

The momentum principle:

$$d\vec{p} = \vec{F}_{net} dt$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{p}_{new} = \vec{p}_{old} + \vec{F}_{net} \Delta t$$

$$\vec{r}_{new} = \vec{r}_{old} + \frac{1}{\sqrt{1 + \left(\frac{p}{mc}\right)^2}} \left(\frac{\vec{p}}{m} \right) \Delta t$$

Equations of motion in 1D for constant acceleration and low velocities ($v \ll c$):

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = \frac{dx(t)}{dt} = v_0 + at$$

$$a(t) = \frac{dv(t)}{dt} = a = \text{constant}$$

Requirement for uniform circular motion:

$$F_r = \frac{mv^2}{r}$$

Rotational motion:

$$d = \theta r$$

$$v = \omega r \quad \omega = \frac{d\theta}{dt}$$

$$a = \alpha r \quad \alpha = \frac{d\omega}{dt}$$

Gravitational force:

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}$$

$$\vec{F} = m\vec{g} \quad (\text{close to the surface of the Earth})$$

Electrostatic force:

$$\vec{F} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

Harmonic motion:

$$F = -kx$$

$$x(t) = x_{\max} \cos(\omega t + \phi) \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

Damped harmonic motion:

$$x(t) = x_m e^{-\frac{bt}{2m}} e^{it\sqrt{\frac{k}{m}}}$$

Driven harmonic motion:

$$x(t) = \frac{F_0}{\omega_0^2 - \omega^2} \cos(\omega t + \phi)$$

Stress and strain:

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$Y = \frac{k_s}{d}$$

Work done by a force:

$$W = \vec{F} \cdot \vec{d} \quad \text{constant force}$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \quad \text{variable force}$$

Power:

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Work-energy theorem:

$$\Delta E_{\text{system}} = W$$

$$E_{\text{system}} = (E_1 + E_2 + E_3 + \dots) + U$$

Relativistic energy relations:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 + K$$

$$E^2 - (pc)^2 = (mc^2)^2$$

$$K = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 \xrightarrow{v \ll c} \frac{1}{2}mv^2$$

Potential energy:

$$\Delta U = -W_{\text{internal}} = -\int \vec{F} \cdot d\vec{r}$$

$$\vec{F} = -\vec{\nabla}U = \begin{pmatrix} -\frac{\partial U}{\partial x} \\ -\frac{\partial U}{\partial y} \\ -\frac{\partial U}{\partial z} \end{pmatrix}$$

$$U_{\text{gravity}} = -G \frac{m_1 m_2}{r}$$

$$U_{\text{gravity}} = mgh$$

$$U_{\text{electric}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

Heat capacity:

$$\Delta E_{\text{thermal}} = mC\Delta T$$

Friction forces:

$$f_s \leq \mu_s N$$

$$f_k = \mu_k N$$

Drag force (air):

$$\vec{F}_{\text{air}} = -\frac{1}{2} C \rho A v^2 \hat{v}$$

Energy levels for the Hydrogen atom:

$$E_N = \frac{-13.6}{N^2} \text{ eV}, N = 1, 2, 3, \dots$$

Vibrational energy levels:

$$E_N = E_0 + N\hbar\omega_0 = E_0 + N\hbar\sqrt{\frac{k_s}{m}}, N = 0, 1, 2, \dots$$

Energy and wavelength of light:

$$E_{\text{photon}} = \frac{hc}{\lambda_{\text{light}}}$$

Center of mass:

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\vec{r}_{\text{cm}} = \frac{\int \vec{r} dm}{\int dm} = \frac{1}{M} \int \vec{r} dm$$

Motion of the center of mass:

$$M\vec{a}_{\text{cm}} = \vec{F}_{\text{net,ext}}$$

Gravitational potential energy of a multi-particle system:

$$U = Mg y_{\text{cm}}$$

Kinetic energy of a multi-particle system:

$$K = K_{trans} + K_{rel} = \frac{1}{2} M v_{cm}^2 + K_{rel}$$

Impulse of a force:

$$\vec{J} = \int \vec{F} dt$$

Momentum and impulse:

$$\vec{J} = \vec{p}_f - \vec{p}_i$$

Conservation of linear momentum:

$$\Delta \vec{P}_{system} + \Delta \vec{P}_{surroundings} = 0$$

Elastic collision in one dimension (mass 2 at rest before the collision):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Completely inelastic collision in one dimension (mass 2 at rest before the collision):

