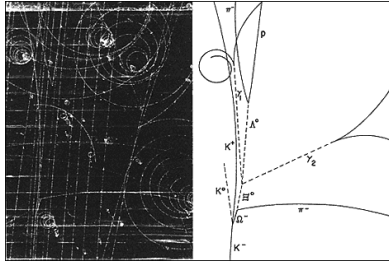


Physics 141. Lecture 2.



Credit: Brookhaven National Laboratory, NY, USA
<http://www.bnl.gov/bnlweb/history/Omega-minus.asp>

The bubble chamber picture of the first omega-minus. An incoming K^- meson interacts with a proton in the liquid hydrogen of the bubble chamber and produces an omega-minus, a K^0 and a K^- meson which all decay into other particles. Neutral particles which produce no tracks in the chamber are shown by dashed lines. The presence and properties of the neutral particles are established by analysis of the tracks of their charged decay products and application of the laws of conservation of mass and energy.

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1

Physics 141.

• Today's topics:

- Physics 141 website and homework
- Introduction to Poll Everywhere
- Complete the discussion of error analysis

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2

Physics 141 Course Information.

• The Physics 141 homepage provides up-to-date course information:

- All course handouts are posted in pdf format on our webpage.
- The webpage contains a link to our homework system.
- All lecture presentations will be available on the webpage shortly before or after class. You can even relive these fascinating lectures by watching the videos.

• The URL of our homepage is:

- <http://teacher.pas.rochester.edu/Phy141/Phy141HomePage.shtml>

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The 141 Home Page. Let's have a look.
<http://teacher.pas.rochester.edu/PHY141/Phy141HomePage.shtml>

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Physics 141. Recitations and office hours.

- **Recitations start this week.**
 - Tuesdays: 3:25 pm - 4:40 pm (B&L 269, Nathan Skerrett)
 - Tuesdays: 4:50 pm - 6:05 pm (B&L 203H, Aidan Bachmann)
 - Wednesdays: 2:00 pm - 3:15 pm (B&L 269, Guilherme Fiusa)
 - Wednesdays: 6:15 pm - 7:30 pm (Hylan 202, Guilherme Fiusa)
- **Attendance of recitations is strongly recommended, but it is not required.**
- **Office hours start this week:**
 - My office: Thursdays between 11:30 am and 1:30 pm (B&L 203A).
 - Office hours of the TAs:
 - Fiusa: Wednesdays 5 - 6 pm and Fridays 11 am - 12 pm, B&L 304.
 - Bachmann: Wednesdays 10-11 am, POA.
 - Skerrett: Thursdays 9 - 10 pm, POA.

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Physics 141. Homework assignments.

- The Physics 141 homework assignments will in general have three components:
 - A WeBWork component (focused on analytical problem solving).
 - A written component (detailed written solutions to be handed in)
 - An optional (extra credit) programming component (focused on solving problems without nice analytical solutions). For most programming problems a skeleton program will be provided.
- All homework components can be accessed via our homepage.
- Homework set 1 is due on Friday 9/8/23 at noon.
- You are encouraged to work together on the assignments!



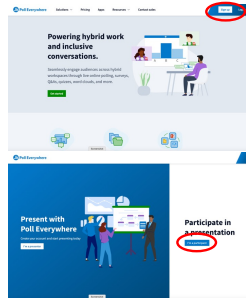
Note: you can make your life much easier if you get used to using spreadsheets. Check out the videos!

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Introduction to Poll Everywhere. <https://www.poll.everywhere.com>

- Poll Everywhere is used to carry out in-class quizzes and concept tests.
- Quizzes will be recorded for credit. The concept tests are not.
- To participate and receive credit for the quizzes, you need an PollEverywhre account. **Make sure you use your U of R email address to setup your account.**
- We will try it out today.



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Introduction to Poll Everywhere. <https://www.poll.everywhere.com>



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8

Error Analysis - a quick summary. Type of Errors.

- **Statistical errors:**
 - Results from a random fluctuation in the process of measurement. Often quantifiable in terms of "number of measurements or trials". Tends to make measurements less precise.
- **Systematic errors:**
 - Results from a bias in the observation due to observing conditions or apparatus or technique or analysis. Tend to make measurements less accurate.

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The Gaussian Distribution: its mean and its standard deviation.

1σ is roughly the half-width at half-maximum of the distribution.

If you carry out one measurement of the length, there will be a 68.3% probability that the outcome of this measurement lies between $\mu - \sigma$ and $\mu + \sigma$.

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

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How to determine the mean μ and width σ of a distribution based on N measurements?

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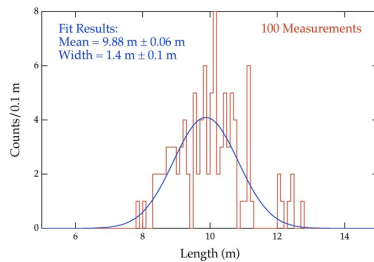
How to determine the mean μ and width σ of a distribution based on N measurements?

