Quantum Mechanics I - Module 5

1. There are two theorems concerning commutators that can come in handy:

   - If A, B, and C are operators, then: $[AB, C] = A[B, C] + [A, C]B$.
   - For any operators A and B: $[A, B] = -[B, A]$.

   (a) Work out the commutator $[x, p_x^2]$ by the brute force method, constructing the square of the momentum operator and writing it all out.

   (b) Work out the same commutator by using the above theorems and the fact that $[x, p_x] = i\hbar$.

2. Consider the infinite square well extending from $x = 0$ to $x = L$.

   (a) Write down a normalized expression for the fifth state $\psi_5(x)$ that is a solution of the time independent Schrödinger equation.

   (b) Write down an expression for the full solution of Schrödinger's equation for the same state $\Psi_5(x, t)$.

   (c) Calculate the expectation value of the kinetic energy for this state.

3. A particle of mass m and energy $E = \hbar^2 k^2 / 2m$ is incident from the left on a potential

$$V(x) = \begin{cases} \dfrac{\hbar^2 k^2}{2m}, & 0 \le x \le L \\ 0, & \text{otherwise} \end{cases} .$$

   In other words, the height of the potential barrier happens to coincide exactly with the energy of the particle.

   (a) Show that for $0 \le x \le L$ the wavefunction is of the form $a+bx$ where a and b are constants.

   (b) Calculate the reflection and transmission coefficients and show that their sum is one.

   (c) Show that the transmission coefficient goes to one for $k \to 0$ and for $L \to 0$.