$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

Moment of inertia:

$$I = \sum_i m_i r_i^2 \quad \text{Discrete mass distribution}$$

$$I = \int_{Volume} r^2 dm \quad \text{Continuous mass distribution}$$

Kinetic energy of a rotating rigid object:

$$K = \frac{1}{2} I \omega^2$$

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Newton's "second" law for rotational motion:

$$\vec{\tau} = I \vec{\alpha}$$

Angular momentum of a single particle:

$$\vec{L} = \vec{r} \times \vec{p}$$

Angular momentum of a rotating rigid object:

$$\vec{L} = I \vec{\omega}$$

The angular momentum principle:

$$\frac{d\vec{L}}{dt} = (\vec{r} \times \vec{F})_{net, ext} = \vec{\tau}_{net, ext}$$

Number of micro states:

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$$

Definition of entropy S :

$$S = k \ln \Omega$$

Definition of temperature T :

$$\frac{1}{T} = \frac{dS}{dE_{int}}$$

The Boltzmann distribution:

$$P(\Delta E) \propto e^{-\Delta E/kT}$$

The Maxwell-Boltzmann velocity distribution:

$$P(v) = 4\pi \left(\frac{M}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2} M v^2 / (kT)}$$

Root-mean-square speed:

$$v_{rms} = \sqrt{v^2} = \sqrt{\frac{3kT}{M}}$$

Average translational kinetic energy of an ideal gas:

$$\bar{K}_{trans} = \frac{3}{2} kT$$

Rate of thermal energy transfer:

Mean-free path d :

$$\frac{N}{V} \left[\pi (R+r)^2 d \right] \approx 1$$

Number of gas molecules hitting an area A per second:

$$\text{molecules/s} = \frac{1}{4} \frac{N}{V} A \bar{v}$$

Efficiency of a heat engine:

$$\text{efficiency} = \frac{|W|}{Q_H} = 1 - \frac{T_L}{T_H}$$

Quality factor of a heat pump:

Ideal gas law:

$$pV = NkT$$

$$K = \frac{|Q_C|}{|W|}$$

Work done by a gas:

$$W = \int_{V_1}^{V_2} p dV$$

First law of thermodynamics:

$$\Delta E_{\text{system}} = W + Q$$

Isothermal compression:

$$W = NkT \ln \left(\frac{V_1}{V_2} \right)$$

Adiabatic compressions:

$$pV^\gamma = pV^{C_p/C_v} = \text{constant}$$

Heat capacity C defined:

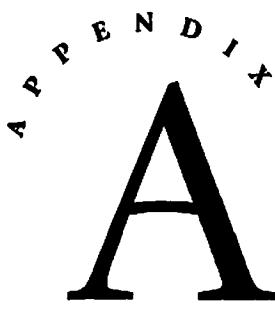
$$\Delta Q = C \Delta T$$

Heat capacities per molecule for ideal gasses:

$$C_V = \frac{3}{2}k \quad \text{monatomic gas}$$

$$C_V \geq \frac{3}{2}k \quad \text{other gases}$$

$$C_p = C_V + k$$



Mathematical Formulas

A-1 Quadratic Formula

If $ax^2 + bx + c = 0$
then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

A-2 Binomial Expansion

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

$$(x+y)^n = x^n \left(1 + \frac{y}{x}\right)^n = x^n \left(1 + n\frac{y}{x} + \frac{n(n-1)}{2!} \frac{y^2}{x^2} + \dots\right)$$

A-3 Other Expansions

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

$$\tan \theta = \theta + \frac{\theta^3}{3} + \frac{2}{15}\theta^5 + \dots \quad |\theta| < \frac{\pi}{2}$$

In general: $f(x) = f(0) + \left(\frac{df}{dx}\right)_0 x + \left(\frac{d^2f}{dx^2}\right)_0 \frac{x^2}{2!} + \dots$

A-4 Exponents

$$(a^n)(a^m) = a^{n+m} \qquad \frac{1}{a^n} = a^{-n}$$

$$(a^n)(b^n) = (ab)^n \qquad a^n a^{-n} = a^0 = 1$$

$$(a^n)^m = a^{nm} \qquad a^{\frac{1}{2}} = \sqrt{a}$$

A-5 Areas and Volumes

Object	Surface area	Volume
Circle, radius r	πr^2	—
Sphere, radius r	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Right circular cylinder, radius r , height h	$2\pi r^2 + 2\pi r h$	$\pi r^2 h$
Right circular cone, radius r , height h	$\pi r^2 + \pi r \sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$

A-8 Vectors

Vector addition is covered in Sections 3-2 to 3-5.