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_{N-1} + x_N}{N} = \frac{1}{N} \sum_{i=1}^N x_i = \mu$$

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How to determine the mean μ and width σ of a distribution based on N measurements?



The "standard deviation" is a measure of the error in each of the N measurements:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}}$$

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How to determine the mean μ and width σ of a distribution based on N measurements?

- The standard deviation is equal to $\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}}$
- But μ is unknown. So we will use the mean (which is your best estimate of μ). We also change the denominator to increase the error slightly due to using the mean.
- This is the form of the standard deviation you use in practice: $\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}}$
- Note: This quantity cannot be determined from a single measurement.

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}}$$

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What matters? The standard deviation or the error in the mean?

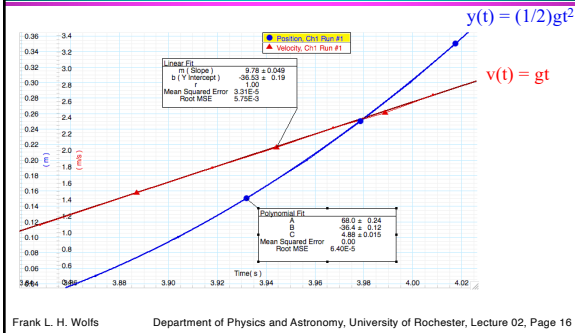
- The standard deviation is a measure of the error made in each individual measurement.
- Often you want to measure the mean and the error in the mean.
- Which should have a smaller error, an individual measurement or the mean?
- The answer the mean, if you do more than one measurement:

$$\sigma_m = \frac{\sigma}{\sqrt{N}}$$

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Applying this in our laboratory.
Laboratory 1: measuring g.



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Applying this in our laboratory.
Laboratory 1: measuring g.

- Student 1: 9.0 m/s²
- Student 2: 8.8 m/s²
- Student 3: 9.1 m/s²
- Student 4: 8.9 m/s²
- Student 5: 9.1 m/s²

What is the best estimate of the gravitational acceleration measured by these students?

$$\bar{a} = \frac{9.0 + 8.8 + 9.1 + 8.9 + 9.1}{5} = 8.98 \frac{m}{s^2}$$

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Applying this in our laboratory.
Laboratory 1: measuring g.

- Student 1: 9.0 m/s²
- Student 2: 8.8 m/s²
- Student 3: 9.1 m/s²
- Student 4: 8.9 m/s²
- Student 5: 9.1 m/s²

What is the best estimate of the standard deviation of the gravitational acceleration measured by these students?

$$\sigma = \sqrt{\frac{(9.0-9.0)^2 + (8.8-9.0)^2 + (9.1-9.0)^2 + (8.9-9.0)^2 + (9.1-9.0)^2}{5-1}}$$

$$= 0.12 \frac{m}{s^2}$$

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Applying this in our laboratory.
Laboratory 1: measuring g.

- Student 1: 9.0 m/s²
- Student 2: 8.8 m/s²
- Student 3: 9.1 m/s²
- Student 4: 8.9 m/s²
- Student 5: 9.1 m/s²

$$\sigma_m = \frac{0.12}{\sqrt{5}} = 0.054 \frac{m}{s^2}$$

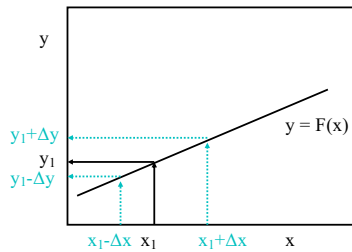
Note: this procedure is valid if you can assume that all your measurements have the same measurement error.

Final result: $g = 8.98 \pm 0.05 \text{ m/s}^2$. Does this agree with the accepted value of 9.8 m/s^2 ?

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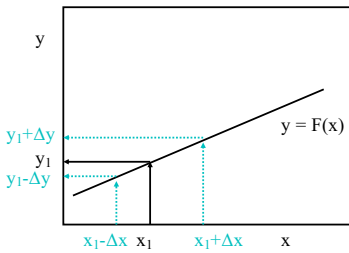
How does an error in one measurable affect the error in another measurable?



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How does an error in one measurable affect the error in another measurable?



The degree to which an error in one measurable affects the error in another is driven by the functional dependence of the variables (or the slope: dy/dx)

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How does an error in one measurable affect the error in another measurable?

- But most physical relationships involve multiple measurables!

$$x = x_o + v_o t + \frac{1}{2} a t^2$$

$$F = Ma$$

$$P = Mv$$

- We must take into account the dependence of the parameter of interest, f , on each of the contributing quantities, x, y, z, \dots : $f = F(x, y, z, \dots)$

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Error propagation.
Partial derivatives.

