \Rightarrow$$

$$\frac{p^2}{m^2} = v^2 + \frac{p^2}{m^2 c^2} v^2 = \left(1 + \frac{p^2}{m^2 c^2}\right) v^2 \Rightarrow v = \frac{\frac{p}{m}}{\sqrt{\left(1 + \left(\frac{p}{mc}\right)^2\right)}}$$

Applying the momentum principle.

- The new position of the object can now be expressed in terms of its old position and its velocity:

$$\vec{r}_{new} = \vec{r}_{old} + \vec{v}_{old} \Delta t = \vec{r}_{old} + \frac{1}{\sqrt{1 + \left(\frac{p_{old}}{mc}\right)^2}} \frac{\vec{p}_{old}}{m} \Delta t$$

- For speeds much smaller than the speed of light this relation becomes:

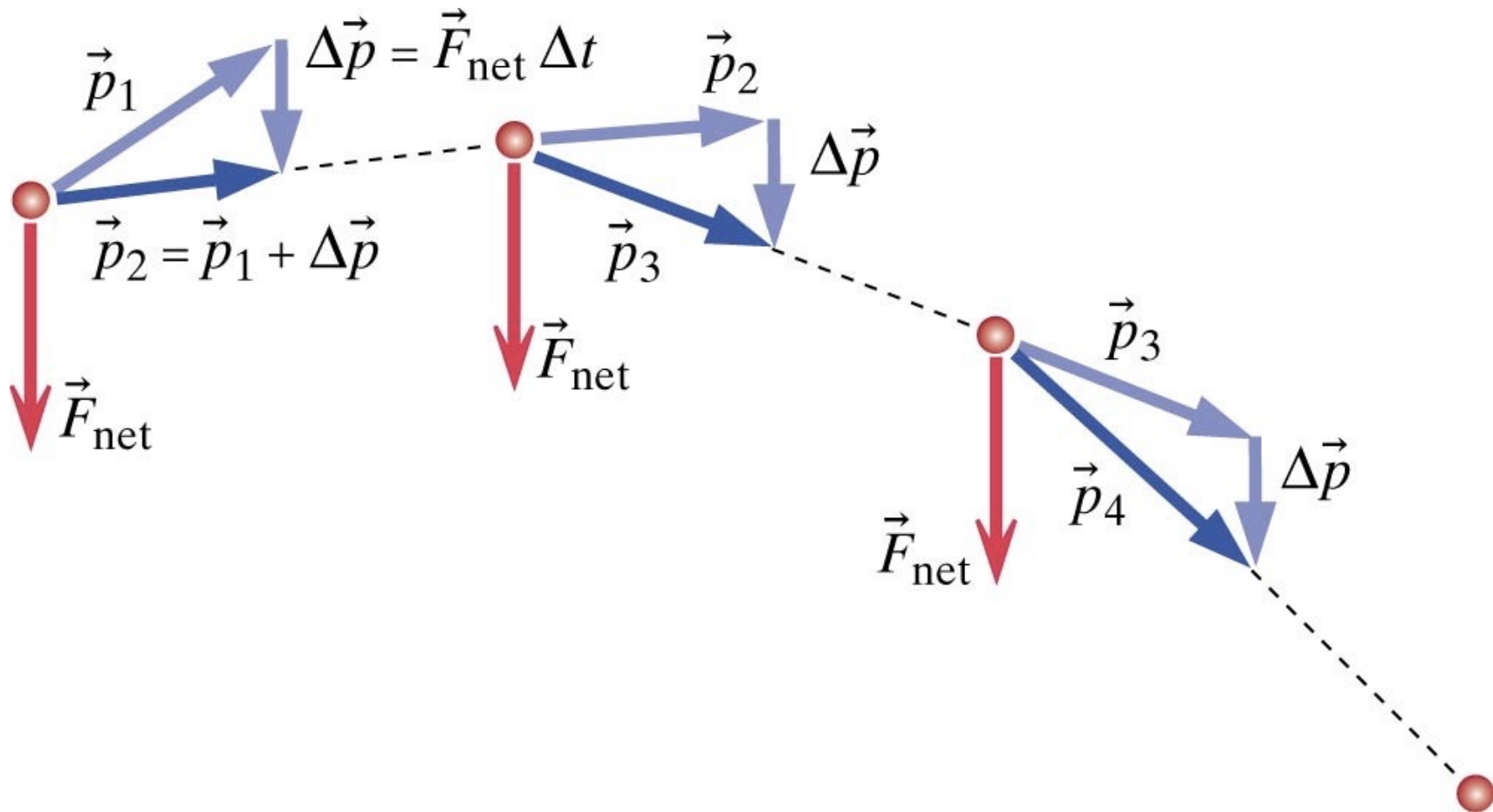
$$\vec{r}_{new} = \vec{r}_{old} + \frac{\vec{p}_{old}}{m} \Delta t$$

- These expressions form the core of most numerical simulations of dynamic systems.

Applying the momentum principle.

- To study the time evolution of a system we do the following:
 - Define the initial conditions (position and linear momentum).
 - Using the current linear momentum and the current position, determine the new position after a time interval Δt .
 - Using the interaction at the current position, calculate the change in the linear momentum during a time interval Δt .
 - Using the change in the linear momentum, calculate the new linear momentum at the new position.
 - Repeat these steps.
- Note: care should be taken to ensure that the step size Δt is small enough to ensure that the force is constant, but not too small to ensure that the calculation finished in our lifetime.

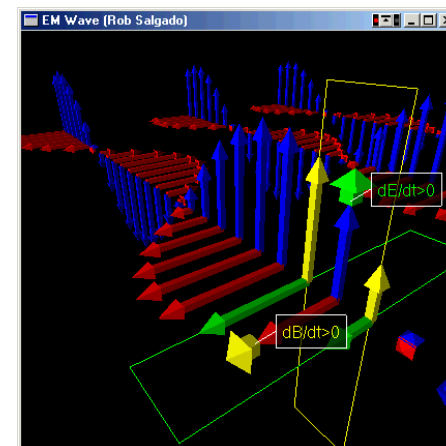
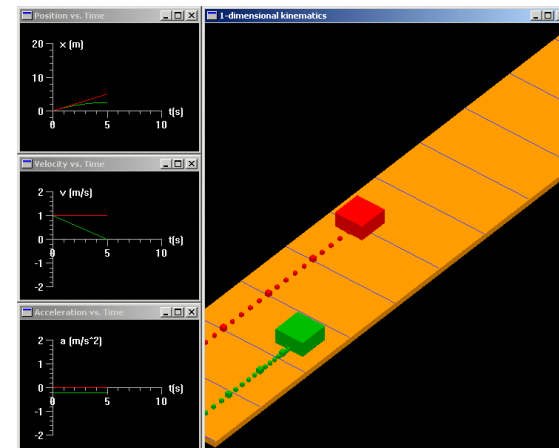
Applying the momentum principle.



VPython Tutorial.

(<http://www.vpython.org/>)

- Many important mechanics effects, for which analytical solutions do not exist, can be studied using programming languages such as VPython.
- Becoming familiar with the principles of programming is not hard, especially for a group of honors students like you.
- This skill will not only benefit you for this course (by allowing you to study realistic systems), it will benefit you in many other science courses.



Examples from
<http://www.physics.syr.edu/%7Eesalgado/software/vpython/>

That's all (for today)!

Next lecture: Quiz on Chapter 2.



Images from Hubble:
Neptune's dynamic
atmosphere and its
satellites.

<http://hubblesite.org/newscenter/newsdesk/archive/releases/2005/22/video/a>