Vector multiplication is covered in Sections 3-3, 7-2, and 11-2.

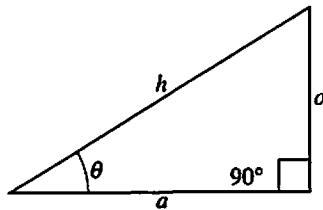


FIGURE A-5

FIGURE A-6

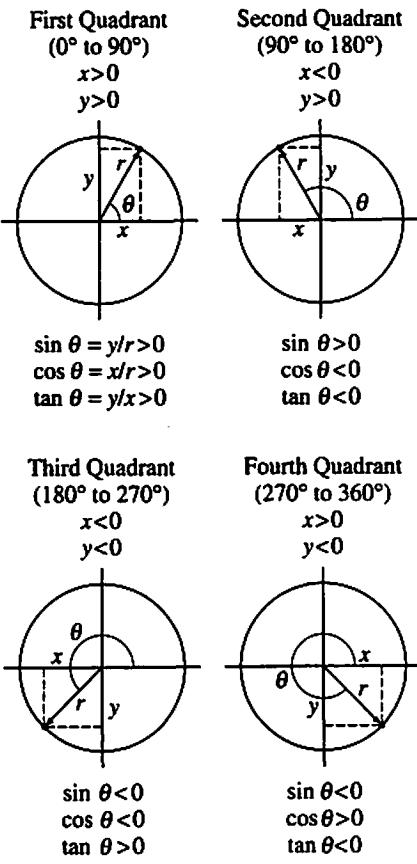


FIGURE A-7

A-9 Trigonometric Functions and Identities

The trigonometric functions are defined as follows (see Fig. A-5, o = side opposite, a = side adjacent, h = hypotenuse. Values are given in Table A-2):

$$\begin{aligned}\sin \theta &= \frac{o}{h} & \csc \theta &= \frac{1}{\sin \theta} = \frac{h}{o} \\ \cos \theta &= \frac{a}{h} & \sec \theta &= \frac{1}{\cos \theta} = \frac{h}{a} \\ \tan \theta &= \frac{o}{a} = \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} = \frac{a}{o}\end{aligned}$$

and recall that

$$a^2 + o^2 = h^2 \quad [\text{Pythagorean theorem}].$$

Figure A-6 shows the signs (+ or -) that cosine, sine, and tangent take on for angles θ in the four quadrants (0° to 360°). Note that angles are measured counterclockwise from the x axis as shown; negative angles are measured from below the x axis, clockwise: for example, $-30^\circ = +330^\circ$, and so on.

The following are some useful identities among the trigonometric functions:

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \sec^2 \theta - \tan^2 \theta &= 1, \quad \csc^2 \theta - \cot^2 \theta = 1 \\ \sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\ \sin(180^\circ - \theta) &= \sin \theta \\ \cos(180^\circ - \theta) &= -\cos \theta \\ \sin(90^\circ - \theta) &= \cos \theta \\ \cos(90^\circ - \theta) &= \sin \theta \\ \sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta \\ \sin \frac{1}{2}\theta &= \sqrt{\frac{1 - \cos \theta}{2}}, \quad \cos \frac{1}{2}\theta = \sqrt{\frac{1 + \cos \theta}{2}}, \quad \tan \frac{1}{2}\theta = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \\ \sin A \pm \sin B &= 2 \sin\left(\frac{A \pm B}{2}\right) \cos\left(\frac{A \mp B}{2}\right).\end{aligned}$$

For any triangle (see Fig. A-7):

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \quad [\text{Law of sines}]$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma. \quad [\text{Law of cosines}]$$

Values of sine, cosine, tangent are given in Table A-2.