- The partial derivative with respect to a certain variable is the ordinary derivative of the function with respect to that variable where all the other variables are treated as constants.

$$\left. \frac{\partial F(x, y, z, \dots)}{\partial x} = \frac{dF(x, y, z, \dots)}{dx} \right]_{y, z, \dots \text{const}}$$

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Error propagation.
Partial derivatives: an example.

$$F(x, y, z) = x^2 y z^3$$

$$\frac{\partial F}{\partial x} = 2xy z^3$$

$$\frac{\partial F}{\partial y} = x^2 z^3$$

$$\frac{\partial F}{\partial z} = x^2 y 3z^2$$

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Error propagation. The formula!

- Consider that a parameter of interest $f = F(x, y, z, \dots)$ depends on the measured parameters x, y, z, \dots
- The error in f , σ_f , depends on the function F , the measured parameters x, y, z, \dots , and their errors, $\sigma_x, \sigma_y, \sigma_z, \dots$, and can be calculated using the following formula:

$$\sigma_f = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial F}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial F}{\partial z}\right)^2 \sigma_z^2 + \dots}$$

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The formula for error propagation. An example.

A pitcher throws a baseball a distance of 30 ± 0.5 m at 40 ± 3 m/s (~90 mph). From this data, calculate the time of flight of the baseball.

$$t = \frac{d}{v}$$

$$\frac{\partial F}{\partial d} = \frac{1}{v}$$

$$\frac{\partial F}{\partial v} = -\frac{d}{v^2}$$

$$\sigma_t = \sqrt{\left(\frac{1}{v}\right)^2 \sigma_d^2 + \left(-\frac{d}{v^2}\right)^2 \sigma_v^2}$$

$$= \sqrt{\left(\frac{0.5}{40}\right)^2 + \left(\frac{30}{40^2}\right)^2 3^2} = 0.058 \Rightarrow$$

$$t = 0.75 \pm 0.06 \text{ s}$$

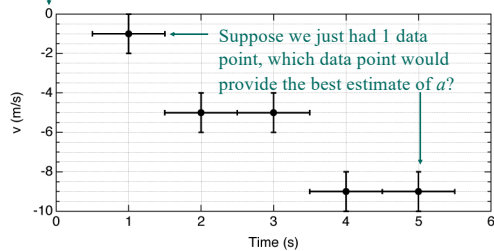
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Another example of error propagation.

If $t = 0, v = 0.$

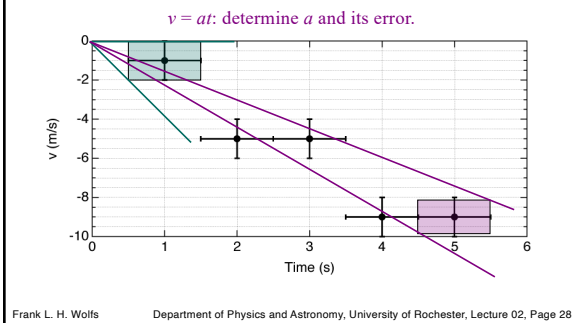
$v = at$: determine a and its error.



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Another example of error propagation.

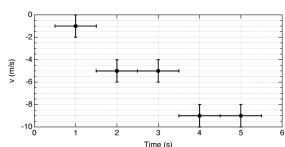


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Another example of error propagation.

- For each data point we can determine $a (= v/t)$ and its error:

$$\sigma_a = \sqrt{\left(\frac{1}{t}\sigma_v\right)^2 + \left(\frac{v}{t^2}\sigma_t\right)^2} = \frac{v}{t} \sqrt{\left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{\sigma_t}{t}\right)^2}$$



- We see that the error in a is different for different points. Simple averaging will not be the proper way to determine a and its error.

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The weighted mean.

- When the data have different errors, we need to use the weighted mean to estimate the mean value.
- This procedure requires you to assign a weight to each data point:

$$w_i = \frac{1}{\sigma_i^2}$$

- Note: when the error decreases the weight increases.
- The weighted mean and its error are defined as:

$$\bar{y} = \frac{\sum_{i=1}^N w_i y_i}{\sum_{i=1}^N w_i} \quad \sigma_{\bar{y}} = \sqrt{\frac{1}{\sum_{i=1}^N w_i}}$$

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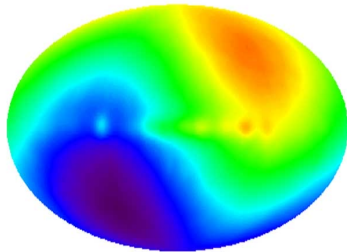
This is the start of your learning curve on how to deal with experimental errors.

- Certainly there is a lot more about statistical treatment of data than we can cover in part of one lecture.
- A true understanding comes with practice, and this is what you will do in the laboratory (and on WeBWorK set 1). Good luck!
- More details can be found in our "Data Reduction and Error Analysis" book: Chapter 1 (Uncertainties in Measurements) and Chapter 3 (Error Analysis). The terminology used in fitting data is explained in detail in several Chapters in this book.

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We are done for today!
Please review Chapter 1 before the next class.



CMBR Dipole: Speeding Through the Universe
Credit: DMR, COBE, NASA, Four-Year Sky Map

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