B

Derivatives and Integrals

Derivatives: General Rules

(See also Section 2-3.)

$$\begin{aligned}\frac{dx}{dx} &= 1 \\ \frac{d}{dx}[af(x)] &= a \frac{df}{dx} \quad [a = \text{constant}] \\ \frac{d}{dx}[f(x) + g(x)] &= \frac{df}{dx} + \frac{dg}{dx} \\ \frac{d}{dx}[f(x)g(x)] &= \frac{df}{dx}g + f \frac{dg}{dx} \\ \frac{d}{dx}[f(y)] &= \frac{df}{dy} \frac{dy}{dx} \quad [\text{chain rule}] \\ \frac{dx}{dy} &= \frac{1}{\left(\frac{dy}{dx}\right)} \quad \text{if } \frac{dy}{dx} \neq 0.\end{aligned}$$

Derivatives: Particular Functions

$$\begin{aligned}\frac{da}{dx} &= 0 \quad [a = \text{constant}] \\ \frac{d}{dx}x^n &= nx^{n-1} \\ \frac{d}{dx}\sin ax &= a\cos ax \\ \frac{d}{dx}\cos ax &= -a\sin ax \\ \frac{d}{dx}\tan ax &= a\sec^2 ax \\ \frac{d}{dx}\ln ax &= \frac{1}{x} \\ \frac{d}{dx}e^{ax} &= ae^{ax}\end{aligned}$$

Indefinite Integrals: General Rules

(See also Section 7-3.)

$$\begin{aligned}\int dx &= x \\ \int af(x) dx &= a \int f(x) dx \quad [a = \text{constant}] \\ \int [f(x) + g(x)] dx &= \int f(x) dx + \int g(x) dx \\ \int u dv &= uv - \int v du \quad [\text{integration by parts: see also B}]\end{aligned}$$

Indefinite Integrals: Particular Functions

(An arbitrary constant can be added to the right side of each equation.)

$$\int a \, dx = ax \quad [a = \text{constant}]$$

$$\int \frac{dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}}$$

$$\int x^m \, dx = \frac{1}{m+1} x^{m+1} \quad [m \neq -1]$$

$$\int \frac{x \, dx}{(x^2 \pm a^2)^{\frac{3}{2}}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$\int x e^{-ax} \, dx = -\frac{e^{-ax}}{a^2} (ax + 1)$$

$$\int \tan ax \, dx = \frac{1}{a} \ln|\sec ax|$$

$$\int x^2 e^{-ax} \, dx = -\frac{e^{-ax}}{a^3} (a^2 x^2 + 2ax + 2)$$

$$\int \frac{1}{x} \, dx = \ln x$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left(\frac{x-a}{x+a} \right) \quad [x^2 > a^2]$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$$

$$= -\frac{1}{2a} \ln \left(\frac{a+x}{a-x} \right) \quad [x^2 < a^2]$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) = -\cos^{-1} \left(\frac{x}{a} \right) \quad [\text{if } x^2 \leq a^2]$$

A Few Definite Integrals

$$\int_0^\infty x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^2 e^{-ax^2} \, dx = \sqrt{\frac{\pi}{16a^3}}$$

$$\int_0^\infty e^{-ax^2} \, dx = \sqrt{\frac{\pi}{4a}}$$

$$\int_0^\infty x^3 e^{-ax^2} \, dx = \frac{1}{2a^2}$$

$$\int_0^\infty x e^{-ax^2} \, dx = \frac{1}{2a}$$

$$\int_0^\infty x^{2n} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

Integration by Parts

Sometimes a difficult integral can be simplified by carefully choosing the functions u and v in the identity:

$$\int u \, dv = uv - \int v \, du. \quad [\text{Integration by parts}]$$

This identity follows from the property of derivatives

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

or as differentials: $d(uv) = u \, dv + v \, du$.

For example $\int x e^{-x} \, dx$ can be integrated by choosing $u = x$ and $dv = e^{-x} \, dx$ in the "integration by parts" equation above:

$$\begin{aligned} \int x e^{-x} \, dx &= (x)(-e^{-x}) + \int e^{-x} \, dx \\ &= -xe^{-x} - e^{-x} = -(x+1)e^{-x}. \end{aligned}$$